MATHEMATICAL MODEL OF MULTICRITERIA OPTIMIZATION FOR PROJECT OF REENGINEERING LARGE-SCALE MONITORING SYSTEMS

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The analysis of modern projects and publications devoted to the design of large-scale monitoring systems provided an opportunity to identify a set of indicators which impact the structural and topological characteristics of this class of systems. It has been proposed objectives for optimization, which take into account cost, reliability, operativeness, survivability and can improve the efficiency of the results of the solution of practical problems of reengineering.

The target of large-scale monitoring systems (LSMS) is provide information about territorially distributed on a large area controlled facilities, with a defined time and cost efficiency. Examples include the system of radiation, ecological, hydrometeorological, geological, economic, medical, astronomical and other types of monitoring.

During process of reengineering (redesign) of such systems it is necessary resolve the complex problems of structural, topological, parametric and process optimization. This leads to the need for a complex set of mathematical models, methods, algorithms and software.

The analysis of problem demonstrated that structural and topological implementation has greatest impact on performance and cost of monitoring system. This defines importance of structural and topological optimization problem for projects of reengineering large-scale monitoring systems.

Targets of LSMS defines the set of efficiency criteria (objectives) which will use for optimization of functional and cost indicators for current system. In practice, as particular criteria traditionally used indicators of the cost of re-engineering (or adjusted cost), operativeness and reliability, such approach not allowed to get most efficient variant of reengineering. It actualizes the problem of developing a universal mathematical model of multicriteria problem for reengineering topological structures of LSMS, considering set of criterias and constraints on cost, reliability, operativeness, survivability.

We consider the three-tier centralized LSMS, which consist of data acquisition subsystem (monitoring stations, the elements), subsystem for pretreatment (nodes), subsystem for processing and dissemination of information (center, root node).

Requests for information from the center through the nodes are sent to the elements, and received from the elements information flows through the nodes in the center. We assume that the whole set of monitored facilities to a predetermined magnification is under the supervision of the system elements $I = \{i\}, i = \overline{1,n}$, system nodes can be placed only on the basis of its elements, the elements are connected to a node with minimum cost.

Thus, the set of feasible solutions of the problem $s \in S$ depends on the following:

$$S = \{s\} = \begin{cases} x = [x_{ij}],\ x_{ij} \in \{0,1\},\ i,j = \overline{1,n},\ x_{11} = 1; \\ \sum\limits_{i=1}^{n} x_{ij} \ge 1,\ \forall j = \overline{1,n};\ \sum\limits_{j=1}^{n} x_{ij} \ge 1,\ \forall i = \overline{1,n}; \\ \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{n} x_{ij} = 2 \cdot \left(n + \sum\limits_{i=1}^{n} x_{ii} \right); \\ x_{ii} = 1 \to x_{i1} = 1 \wedge x_{1i} = 1,\ \forall i = \overline{1,n}; \\ x_{ii} = 1 \wedge x_{ij} = 1 \to ij = \arg \min\limits_{1<i'<j} c_{i'j}\ \forall j \le n,\ i,j = \overline{1,n}, \end{cases} \tag{1}$$

where S – the set of feasible variants of topological structures LSMS; s – variant of the topological structure; $x = [x_{ij}],\ i,j = \overline{1,n}$ – symmetric matrix of connections (x_{ij} – binary variable, $x_{ij} = 1$ if element i and j are connected to each other; $x_{ij} = 0$ – in other case; $x_{ii} = 1$ system node based on i-th element, $x_{ii} = 0$ – in other case, $i = \overline{1,n}$; system center based on element $i = 1$; n – amount of the system elements; $c_{i'j},\ i', j = \overline{1,n},$ – cost of connection between i' and j.

The cost of LSMS consists of the costs of the center C_C, nodes C_U, elements C_E, connection between nodes and center C_{UC} between elements and nodes C_{EU}:

$$C = C_C + C_U + C_{UC} + C_E + C_{EU} \tag{2}$$

Costs for the existing variant of LSMS $C(a), a \in S$ and costs of the optimal variant LSMS with the new functioning conditions $C(b), b \in S$ calculates with (2). A desirable target is to minimize the additional cost $\Delta C(a,b)$ This difference does not include the possibility of reuse part of the topological structure of the existing system $a \in S$ .Given this, criteria for the additional costs of reengineering LSMS $k_1(a,s) \to \min\limits_{s \in S}$ with the possibility of reuse part of the topological structure of the existing system $a \in S$ can be represented as:

$$k_1(a,s) = \Delta C(a,s) = \sum_{i=1}^{n} [c_i(1-x'_{ii})x_{ii} + d_i x'_{ii} x_{ii} + e_i(1-x'_{ii})\, x_{ii} - g_i x'_{ii} x_{ii}] +$$
$$+ \sum_{i=1}^{n} \sum_{j=i}^{n} [c_{ij}(1-x'_{ij})x_{ij} + d_{ij} x'_{ij} x_{ij} + e_{ij}(1-x'_{ij})x_{ij} - g_{ij} x'_{ij} x_{ij}] \to \min_{s \in S} \quad (3)$$

where c_i – costs of creation elements, nodes, center in new structure, $i = \overline{1,n}$; x'_{ij} and x_{ij} – respectively elements of adjacency matrix (connections) between elements, nodes and center in the existing structure $x' = [x'_{ij}]$ and structure after reengineering $x = [x_{ij}]$ ($x'_{ij} = 1$ or $x_{ij} = 1$, if element i and j are connected to each other; $x'_{ij} = 0$ or $x_{ij} = 0$ – in other case); d_i – costs of modernization element, node, or center in new structure $i = \overline{1,n}$; e_i – costs of dismantling nodes of existing structure $i = \overline{1,n}$; g_i – cost of resources, which could be reused (or traded) after dismantling equipment of nodes $i = \overline{1,n}$; c_{ij} $i, j = \overline{1,n}$ – cost of connections between i and j ; S – feasible set of variants of topological structure LSMS (1).

When assessing operativeness of LSMS for the similar nodes and connections should be aware, that time of requests and responses between center and node $\tau_i^{CU}(s)$, between element and node $\tau_i^{EU}(s)$, and also time of processing requests and responses inside nodes $\tau_i^{U1}(s)$, $\tau_i^{U2}(s)$, $i = \overline{1,n}$ depends on amount of the elements, connected to each node (in the topological structure of LSMS), and time for generating request in the center τ_i^{C}, receiving information by system element τ_i^{E} and delivery response between element and node τ_i^{EU} not depends on topological structure of LSMS $i = \overline{1,n}$.

A desirable target for LSMS is minimization of maximal response time about state of monitored facility:

$$k_2(s) = \max_{1 \le i \le n} \left[\tau_i^{C} + \frac{\alpha_i}{g_{ii}} + \tau_i^{E} + \frac{\beta_i}{g_{ii}} + \left(\frac{\alpha_i}{g_i} + \frac{\alpha_i}{h_i^1} + \frac{\beta_i}{h_i^2} + \frac{\beta_i}{g_i} \right) \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} x_{ii} \right] \to \min_{s \in S}, \quad (4)$$

where g_i and g_{ij} – bandwidth of connection center-node, and node-element; h_1 and h_2 – speed of processing request and response in system nodes.

When assessing reliability of LSMS must be accounted reliability of the center, reliability of all elements, nodes and connections between them:

$$k_3(s) = k_3^C(s) \cdot k_3^U(s) \cdot k_3^E(s) \cdot k_3^{CU}(s) \cdot k_3^{UE}(s), \quad (5)$$

where $k_3(s)$, $k_3^C(s)$, $k_3^U(s)$, $k_3^E(s)$, $k_3^{CU}(s)$, $k_3^{UE}(s)$ – respectively readiness factors of topological structure in total, technical tools of top level (center), technical tools of middle level (nodes), technical tools of bottom level (elements), connections on top level (center-nodes), connection on bottom level (nodes-elements).

Taking into account the topological structure of LSMS expression (5) represent:

$$k_3(s) = k^C \cdot (k^U)^u \cdot (k^E)^n \cdot (k^{CU})^n \cdot (k^{BL})^n \to \max_{s \in S} \quad (6)$$

where u – amount of nodes in system; S – the set of feasible variants of topological structures LSMS (1).

As an indicator of survivability we use the value of the share of items associated with the center in functioning system in case individual equiprobable damage (damage on center, node, element or connection):

$$k_4(s) = \left\{ \min_{1 \le j \le n} \left\{ \frac{n - \sum_{j=1}^{n} \sum_{i=j}^{n} x_{ij} x_{jj}}{n} \right\} \right\} \to \max_{s \in S}. \quad (7)$$

The proposed mathematical model of multicriteria problem of reengineering topological structures LSMS includes formalized criteria for costs $k_1(a,s) \to \min_{s \in S}$ (3), operativeness $k_2(s) = \{ \max_{1 \le i \le n} \tau_i \} \to \min_{s \in S}$ (where τ_i – response delivering time from the i-th element) (4), reliability $k_3(3) \to \max_{s \in S}$ (6), and $k_4(s) = \{ \min_{1 \le j \le n} [k_{4j}^{CU}(s), k_{4j}^U(s)]\} \to \max_{s \in S}$ survivability (where $k_{4j}^{CU}(s)$ and $k_{4j}^U(s)$ – indicators of survivability in the case of damage of communication between center-node and node on base j-th element (7).

From the proposed multicriteria models can be obtained (by excluding the relevant particular criteria and constraints) almost all important tasks reengineering model for one, two or three particular criteria. This can be implemented, for example, by convolution with particular criteria of objective function

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