Parametric Identification of Utility Functions in Multicriterion Task Model Models

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***Abstract*—For models of multicriteria estimation of design and management decisions based on the Kolmogorov-Gabor polynomial, a modification of the function of the utility of variants by partial criteria is proposed. The mathematical model of the problem of parametric synthesis of known utility functions is formulated, the method of its solution is proposed, and estimates of their accuracy are obtained. It is established that the highest accuracy of approximation of DM estimates has been proposed modification of the utility function of partial criteria.**

***Анотація*—Для моделей багатокритеріального оцінювання проектних і управлінських рішень, що побудовані на основі полінома Колмогорова-Габора, запропонована модифікація функції корисності варіантів за частковими критеріями. Сформульована математична модель задачі параметричного синтезу відомих функцій корисності, запропоновано метод її розв'язання та отримано оцінки їх точності. Встановлено, що найвищу точність апроксимації оцінок ОПР має запропонована модифікація функції корисності часткових критеріїв.**

***Keywords*—task of multicriteria assessment, model, utility function, parametric synthesis.**

***Ключові слова*—задача багатокритеріального оцінювання, модель, функція корисності, параметричний синтез.**

I. Introduction

The problem of decision-making is an integral part of all stages and stages of life cycles of anthropogenic objects. Traditionally, decisions are made by person or group of decision-makers (DM). The importance of adopting effective solutions in automated design or management systems leads to the need for formalization of these procedures and the creation of appropriate selection methods based on a plurality of indicators [1]. The methodology for solving such problems is based on the theory of decision-making [2–3]. In this case, the choice of the best solution from the set of effective only in the simplest situations can be DM without the use of formal methods or using the method of analysis of the hierarchy. To automate the procedures for evaluating alternative solutions, it is necessary to involve additional information on the value of individual formalized properties (partial criteria) and their meanings. At the same time, one of the priority tasks is the problem of synthesis of models for scalar multicriteria estimation of variants. This determines the relevance of the task of developing a mathematical model for evaluation and selection of solutions by a set of partial criteria.

II. Analysis of the current state of the problem

The solution of optimization problems by a plurality of partial criteria is based on the theory of utility within the framework of the orthogonal (ordering of alternatives to DM) or cardinal (quantitative generalized efficiency criterion) approaches [4]. In systems of automated design and management there is a need for quantitative estimate $P(q,x)$ for each of the options x, belonging to the set of admissible $x \in X$ (where q – vector of parameters of the general utility function). In this case, for all variants of the set of permissible $x,y \in X$ the conditions are satisfied: strict $x \succ y \leftrightarrow P(q,x) > P(q,y)$, non-strict $x \sim y \leftrightarrow P(q,x) = = P(q,y)$ preferences or their equivalence $x \tilde{\succ} y \leftrightarrow P(q,x) \geq P(q,y)$.

Due to the incomplete certainty of the requirements for the properties of multicriteria solutions as a total utility function (TUF), use the function of belonging to the fuzzy set "Best option" x^o, which can be presented as a set of ordered pairs [5]:

$$x^o = \{ < x, P(q,x) > \}, \quad (1)$$

where $x \in X$ - solutions from the set of permissible ones; $P(q,x)$ - value of the function (degree) of the decision of the $x \in X$ fuzzy set "Best option" (1).

Determination of the metric for ranking of solutions from the set of admissible $x \in X$ is carried out using additive, multiplicative, or combined functions $P(q,x)$ [6]. The most universal TUF $P(q,x)$ is a function built on the basis of the Kolmogorov-Gabor polynomial [7]:

$$P(q,x) = \sum_{i=1}^{m} \lambda_i \cdot \xi_i(x) + \sum_{i=1}^{m} \sum_{j=i}^{m} \lambda_{ij} \cdot \xi_i(x) \cdot \xi_i(x) + \ldots, \quad (2)$$

where m - number of partial criteria; λ_i, λ_{ij} - weight coefficients of partial criteria $k_i(x)$ and their multiplications, $\lambda_i \geq 0,\ \lambda_{ij} \geq 0$, $i = \overline{1,m}$; $\xi_i(x)$ - partial criterion utility function (PCUF) $k_i(x)$, $i = \overline{1,m}$.

At the same time to the (PCUF) $\xi_i(x)$, $i = \overline{1,m}$ a number of requirements are put forward [7–8]: monotony, dimensionality; same range of change (from 0 to 1); invariance to the type of extremum; realization of linear, convex, concave, *S*- and *Z*-like dependencies on the values of partial criteria $k_i(x)$, $i = \overline{1,m}$. In addition, it is desirable that the utility function of all partial criteria should have the same form and differ only in the values of the parameters. This would reduce the task of their choice to the parametric optimization problem.

If in the estimation model $P(q,x)$ (2) a vector $q^{(1)}$ is determined, the components of which are weighted coefficients of partial criteria $\lambda_i \geq 0,\ \lambda_{ij} \geq 0$, $i = \overline{1,m}$ and vector $q^{(2)}$, whose components are the parameters of the function of the utility of partial criteria $\xi_i(x)$, $i = \overline{1,m}$, then the task of choosing the best solution for a set of criteria can be reduced to the problem of optimizing the type:

$$x^o = arg \max_{x \in X,\, q \in Q} P(q,x), \quad (3)$$

where Q - set of permissible values of model parameters (2).

Due to its complexity, the task of determining the vector of parameters of the model of multicriteria assessment $P(q,x)$ (2) is traditionally solved in two stages. In the first stage, by means of approximation of experts' assessments, the parameters are determined PCUF $q^{(1)}$. At the second stage by expert methods by means of comparative identification, ranking, ranking of attributes, successive advantages or pair comparisons, weight coefficients of partial criteria are determined λ_i, λ_{ij}, $i = \overline{1,m}$, which is the coordinates of the vector of model parameters To do this, in general, it is necessary to determine the similarity criterion, the set of informative input data, the structure and parameters of the function $P(q,x)$, assess its accuracy (adequacy to the benefits of DM). As the identification criteria (depending on the conditions of the task), a minimum of total, total quadratic, maximum, absolute or relative error of estimation of the general utility, the maximum of the function of correctness of choice or the minimum of the error of restoring the order of alternatives is used $x \in X$ [9]. A modern alternative to the expert evaluation of weighting factors of partial criteria λ_i, λ_{ij}, $i = \overline{1,m}$ is the technology of comparative identification, and one of the most common functions of utility of partial criteria is the function [7]:

$$\xi_i(x) = \left(\frac{k_i(x) - k_i^-}{k_i^+ - k_i^-} \right)^{\alpha_i}, \quad (4)$$

where α_i - parameter defining the type of dependence (4) (with $\alpha_i = 1$ realized linear, with $0 < \alpha_i < 1$ – convex, with $\alpha_i > 1$ – concave dependences).

The disadvantage of the PCUF $\xi_i(x)$ of the form (4) is its inability to realize *S*- and *Z*-like dependencies on the values of partial criteria $k_i(x)$, $i = \overline{1,m}$, that more adequately describe the situation of choice of multicriteria solutions using fuzzy mathematics. All the above-named requirements satisfy the functions given and investigated in the works [5, 8]. However, they have different accuracy of representing the advantages of DM, and the algorithms for their parametric synthesis and the calculation of values have significantly different time constraints.

An overview of the current state of the problem of multi-criteria evaluation automation and selection [1–9] shows that until now, the tasks of structural, parametric or structural-parametric synthesis in one of the classes of additive, multiplicative or mixed functions of general utility, constructed on the basis of universal PCUF (4) or similar to them, using heuristic or expert methods of their synthesis. This does not guarantee the receipt of adequate evaluation models for automated systems supporting the adoption of multicriteria management or design decisions.

III. Research results

As a basis for the general utility function (2) proposed to use a universal PCUF, that allows for the implementation of linear, convex, concave, *S*- and *Z*-like dependencies on the values of partial criteria [10]:

$$\xi(x)=\begin{cases}\bar{a}\cdot\left(\dfrac{\bar{k}(x)}{\bar{k}_a}\right)^{\alpha_1}, & 0\le\bar{k}(x)\le\bar{k}_a;\\ \bar{a}+(1-\bar{a})\cdot\left(\dfrac{\bar{k}(x)-\bar{k}_a}{1-\bar{k}_a}\right)^{\alpha_2}, & \bar{k}_a<\bar{k}(x)\le 1,\end{cases} \quad (5)$$

$$\bar{k}(x)=\frac{k(x)-k^-}{k^+-k^-},\ i=\overline{1,m}, \quad (6)$$

where $\bar{k}_a$, $\bar{a}$ - normalized values of coordinates of the point of gluing PCUF (4), $0\le\bar{k}_a\le 1$, $0\le\bar{a}\le 1$; α_1,α_2 - coefficients determining the type of dependence on the initial and final segments; $k(x)$, k^+, k^- - is the partial criterion for the solution x, the best and worst value of the criterion $k(x)$.

The main problem with the practical use of universal PCUF type (5) is the complexity of the procedure for selecting the values of its parameters $q^{(1)}$. The source of information for choosing $q^{(1)}$ values is experts. Assume that the expert (or any other way) has been able to determine for a number of *n* values of the *i*-th partial criterion $k_{i1}=k_i(x_1)$, $k_{i2}=k_i(x_2),\ldots,\ k_{in}=k_i(x_n)$ the value of their utility (value) from the point of view of DM $\xi_i[k_i(x_1)]=\tilde{\xi}_{i1}$, $\xi_i[k_i(x_2)]=\tilde{\xi}_{i2},\ldots,\ \xi_i[k_i(x_n)]=\tilde{\xi}_{in}$.

Perform the transformation of the values of partial criteria by the scheme (6), denoting the obtained values through $\bar{k}_{ij}=\omega[k_i(x_j)]$, $j=\overline{1,n}$. Then the choice of the best values of the parameters of the PCUF $q_i^{(1)}=[a_{1i},a_{2i},k_{ia},a]$, $i=\overline{1,m}$ can be done by one of the indicators of its deviations from the whole set of given points $\{<\bar{k}_{ij},\tilde{\xi}_{ij}>\}$, $j=\overline{1,n}$ (fig. 1):

$$R[a_{1i},a_{2i},k_{ia},a,K_i(x),\Xi_i(x)]\rightarrow\min_{a_{1i},a_{2i},k_{ia},a}, \quad (7)$$

where $K_i(x)=\{k_{ij}\}$, $\Xi_i(x)=\{\tilde{\xi}_{ij}\}$, $i=\overline{1,m}$, $j=\overline{1,n}$.

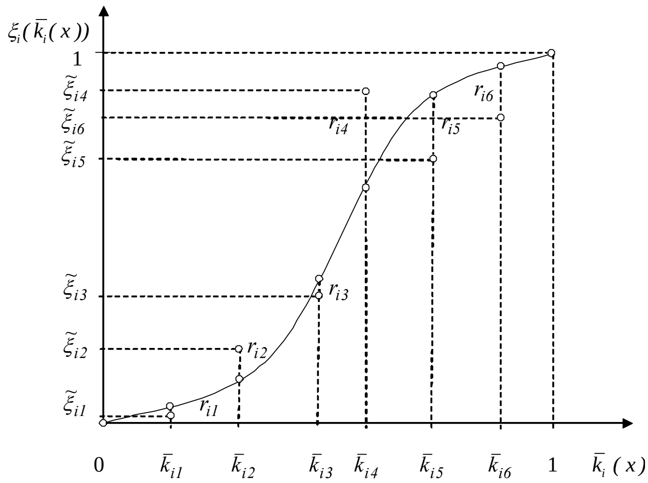

Fig. 1. Approximation of input data in the form (5): r_{ij} – error of approximation at the point $\bar{k}_{ij}$ (option x_j according to criterion k_i)

In the absence of information on the accuracy of determining the values of $\Xi_i(x)=\{\tilde{\xi}_{ij}\}$, $i=\overline{1,m}$, $j=\overline{1,n}$ for this purpose, the criteria of the minimum sum of the squares of deviations (8), the maximum deviation (9) and the sum of the deviations modules (10) can be used:

$$R=[\xi_{ij}(a_{1i},a_{2i},\bar{k}_{ia},a)-\tilde{\xi}_{ij}]^2\rightarrow\min_{a_{1i},a_{2i},k_{ia},a}, \quad (8)$$

where $\xi_{ij}(a_{1i},a_{2i},\bar{k}_{ia},a)$ - the value of the utility function of the partial criterion $k_i(x_j)$, $i=\overline{1,m}$ (5) with parameter values $[a_{1i},a_{2i},\bar{k}_{ia},a]=q_i^1$ for option x_j;

$$R=\max_{1\le j\le n}\left\{\left|\xi_{ij}(a_{1i},a_{2i},\bar{k}_{ia},a)-\tilde{\xi}_{ij}\right|\right\}\rightarrow\min_{a_{1i},a_{2i},k_{ia},a}, \quad (9)$$

$$R=\sum_{j=1}^{n}\left|\xi_{ij}(a_{1i},a_{2i},\bar{k}_{ia},a)-\tilde{\xi}_{ij}\right|\rightarrow\min_{a_{1i},a_{2i},k_{ia},a}. \quad (10)$$

Particular features of the parametric synthesis of PCUF (5) are the relatively low degree of accuracy of the output data (given by experts or DM) and the multi-extremity of the target function (7), which may take the form (8), (9) or (10). Proceeding from this, for its solution, the method of nets is chosen.

A series of experimental studies was carried out to approximate the estimates of DM on the usefulness of the values of partial criteria using the functions of Gauss (11), logistic (12), Harrington (13), modified Gauss function (14) [8], power gluing (15)–(16) [7, 11–12] and function (5):

$$\xi(x)=\exp\left[-\frac{(\bar{k}(x)-1)^2}{c}\right], \quad (11)$$

$$\xi(x)=\frac{1}{1+exp\left[-\frac{(\bar{k}(x)-a)}{b}\right]}, \quad (12)$$

$$\xi(x)=exp\left\{-exp\left[(g\cdot\bar{k}(x)-a)\right]\right\}, \quad (13)$$

$$\xi(x)=exp\left[-\frac{(\bar{k}(x)-1)^{2\cdot\alpha}}{c}\right], \quad (14)$$

$$\xi(x)=\begin{cases}2^{p-1}\cdot\left[\bar{k}(x)\right]^{p}, & 0\le\bar{k}(x)\le 0.5;\\ 1-2^{p-1}\cdot\left[\frac{0.5-\bar{k}(x)}{0.5}\right]^{p}, & 0.5<\bar{k}(x)\le 1,\end{cases} \quad (15)$$

$$\xi(x)=\begin{cases}\bar{a}\cdot(b_1+1)\cdot\left(1-\left(b_1/\left(b_1+\frac{\bar{k}(x)}{\bar{k}_a}\right)\right)\right),\\ 0\le\bar{k}(x)\le\bar{k}_a;\\ \bar{a}+(1-\bar{a})\cdot(b_2+1)\cdot\left(1-\left(b_2/\left(b_2+\frac{\bar{k}(x)-\bar{k}_a}{1-\bar{k}_a}\right)\right)\right),\\ \bar{k}_a<\bar{k}(x)\le 1,\end{cases} \quad (16)$$

where a,b,b_1,b_2,c,g,p - parameters that determine the appearance of function dependencies (11)–(16).

According to research results, the modified Gaussian function (14), the power gluing (15) and the Gaussian function (11) relatively quickly change their values when entering the zones of insensitivity (the approximation of the values of partial criteria to the best $\bar{k}(x)\to 1$ and the worst values of $\bar{k}(x)\to 0$, which can lead to significant errors in the multicriteria assessment of the properties of solutions; the highest accuracy of approximation of DM estimates has the function of the utility of partial criteria (5); the relatively low errors are the power gluing (16), the Harrington function (13), and the logistic function (12).

IV. Conclusions

According to the results of the analysis of the current state of the problem of multicriteria assessment and choice of solutions, it is established that: the most adequate models of such problems are constructed on the basis of the Kolmogorov-Gabor polynomial; preferences of DM in many cases are described by S- and Z-like dependencies on the values of quality indices; To evaluate the utility of partial criteria in existing models, functions that do not allow S- and Z-like dependencies to be implemented are used. For the approximation of DM estimates regarding to the value of the values of partial criteria in the fuzzy set theory, Gauss, logistic, and Harrington functions and power gluing are used.

The mathematical model of the problem of parametric synthesis of utility functions is formulated, the method of its solution is proposed and estimates of their accuracy are obtained. It is established that the highest accuracy of approximation of DM estimates has been proposed modification of the utility function of partial criteria. The obtained results can be used in systems of designing, management, artificial intelligence. Their practical use will improve the quality of project or management decisions that are accepted for a variety of indicators.

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