Synchronized Oscillators With the Phase-Negative Feedback

Vladimir Rapin

***Abstract*—A novel idea, a phase-negative feedback for fundamentally injected *LC* oscillators, is discussed, and a new method of linear approximation of their shortened equations has been presented.**

***Index Terms*—Approximation, feedback, LC oscillator, shortened equations.**

I. Introduction

The injection-locked *LC* oscillators can perform a wide variety of functions [1]–[4]; however, their potentialities were not realized because of the oscillator analysis difficulties and for lack of technical innovations. To study the oscillators, several methods have been developed, for example, see [4]–[14], but in most cases the investigation results in solving very complex sets of nonlinear shortened differential equations. They are studied by numerical methods, which take a great deal of time and effort, as a topological approach, even if simple systems of oscillators are taken into consideration [14]. The phase feedback is known and is useful in various fields, but it is not used in synchronized oscillators and their systems. Thus, the purpose of the paper is to show the advantages of the oscillators with the phase-negative feedback and a new analytical method for their investigation.

II. Phase-Negative Feedback

The phase-negative feedback is formed by introducing feedback signal phase into a synchronizing signal of oscillator. The main procedure is as follows. Let $u_e = U_e \cos(\omega_c t + \varphi_0)$ be the original synchronizing signal, and the feedback signal is given by $u_f = U_f \cos(\omega_c t + \varphi_f)$. Then the synchronizing signal is squared, and we obtain $u_1 = U_1 \cos(2\omega_c t + 2\varphi_0)$ after the elimination of the constant term. The direct synchronizing signal u_c is the first harmonic of the product $u_1 \times u_f$, $u_c = U_c \cos(\omega_c t + \psi)$, $\psi = 2\varphi_0 - \varphi_f$, and the other harmonic is filtered out. In the general case, for each of N oscillators of the system with the phase feedback, its signal and the signals of other oscillators, except one, are the feedback signals. This signal is the only original synchronizing signal. For one of the oscillators, the original synchronizing signal is the external signal. Repeating the above process for each feedback signal at first with the original synchronizing signal and then with the synchronizing signal found on the previous step, the phase of the direct synchronizing signal of the jth-system oscillator can be obtained as

$$\psi_j = \underset{r \neq j}{m_{jr}}\, \varphi_r - \sum_{\substack{n=1 \\ n \neq r}}^{N} m_{jn}^0 \varphi_n$$

where $0 \le r \le N, 1 \le j \le N, \varphi_r$, and φ_n are the phases of the original synchronizing and feedback signals and m_{jr} and m_{jn}^0 are positive integers. For the external synchronizing signal phase, $r = 0$. This is the system of general structure, as each oscillator is coupled with all others. The systems of other structures are obtained from this one.

Manuscript received May 2, 2000; revised February 10, 2001. This paper was recommended by Associate Editor P. V. A. Mohan.

The author is with AV Geroev Stalingrada, Kharkov 61162, Ukraine (e-mail: vrapin@ukr.net).

Publisher Item Identifier 10.1109/TCSI.2002.800615.

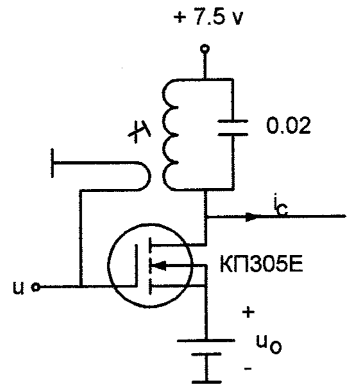

Fig. 1. Circuit diagram of the synchronized oscillator.

III. The Model of the Synchronized Oscillator System With the Phase-Negative Feedback

To be specific, we shall consider a nonautonomous system of N fundamentally injected oscillators. The circuit diagram of one of the oscillators is shown in Fig. 1. The obtained results of course can also be used for other types of oscillators. We shall consider that the transistors are nonlinear and inertialess, and other elements are linear. $i_{jc} = I_{jc}\cos(\omega_c t + \psi_j)$ is a direct synchronizing signal where $I_{jc} = const$. The resonant frequencies of the oscillator tanks ω_0 are identical. A nonlinear characteristic of the oscillator nonlinear device is approximated by the expression $i = a_{j0} + a_{j1}u_{jd} + a_{j2}u_{jd}^2 + a_{j3}u_{jd}^3 + a_{j4}u_{jd}^4$ where $u_{jd} = u_j + u_{j0}$, u_j is the voltage at the gate of the transistor, and u_{j0} is constant and is a dc bias point. Then the set of Van der Pol's type fundamental differential equations of the nonautonomous system is as follows:

$$\frac{d^2u_j}{d\tau^2} - \varepsilon_j \frac{\omega_0}{\omega_c}\left(1 - 2\beta_j u_j - 3\gamma_j u_j^2 - 4\delta_j' u_j^3\right)\frac{du_j}{d\tau} + \frac{\omega_0^2}{\omega_c^2}u_j = k_j R_j \delta_j \frac{\omega_0}{\omega_c}\frac{di_{jc}}{d\tau}, \qquad 1 \le j \le N$$

where $\tau = \omega_c t$, $\varepsilon_j = \delta_j \alpha_j$ is small parameter, $\alpha_j = (k_j R_j \alpha_{j0} - 1)$, $\beta_j = \beta_{j0}/\alpha_{j0}'$, $\gamma_j = \gamma_{j0}/\alpha_{j0}'$, $\alpha_{j0} = a_{j1} + 2a_{j2}u_{j0} + 3a_{j3}u_{j0}^2 + 4a_{j4}u_{j0}^3$, $\delta_{j0} = a_{j4}$, $\beta_{j0} = a_{j2} + 3a_{j3}u_{j0} + 6a_{j4}u_{j0}^2$, $\gamma_{j0} = a_{j3} + 4a_{j4}u_{j0}$, $\alpha_{j0}' = 1/(k_j R_j) - \alpha_{j0}$, $\delta_j' = \delta_{j0}/\alpha_{j0}'$, $\delta_j = 1/Q_j$, R_j, Q_j are the resonant resistance and Q -factor of the tank, $k_j = M_j/L_j$ is the feedback factor of the oscillator, and L_j, M_j are the tank and mutual inductances.

We shall seek the periodic solutions of these equations in the form: $u_j = A_j \cos(\omega_c t + \varphi_j)$, where A_j and φ_j are slowly varying functions of time. In this case, the system model is a set of the shortened equations

$$\begin{aligned} \frac{dy_j}{d\tau} - \frac{\varepsilon_j}{2}\left(1 - y_j^2\right)y_j &= \frac{\varepsilon_j B_j}{2\alpha_1}\cos\theta_j \\ \frac{d\theta_j}{d\tau} + \frac{\varepsilon_j B_j}{2\alpha_j y_j}\sin\theta_j &= -\frac{\Delta\omega}{\omega_0} - \frac{d\psi_j}{d\tau}, \qquad 1 \le j \le N \end{aligned} \tag{1}$$

obtained by means of the averaging method [11]–[13] where $\theta_j = \varphi_j - \psi_j$, $y_j = A_j/A_{j0} > 1$ is a dimensionless amplitude, A_j and $A_{j0} = \sqrt{4\alpha_{j0}'/(3\gamma_{j0})}$ are signal amplitudes of the synchronized and free running oscillator, respectively, $B_j = I_{jc}/I_{j0}$, $I_{j0} = A_{j0}/(R_j k_j)$, $\Delta\omega/\omega_0 = (\omega_c - \omega_0)/\omega_0$, and ω_0 is a resonant frequency of the tank.

Assume that the oscillators are synchronized already. For their investigation, a method of linear approximation of the shortened equations is proposed. Let $\theta_j = F((\Delta\omega/\omega_0)_{jn})$ be a phase characteristic of the synchronized oscillator, which is computed from the set (1), recast as follows:

$$y_j^3 - y_j = \frac{B_j}{\alpha_j}\cos\theta_j - \frac{2}{\varepsilon_j}\frac{dy_j}{d\tau}$$
$$\theta_j = \arcsin\left(-\left(\frac{\Delta\omega}{\omega_0}\right)_{jn} y_j\right) \tag{2}$$

where $(\Delta\omega/\omega_0)_{jn} = (\Delta\omega/\omega_0 + d\psi_j/d\tau + d\theta_j/d\tau)2\alpha_j/(\varepsilon_j\beta_j)$ is a normalized detuning. $-1 \leq (\Delta\omega/\omega_0)_{jn} \leq 1$. $B_j \ll 1$. In the stationary state, $d\theta_j/d\tau = dy_j/d\tau = d\psi_j/d\tau = 0$ and the phase characteristic can be easily computed. The first equation in (2) has only one root for $y_j > 1$. For dynamic consideration, the first equation of (1) should be rewritten as

$$\frac{dy_j}{d\tau} = \frac{\varepsilon_j B_j}{2\alpha_j}\cos\theta_j - \frac{\varepsilon_j}{2}\left(y_j^3 - y_j\right). \tag{3}$$

The phase characteristic in this case can be also computed, since the range of values of normalized detuning is known, the influence of the transient process of change of y_j is small, especially for small phase shifts (if $\cos\theta_j \approx 1$), and we can consider that the process of change of y_j is transient-free. The confirmation is in Section V. The reason lies in slow change θ_j, $\cos\theta_j$ and faster change y_j^3, ($y_j > 1$). This means that the term $\varepsilon_j(y_j^3 - y_j)/2$ of (3) follows fast enough the term $\varepsilon_j\beta_j/(2\alpha_j)\cos\theta_j$ and the value of derivative $dy_j/d\tau$ is small. y_j can be represented by its stationary value for a given value of θ_j, i.e., y_j is calculated, assuming that $dy_j/d\tau = 0$ So, if we deal with the slowly varying functions, we shall consider that the phase characteristics for the stationary state and dynamics are identical. They are odd functions and only a range of negative or positive detunings can be considered. The phase characteristic evolve from the **arcsin** type for small levels of the synchronizing signals, when the greatest value of the dimensionless amplitude $y_{j\max} = y_{j0} \approx 1$ toward a linear function if $y_{j\max} = y_{j0} \approx \pi/2$ because of the changes of amplitude of oscillations with the detuning. The phase characteristic can be approximated by the linear function

$$\theta_{ja} = \theta_j = \frac{-\left(\frac{\Delta\omega}{\omega_0}\right)_{jn} y_{j0}}{(1-\Delta_j)}. \tag{4}$$

Δ_j is determined so as to minimize the errors of the approximation. This equation takes into account the variation of the amplitude of oscillations as a first approximation. In our case, $y_{j0} < \pi/2$. y_{j0} is computed from the first equation in (2) if $\theta_j = 0$ and $dy_j/d\tau = 0$. From (4), we get the phase shift θ_{ja}. The reference phase shift θ_j is obtained from the phase characteristic. The even function of the detuning $\delta\theta_{ja} = (\theta_{ja} - \theta_j)/\theta_j$ is the error of approximation and to get Δ_j the largest range of detunings of the same sine is used. In practice, $(\Delta\omega/\omega_0)_{jn}^{(1)} = 0$, and $\left|(\Delta\omega/\omega_0)_{jn}^{(2)}\right| < 1$ are the endpoints of this range. Errors at these points are as follows:

$$\delta\theta_{ja}^{(2)} = \frac{\left(\theta_{ja}^{(2)} - \theta_j^{(2)}\right)}{\theta_j^{(2)}} = \frac{-\left(\frac{\Delta\omega}{\omega_0}\right)_{jn}^{(2)} y_{jo}}{\left[(1-\Delta_j)\theta_j^{(2)}\right]} - 1 < 0$$
$$\delta\theta_{ja}^{(1)} = \frac{\left(\theta_{ja}^{(1)} - \theta_j^{(1)}\right)}{\theta_j^{(1)}}$$
$$= \lim\left\{\frac{-(\frac{\Delta\omega}{\omega_0})_{jn} y_{j0}}{[(1-\Delta_j)(\arcsin(-(\frac{\Delta\omega}{\omega_0})_{jn} y_j))]} - 1\right\}$$
$$= \frac{1}{(1-\Delta_j)} - 1 > 0$$

if $(\Delta\omega/\omega_0)_{jn} \to 0$. Let the approximation be optimum if $\delta\theta_{ja}^{(1)} = -\delta\theta_{ja}^{(2)}$, i.e., the absolute values of errors are identical. Then, we obtain

$$\Delta_j = 0.5 + \frac{\frac{y_{j0}}{2}\left(\frac{\Delta\omega}{\omega_0}\right)_{jn}^{(2)}}{\theta_j^{(2)}}. \tag{5}$$

$0 < \Delta_j < 0.181$ if $1 < y_{j0} < \pi/2$. In this case, the whole range of the detunings we deal with is symmetrical with the endpoints $\pm(\Delta\omega/\omega_0)_{jn}^{(2)}$. So, the shortened equations of the synchronized oscillator is approximated by (4), therefore the model of the synchronized oscillator system (1) can be approximated by (4). Using the expression for a normalized detuning, this set can be rewritten as

$$\frac{d\theta_j}{d\tau} + \xi_j(1-\Delta_j)\theta_j = -\frac{\Delta\omega}{\omega_0} - \frac{d\psi_j}{d\tau}, \qquad 1 \leq j \leq N \tag{6}$$

where $\xi_j = \varepsilon_j B_j/(2\alpha_j y_{j0})$. The coefficients $\xi_j(1-\Delta_j)$ should be identical for identical system oscillators with the identical amplitudes of the direct synchronizing signals, as well as the terms Δ_j. Now, to analyze the oscillators with the phase negative feedback, a new variable $\theta_j^0 = \varphi_j - \varphi_0$ will be used.

IV. Examples

Let us consider an oscillator amplifier of PM and FM signals shown in Fig. 1. Let $u_e = U_e\cos(\omega_c t + \varphi_0)$ be a small external synchronizing signal, and let $u_1 = U_1\cos(\omega_c t + \varphi_1)$ be a considerable signal of the oscillator. The synchronized oscillator will be an amplifier if $\varphi_1 \approx \varphi_0$. If the oscillator signal is the feedback signal then $i_c = I_c\cos(\omega_c t + 2^n\varphi_0 - (2^n - 1)\varphi_1)$ is the direct synchronizing signal obtained by means of the procedure of Section II where $I_c = const$. The model of this oscillator is the set (1) where $N = 1$, $\theta_1 = 2^n\theta_1^0$, $\theta_1^0 = \varphi_1 - \varphi_0$, i.e.,

$$\frac{dy_1}{d\tau} - \frac{\varepsilon_1}{2}\left(1 - y_1^2\right)y_1 = \frac{\varepsilon_1 B_1}{2\alpha_1}\cos(2^n\theta_1^0)$$
$$\frac{d\theta_1^0}{d\tau} + \frac{\varepsilon_1 B_1}{2\alpha_1 y_1}\sin(2^n\theta_1^0) = -\frac{\Delta\omega}{\omega_0} - \frac{d\varphi_0}{d\tau}. \tag{7}$$

For stability investigation equations for small perturbations δy_1 and $\delta\varphi_1$ are obtained from (7) as

$$\frac{d(\delta y_1)}{d\tau} = a\delta y_1 + b\delta\varphi_1$$
$$\frac{d(\delta\varphi_1)}{d\tau} = d\delta y_1 + c\delta\varphi_1$$

where

$$a = -\frac{\varepsilon_1}{2}\left(3y_1^2 - 1\right), b = -\frac{\varepsilon_1 B_1}{2\alpha_1}2^n\sin\theta_1$$
$$c = -\frac{\varepsilon_1 B_1}{2y_1\alpha_1}2^n\cos\theta_1, d = \frac{\varepsilon_1 B_1}{2y_1^2\alpha_1}\sin\theta_1.$$

$\lambda^2 - (a+c)\lambda + ac - bd = 0$ is the characteristic equation. $-(a+c) > 0$ and $(ac - bd) > 0$ if $-\pi/2 < \theta_1 < \pi/2$ or $-\pi/2 \times 2^{-n} < (\varphi_1 - \varphi_0) < \pi/2 \times 2^{-n}$ Then, the oscillations are stable in this range, which

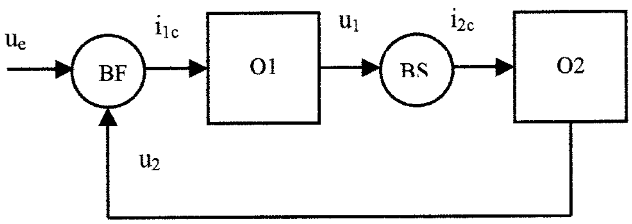

Fig. 2. Block diagram of the synchronized oscillator system. BF $=$ block of feedback, O$\mathbf{1}$ $=$ first oscillator, BS $=$ block of synchronization, O$\mathbf{2}$ $=$ second oscillator.

is consistent with the detuning range $-\varepsilon_1 B_1/(2\alpha_1) < \Delta\omega/\omega_0 < \varepsilon_1 B_1/(2\alpha_1)$. It follows from (6) that

$$d\theta_1^0/d\tau + 2^n\xi_1(1-\Delta_1)\theta_1^0 = -\Delta\omega/\omega_0 - d\varphi_0/d\tau.$$

Assume that $\theta_{1(0)}^0 = \varphi_m$ is the initial condition, $\Delta\omega/\omega_0 = 0$ and $\varphi_0 = \varphi_m\cos(\Omega\tau) = \varphi_m\cos(M_0\tau)$. Then

$$\theta_{1(\tau)}^0 = \frac{\varphi_m M_0}{2^n\sigma\left(1+\left[\frac{(\frac{M_0}{\sigma})}{2^n}\right]^2\right)} \left\{\sin(M_0\tau) - \frac{M_0}{2^n\sigma}\left[\cos(M_0\tau) - \exp(-2^n\sigma\tau\right]\right\} + \varphi_m\exp(-2^n\sigma\tau) \quad (8)$$

where $M_0 = \Omega/\omega_c, \sigma = \xi_1(1-\Delta_1)$. For the oscillator without phase feedback, $n = 0$ i.e.,

$$\theta_{1(\tau)}^0 = \frac{\varphi_m M_0}{\sigma(1+(M_0/\sigma)^2)} \left\{\sin(M_0\tau) - \frac{M_0}{\sigma}\left[\cos(M_0\tau) - \exp(-\sigma\tau\right]\right\} + \varphi_m\exp(-\sigma\tau). \quad (9)$$

The signal phase of the oscillator is described by the expression $\varphi_1 = \theta_{1(\tau)}^0 + \varphi_0$. The phase shift $\theta_{1(\tau)}^0$ is an error, introduced by the oscillator, and the phase feedback reduces it as well as a time of transient process. However, for a considerable decrease in phase shift (large n), the direct synchronizing signal forming device is comlex enough.

Let us consider now an oscillator amplifier shown in Fig. 2. This is a system of two identical synchronized oscillators. $u_e = U_e\cos(\omega_c t + \varphi_0)$ is a small external synchronizing signal, and $u_1 = U_1\cos(\omega_c t + \varphi_1)$ is a signal of the first oscillator O1. $u_2 = U_2\cos(\omega_c t + \varphi_2)$ is a signal of the second oscillator O2 and is a feedback signal. The direct synchronizing signals of the oscillators O1 and O2 are formed in the blocks BF and BS $i_{1c} = I_1\cos(\omega_c t + 2\varphi_0) - \varphi_2, i_{2c} = I_1\cos(\omega_c t + \varphi_1), I_1 = const$. The model of the system is a set of (1) where $N = 2$, $\theta_1 = \theta_1^0 + \theta_2^0, \theta_2 = \theta_2^0 - \theta_1^0, \theta_1^0 = \varphi_1 - \varphi_0, \theta_2^0 = \varphi_2 - \varphi_0$, i.e.,

$$\begin{aligned} \frac{dy_1}{d\tau} - \frac{\varepsilon}{2}\left(1-y_1^2\right)y_1 &= \frac{\varepsilon B}{2\alpha}\cos(\theta_1^0+\theta_2^0) \\ \frac{d\theta_1^0}{d\tau} + \frac{\varepsilon B}{2\alpha y_1}\sin(\theta_1^0+\theta_2^0) &= -\frac{\Delta\omega}{\omega_0} - \frac{d\varphi_0}{d\tau} \\ \frac{dy_2}{d\tau} - \frac{\varepsilon}{2}\left(1-y_2^2\right)y_2 &= \frac{\varepsilon B}{2\alpha}\cos(\theta_2^0-\theta_1^0) \\ \frac{d\theta_2^0}{d\tau} + \frac{\varepsilon B}{2\alpha y_2}\sin(\theta_2^0-\theta_1^0) &= -\frac{\Delta\omega}{\omega_0} - \frac{d\varphi_0}{d\tau}. \end{aligned} \quad (10)$$

For a stationary-state stability investigation, the equations for small perturbations $\delta y, \delta\varphi$ are found from (10) as

$$\begin{aligned} \frac{d(\delta y_1)}{d\tau} &= a\delta y_1 + b(\delta\varphi_1 + \delta\varphi_2) \\ \frac{d(\delta\varphi_1)}{d\tau} &= c(\delta\varphi_1 + \delta\varphi_2) + d\delta y_1 \\ \frac{d(\delta y_2)}{d\tau} &= a\delta y_2 + b(\delta\varphi_2 - \delta\varphi_1) \\ \frac{d(\delta\varphi_2)}{d\tau} &= c(\delta\varphi_2 - \delta\varphi_1) + d\delta y_2 \end{aligned}$$

where

$$a = -\frac{\varepsilon}{2}\left(3y_1^2 - 1\right), b = -\frac{\varepsilon B}{2\alpha}\sin\theta_1$$
$$c = -\frac{\varepsilon B}{2y_1\alpha}\cos\theta_1, d = \frac{\varepsilon B}{2y_1^2\alpha}\sin\theta_1.$$

In the stationary state, the phase shift is defined by the frequency detuning which is identical for the oscillators. Then $\theta_1 = \theta_2$ and $y_1 = y_2$.The characteristic equation is the expression $\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$ where $a_1 = -2(c+a)$, $a_2 = 4ac + a^2 + 2c^2 - 2bd$, $a_3 = 2abd + 4bcd - 4c^2a - 2a^2c$, $a_4 = 2c^2a^2 - 4abcd + 2b^2d^2$. The investigation shows that the range of stable oscillations is $-\pi/2 < \theta_1 < \pi/2$ or $-\varepsilon B/(2\alpha) < \Delta\omega/\omega_0 < \varepsilon B/(2\alpha)$.

Let $\varphi_0 = \varphi_m\cos(\Omega\tau) = \varphi_m\cos(M_0\tau)$. Using the linear approximation of the shortened equations (10), we have

$$\begin{aligned} \frac{d\theta_1^0}{d\tau} + \sigma(\theta_1^0 + \theta_2^0) &= -\frac{\Delta\omega}{\omega_0} + \varphi_m M_0\sin(M_0\tau) \\ \frac{d\theta_2^0}{d\tau} + \sigma(\theta_2^0 - \theta_1^0) &= -\frac{\Delta\omega}{\omega_0} + \varphi_m M_0\sin(M_0\tau) \end{aligned}$$

where $M_0 = \Omega/\omega_c$ and $\sigma = \xi(1-\Delta)$. Let $\theta_{1(0)}^0 = \theta_{2(0)}^0 = \varphi_m$ be the initial conditions. Then

$$\begin{aligned} \theta_{1(\tau)}^0 = &\varphi_m\left[1 + \left(\frac{M_0}{\sigma}\right)^2\frac{\rho_1}{\rho_2}\right]\exp(-\sigma\tau) \\ &\times[\cos(\sigma\tau) - \sin(\sigma\tau)] - \left[\frac{1}{\sigma}\left(\frac{\Delta\omega}{\omega_0}\right)\right. \\ &\left.+\varphi_m\left(\frac{M_0}{\sigma}\right)^2\frac{4}{\rho_2}\right] \\ &\times\exp(-\sigma\tau)\sin(\sigma\tau) + \varphi_m\left(\frac{M_0}{\sigma}\right)^2 \\ &\cdot\left[\frac{-\rho_1}{\rho_2}\cos(M_0\tau) + \frac{M_0}{\sigma}\frac{2}{\rho_2}\sin(M_0\tau)\right] \end{aligned} \quad (11)$$

where $\rho_1 = (M_0/\sigma)^2 - 2, \rho 2 = (M_0/\sigma)^4 + 4$.

The phase of oscillations of the first oscillator is $\varphi_1 = \theta_{1(\tau)}^0 + \varphi_0$. Analyzing (8) and (11), it can be seen that the system of two oscillators reduces the error $\theta_{1(\tau)}^0$ far stronger then the single oscillator. $\theta_{1(\tau)}^0$ close to zero if inertia of the synchronized oscillators is small, i.e., $M_0/\sigma << 1$. This is the new property of this system, obtained by means of the phase-negative feedback. It is quite possible that this system can be used also as a phase-locked loop. This system has a simple direct synchronizing signal forming devices.

The investigation of this system by the new method takes far less time and effort than numerical one. This advantage is reflected strongly when large amount of systems with a different phase feedback should be analyzed to find a system with required property.

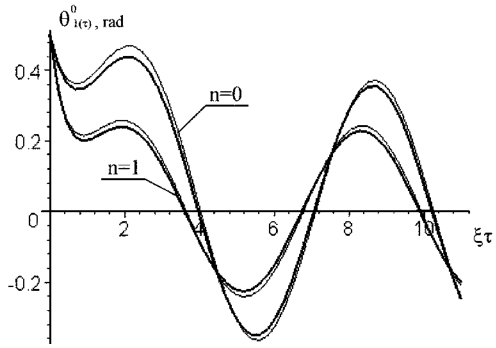

Fig. 3. Phase-shift variations of the synchronized oscillator signal.

V. Experiment

To verify the theoretical results, an experiment in the form of numerical integration of the shortened equations of the oscillator represented in the example, performed with the fourth-order Runger–Kutta algorithm, has been presented. The parameters of the oscillator are as follows. Resonant frequency of the tank $f_0 = 50$ kHz, $\varepsilon = 1.712 \times 10^{-3}$, $B = 0.0494$, $y_0 = 1.2262$, $A_0 = 0.9$ V, $R = 7.5 \times 10^3\ \Omega$, $u_0 = -1$ V, $k = 0.16$ V, $\xi = 4.2704 \times 10^{-4}$, $\alpha = 0.0804$, and $I_c = 37 \times 10^{-6}$ A,. The nonlinear characteristic of the transistor was approximated by the polynomial $i = 1.538 + 1.302u_d - 0.356u_d^2 - 0.502u_d^3 - 0.098u_d^4$ mA. If the working range is symmetrical with the endpoints $(\Delta\omega/\omega_0)_n^{(2)} = \pm 0.95$, then $\theta^{(2)} = \pm 1.445$ and $\Delta_1 = 0.0967$. Variations of the phase shift of the oscillator signal in dynamics in the case of the harmonic phase deviation of the external synchronizing signal for oscillator with feedback for $n = 1$ and without feedback $n = 0$ are shown in Fig. 3. Calculations were made by means of (7)–(9) where $M_0 = \xi$, $\varphi_m = 0.5$, and $\theta_{1(0)}^0 = 0.5$, after the introduction of the new independent variable $\xi\tau$. Bold curves represent the results obtained by solving the shortened equations by means of the numerical method, and other curves were obtained using the proposed method. Analysis shows good agreement between the analytic solution of the shortened equations, obtained by means of the new method, and its numerical solution.

The derivative $dy/d\tau$ is considered to be zero in calculating the phase characteristic by means of (2). Its maximum error $\delta\theta_{\max} = 5\%$ at the end of the locking range as compared with the phase characteristic when $dy/d\tau \neq 0$ for $\Delta\omega = 0$ and $d\psi/d\tau = -0.12\xi\tau$.Thus, the derivative $dy/d\tau$ indeed can be ignored.

VI. Conclusion

The paper describes the fundamentally injected *LC* oscillators with the phase-negative feedback and demonstrates the advantages of the new method in solving the shortened equations. It is shown that the phase-negative feedback reduces the phase shift and the time of transient process. Due to the phase-negative feedback, the system of two oscillators acquires a new property. The phase shift of one of the oscillators is close to zero. This leads to the improvement of the device characteristics, where the oscillators are used. The phase feedback allows us to use many novel properties of oscillators. The new method was developed to obtain a solution of the shortened equations with small expenditure of time and effort and with small errors. It does not permit the evaluation of nonlinear distortions, but it is quite general, is easily applied in practice, and is applicable to many problems in nonlinear mechanics. This method also permitted us to obtain simple and sufficiently accurate analytic expressions. They are necessary to develope simple and sufficiently accurate techniques for the design of oscillator devices.

References

[1] K. Daikoku and Y. Mizushima, "Properties of injection locking in linear oscillator," *Int. J. Electron.*, vol. 31, pp. 279–292, 1971.

[2] T. Isobe and M. Tokida, "Power amplification for FM and PM signals with synchronized IMPATT oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 906–911, Nov. 1970.

[3] N. Fomin, *Theory of Synchronization of Microwave Oscillators with Negative Resistance*. Moscow, Russia: Nayka, 1992.

[4] R. Hunton and A. Weiss, "Synchronization of oscillators," *Proc. IRE*, pp. 1415–1423, Dec. 1947.

[5] M. Jezewski, "An approach to the analysis of injection-locked oscillators," *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 395–401, Mar. 1974.

[6] R. V. Khoklov, "A method of analysis in the theory of sinusoidal self-oscillations," *IRE Trans. Circuit Theory*, vol. CT-7, pp. 400–413, Dec. 1960.

[7] A. Gelb and W. E. Van der Velder, *Multiple-Input Describing Functions and Nonlinear System Design*. New York: McGraw-Hill, 1968.

[8] J. K. Hale, *Oscillations in Nonlinear Systems*. New York: McGraw-Hill, 1963.

[9] L. O. Chua and Y. Tang, "Nonlinear oscillation via Volterra Series," *IEEE Trans. Circuits Syst.*, vol. CAS-29, pp. 150–168, Mar. 1982.

[10] R. Adler, "A study of locking phenomena in oscillators," *Proc. IRE*, vol. 34, pp. 351–357, June 1946.

[11] C. Hayashi, *Nonlinear Oscillations in Physical Systems*. New York: McGraw-Hill, 1964.

[12] N. N. Bogoliubov and Y. A. Mitropolsky, *Asymptotic Methods in the Theory of Nonlinear Oscillations* (in English). Delhi, India: Hindustan Pub. Corp., 1961.

[13] I. M. Kapchinsky, *Teory Kolebany v Radiotekhike* (in English). Tokyo, Japan: Tokyo Tosho, 1959.

[14] P. Arena, S. Baglio, L. Fortuna, and S. Graziani, "Neural networks to generate Poincare' maps," in *Proc. Int. Seminar. Nonlinear Circuits and Systems*, vol. 1, Moscow, Russia, 1992, pp. 171–179.