# Spatial Interpretation of the Notion of Relation and Its Application in the System of Artificial Intelligence

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**Abstract.** This paper considers the point of view on relations that are interpreted as generalized spaces, in contrast to the classical definition of a relation as a subset of the Cartesian product of sets. The connection of generalized spaces with predicates and maps is also considered. Predicates accompanying generalized spaces, projection predicates are introduced. Possible interpretations of generalized spaces are presented. Looking at the attitude as a generalized space leads to the formulation of a number of interesting logical-mathematical problems and to the results that are promising for use in many areas of artificial intelligence.

**Keywords:** Predicate, Quasi-Tolerance, Isomorphism, Cartesian Space, Morphological Structures, Characteristic Predicate.

## 1 Relevance

The development of the newest information technologies, computer facilities, allowing to automate the processes of information transformation in the mode of user requests, is the main direction of modern scientific research in artificial intelligence.

For the effective design of artificial intelligence systems, a formal presentation of information is necessary, which takes into account the polysemy of its structure. As such, predicate algebra has been used for many years, which is a generalization of the algebra of logic and is used in various industries where intellectual systems and interfaces are needed, for example, in [1]. The concept of a predicate used is a generalization of a Boolean function. Any finite relations can now be written in the form of equations of a predicate algebra [2].

Such an application of the formal presentation of arbitrary relationships and the development of their further circuit implementation [2, 3] promotes the development of artificial intelligence systems, the improvement of the computer-aided design process for digital devices, which, among other things, can be part of the language interface [4], computer-based learning systems, expert systems, etc. For example, it allows you to develop IP-core, implementing various functions, including syntactic, morphological, semantic analysis of intellectual processes.
However, the standard representation of relations as a subset of a Cartesian product may not be enough to describe intellectual processes. In this regard, the purpose of this work is to interpret relations as generalized spaces.

## 2 The course of research

Consider some predicate $S(x_1, x_2,..., x_n, y)$, defined on the Cartesian product $A_1 \times A_2 \times ... \times A_n \times B$ of arbitrarily chosen sets $A_1$, $A_2$,..., $A_n$, B. Consider the relation s corresponding to the predicate S, which is given by the equation

$$S\ (x_1, x_2, ..., x_n, y) = 1. \quad (1)$$

It binds the variable y to the variables $x_1$, $x_2$, ..., $x_n$. Now, the object y is considered as a vector of some n-dimensional space S, and the values of the variables $x_1 \in A_1$, $x_2 \in A_2$,..., $x_n \in A_n$, satisfying condition (1), as its coordinates. However, the space S turns out to be not quite ordinary. In classical mathematics, spaces are used that are naturally called Cartesian. Their characteristic property (which the Cartesian coordinate system has) is that each vector of a Cartesian space is determined by a single coordinate representation (set of coordinates) and a single vector corresponds to each set of coordinates. Thus, all the vectors of the Cartesian space and their coordinate representations are connected by a one-to-one correspondence, which makes it possible not to distinguish them from each other. For the space given by equation (1), this property for any predicate S, generally speaking, does not hold. That is why it is called generalized by us.

Thus, each relation generates some generalized space, which is a connection between the vectors y and their coordinate representations $(x_1, x_2, \ldots, x_n)$. Therefore, it is natural to consider the concept of space introduced here as a generalization of the concept of a coordinate system introduced by Descartes, which later developed in classical mathematics into the concept of space.

The construction thus obtained is a space in the generalized sense and does not fully correspond to the classical space. For example, in mathematics the vectors of space can be added, but here it is not true to add elements of the set B. In logical mathematics, the concept of relationship is clearly based on the idea of multidimensionality, which means that we can identify it with the concept of space, by which we can understand any multidimensional formation, i.e. simply coordinate system [5].

Let S $(x_1, x_2,..., x_n, y)$ be some predicate on $A_1 \times A_2 \times ... \times A_n \times B$. We say that the predicate S generates the generalized space S on the Cartesian product

$A = A_1 \times A_2 \times ... \times A_n$ over the set B. In the following, for brevity, we will call generalized spaces simply spaces. The predicate S is called characteristic for the space S. The number n is called the dimension of the space S. The set A is called the coordinate system of the space S, and the set $A_1, A_2, \ldots, A_n$ - its coordinate axes. The elements of the set A are called points or cells (if the set A is finite) of the coordinate system. The set B is called the support of the space S, and its elements $y \in B$ are vectors of the space S. Any point $x = (x_1, x_2, ..., x_n)$ satisfying condition (1) is called the coordinate representation of the vector y.

The space S is characterized by a connection (1) between its vectors $y \in B$ and points $x \in A$ of its coordinate system. For Cartesian spaces, this connection is a bijective mapping of the set A into the set B, therefore each Cartesian space can be characterized, up to isomorphism, by its coordinate system. If, for a generalized space S, we specify only its coordinate system A without specifying the predicate S, then the characteristic of such a space will be incomplete.

$$S\ (x_1, x_2, ..., x_n) = y \tag{2}$$

A in B, corresponding to the predicate S, is called a mapping of the coordinate system A of the space S to its support B [2]. It fully characterizes the space S. The relation s, defined by condition (1), connects each vector $y \in B$ with a set $(x_1, x_2, ..., x_n)$ its coordinates $x_1 \in A_1, x_2 \in A_2, ..., x_n \in A_n$ (one, many or none). Mapping (2) to each set of coordinates $(x_1, x_2, ..., x_n) \in A$ matches vector $y \in B$ (one, many or none).

This is explained by an example. For some values $x_1, x_2 \in \{m, n\}$, $y \in \{1, 2, 3\}$, we define the predicate

$$S\ (x_1, x_2, y) = x_1^n x_2^n y^1 \vee x_1^n x_2^n y^2 \vee x_1^m x_2^n y^2 . \tag{3}$$

The generalized space S is given by the equation s $(x_1, x_2, y) = 1$, which links each vector y to its coordinates $x_1$, $x_2$. Set of coordinates $(x_1, x_2) = (n, n)$ defines two vectors $y = 1$ and $y = 2$ of this space. The set (m, n) defines only vector 2, the sets (n, m) and (m, m) - none. Vector 1 has one (n, n) coordinate representation, vector 2 has two (n, n) and (m, n), vector 3 has none.

Obviously, a given relation generates some generalized space, which is a connection between the vectors y and their coordinate representations $(x_1, x_2)$. Then the mapping s $(x_1, x_2) = y$ is expressed by a system of conditions:

$$x_1^n x_2^n \supset y^1 \vee y^2, \quad x_1^m x_2^n \supset y^2, \quad x_1^n x_2^m \vee x_1^m x_2^m \supset 0. \tag{4}$$

It means that to the set (n, n) the map s associates two vectors 1 and 2, to the set (m, n) one vector 2, to the sets (n, m) and (m, m) one vector. The mapping $s^{-1}(y) = (x_1, x_2)$, the inverse of mapping s, is written in the condition system

$$y^1 \supset x_1^n x_2^n, \quad y^2 \supset x_1^n x_2^n \vee x_1^m x_2^n, \quad y^3 \supset 0, \tag{5}$$

which means that the vector 1 corresponds to a single coordinate representation (n, n); vector 2 - two (n, n) and (m, n); vector 3 - none.

The carrier of the space S defined by predicate (3) is the set B = {1, 2, 3}, the coordinate axes are the sets $A_1 = A_2 = \{m, n\}$, the set A= $\{m, n\}^2$ acts as the coordinate system. The space S can be visually represented as a graph of the map s $(x_1, x_2) = y$ (Fig. 1). The lower left cell of the graph $(x_1, x_2) = (n, n)$ contains two vectors 1 and 2; in the lower right (m, n) - one vector 2; in the remaining cells of the graph there is no vector. Projecting vector 1 on the $x_1$ and $x_2$ axes, we find its unique coordinate representation (n, n); vector 2 has two coordinate representations (n, n) and (m, n). Vector 3 did not fall within the grid, so it does not have a single coordinate representation. If the space S were Cartesian, then all the vectors of the carrier B would be located exactly one time in each cell of the coordinate system without repetitions.

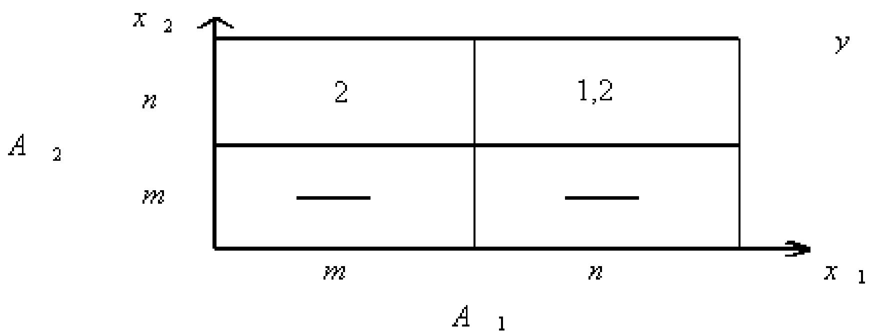


**Fig. 1.** Representation of the mapping s $(x_1, x_2) = y$.

Consider the i-th projection predicate of the space S [2]. By this name we will understand the predicate. $G_i(y, x_i)$ on $B \times A_i$ ( $i=\overline{1,n}$ ), values of which for any $y \in B$ и $x_i \in A_i$ determined by equality:

$$G_i(y, x_i) = \exists x_1 \in A_1 \exists x_2 \in A_2 \ldots \exists x_{i-1} \in A_{i-1} \exists x_{i+1} \in A_{i+1} \ldots \exists x_n \in A_n$$

$$S\,(x_1, x_2, \ldots, x_i, \ldots, x_n, y). \tag{6}$$

For example, for the space given by predicate (3), projection predicates are written in the form:

$$G_1(y, x_1) = x_1^n (y^1 \vee y^2), \quad G_2(y, x_1) = x_2^n (y^1 \vee y^2). \tag{7}$$

The predicate $G_i$ (y, $x_i$) corresponds to the map $g_i$ (y) = $x_i$ from B to $A_i$, called the i-th projector of the space S. The projection predicate $G_i$ (y, $x_i$) connects each point $y \in B$ with its coordinates $x_i \in A_i$ (one , or even one). The projector $g_i$ (y) = $x_i$ of each point $y \in B$ assigns its coordinates $x_i \in A_i$ (one, many or none).

For example, for the space given by predicate (3), the projectors $g_1(y) = x_1$ and $g_2(y) = x_2$ are written in the form:

$$y^1 \supset x_1^n, \quad y^2 \supset x_1^n \vee x_1^m, \quad y^3 \supset 0; \qquad y^1 \vee y^2 \supset x_2^n, \quad y^3 \supset 0. \qquad (8)$$

Move on to examine the internal structures and properties of generalized spaces. For this we need some auxiliary concepts.

The predicate P (x, y) defined on $A \times B$ is called

- well defined on the left, if it satisfies the condition $\forall x \in A \; \exists y \in B \; P(x, y)$;
- well defined on the right if it satisfies the condition $\forall y \in B \; \exists x \in A \; P(x, y)$;
- and well-defined, if it is well defined both left and right.

If the predicate P on $A \times B$ is not well defined, then it can be turned into a well-defined area defined by the operation of natural narrowing of its definition, which is the replacement of area A by area $A' \subseteq A$, characterized by a predicate $A'(x) = \exists y \in B \; P(x, y)$, and area B - area $A'(x) = \exists y \in B \; P(x, y)$ characterized by the predicate $B'(y) = \exists x \in A \; P(x, y)$.

In the process of natural narrowing of the predicate definition domain, all zero rows and columns are excluded from its table. The rows and columns of the predicate table are called zero if they are completely filled with zeros. Zero rows and columns do not carry any useful information and therefore can be excluded from the predicate table without prejudice to the completeness of its characteristics. If necessary, the predicate table can always be expanded to its original size, adding to it the previously excluded from it null rows and columns.

The structure of the spaces under consideration is described by a quasi-tolerant predicate, i.e. the following Theorem 1 and equality (9) are valid.

**Theorem 1.** On the general form of a quasi-tolerant predicate

Let E be a predicate on $B \times B$. Then E is a quasi-tolerance if and only if there exists a set A and such a predicate F on $B \times A$ that for any $x, y \in B$, the equality is true:

$$E(x, y) = \exists u \in A \quad (F(x, u) \wedge F(y, u)). \qquad (9)$$

We introduce the concepts accompanying generalized spaces.

The predicate $E_i$ $(i = \overline{1,n})$ defined on $B \times B$, where B is the carrier of the space S, whose values for any $y_1, y_2 \in B$ are determined by the equality

$$E_i(y_1, y_2) = \exists x_i \in A_i (G_i(y_1, x_i) \wedge G_i(y_2, x_i)) \qquad (10)$$

is called the *i*-th predicate that accompanies the space S. In Expression (10), the predicate $G_i$ is the i-th projection predicate of the space S, defined on $B \times A_i$.

According to Theorem 1 on the general form of the quasi-tolerance predicate, all predicates (10) accompanying the space S are quasi-tolerances, which allows us to formulate the following statement.

**Statement 1. On quasi-tolerances that accompany space.**

a) For each space S on $A_1 \times A_2 \times ... \times A_n$ over B there exists a unique set $E_1, E_2, ..., E_n$ of quasi-tolerances accompanying it;

b) For any set of quasi-tolerances $E_1, E_2, ..., E_n$ on $B \times B$ there are sets $A_1, A_2, ..., A_n$ and the space S on $A_1 \times A_2 \times ... \times A_n$ over B for which predicates $E_1, E_2, ..., E_n$ will be accompanying.

Evidence. Assertion a) is a direct consequence of the definition of the accompanying predicates of the space S and Theorem 1.

We prove assertion b). According to Theorem 1 on the general form of quasi-tolerance, for any quasi-tolerant predicate $E_i$ ($i=\overline{1,n}$) on $B \times B$, there is a set Ai and such a predicate $G_i$ on $B \times A_i$ such that for any $y_1, y_2 \in B$ $E_i (y_1, y_2) = \exists\, x_i \in A_i (G_i (y_1, x_i) \wedge G_i(y_2, x_i))$. Put $S (x_1, x_2, ..., x_n, y) = \bigwedge_{i=1}^{n} G_i (y, x_i)$ for all $x_i \in A_i$ ($i=\overline{1,n}$), $y \in B$. Then the predicate S on $A_1 \times A_2 \times ... \times A_n \times B$ defines a generalized space S for which the predicates $E_1, E_2, ..., E_n$ are accompanying quasi-tolerance. Note that this choice of the sets $A_1, A_2, ..., A_n$ and the spaces S for a given set of $E_1, E_2, ..., E_n$ of quasi-tolerances on $B \times B$ is generally not the only possible one. The statement is proven.

It is certain that the concept introduced is very important. With the help of accompanying quasi-tolerances, one can make quite informative experiments. For example, in the theory of color vision, if we fix one of the colors - x - and give the colors $y_1$ and $y_2$ such that $G (y_1, x) = G (y_2, x) = 1$, then $E (y_1, y_2) = 1$. This means that the colors $y_1$ and $y_2$ are interchangeable, i.e. it is possible to identify whole classes of equivalent colors and study one color from such a class instead of the whole class. Thus, the accompanying quasi-tolerance define the coordinate grid of the space. Note that the connection between generalized spaces and the general form of the second kind of predicate is revealed precisely by the concept of quasi-tolerances that accompany the generalized space and projective predicates of space.

Undoubtedly, the presented new view on relations, which in the approach under study are considered as mappings that carry out directional information processing, is promising for practical application. Therefore, it seems useful to present several possible interpretations of generalized spaces.

Morphological interpretation of space. We study the use of generalized spaces for the description of word formation processes in the Russian language.

Any sentences consist of word forms. The specific form of the word form is determined by a given word and a set of grammatical features. The grammatical features include case, number, gender, person, time, etc. The word form can be studied as a vector y, and the set consisting of a word and grammatical signs can be studied as its coordinate representation.

Usually the word form is uniquely defined by its coordinate representation, and the coordinate representation is uniquely determined by the word form. But sometimes there are ambiguous cases. For example, a set (wet, male, plural) and (wet, neuter, plural) gives rise to one-word form: wet. Such cases emphasize that the concept of the formation of a language does not fit into the concept of Cartesian space because of its ambiguity, but the concept of a generalized one describes all the nuances of a natural language.

We introduce the space carrier B and interpret it as the set of all word forms of adjectives. Let us introduce the axes of the space: $A_1$ - the set of all adjectives represented by its dictionary forms (good, beautiful, etc.); $A_2$ - a genus with three meanings (male, female, neuter); $A_3$ is a number with values unique, plural.

We define the predicate S on $A_1 \times A_2 \times A_3 \times B$ as follows:

$$S(x_1, x_2, x_3, y) = \begin{cases} 1, \text{if all data is consistent,} \\ 0, \text{if not consistent.} \end{cases} \tag{11}$$

Such a predicate S is called a morphological predicate [3, 4]. The morphological predicate (11) connects the word, grammatical signs and word form. A native speaker can always, to the best of his knowledge of the language, realize this predicate. For example,

S (модный, female, singular, модная) = 1,
S (красивый, female, singular, модная) = 0,
S (модный, female, singular, модные) = 0.

The space given by some morphological predicate is called morphological.

The word form is easily distinguished from the text by analyzing spaces and punctuation. Using the morphological predicate, it is relatively easy to perform operations on the word form. Operations may be as follows.

- Word form normalization - transition from word form to the word $g_1$ (y) = $x_1$.
- Word form analysis - finding the gender and the number of the word form $x_2$=$g_2$ (y), x3=g3(y). For example, «грубые» (in English «coarse») - (neuter, plural) or (male, plural). Those. $x_2$=$g_2$ (y), $x_3$=$g_3$ (y) are multivalued mappings, which is very useful for the formalization of the language.
- Synthesis of word form - finding the word form by word and grammatical features S ($x_1$, $x_2$, $x_3$) = y (ambiguous mapping). For example: (pen, male, plural) = pens.

Any natural language is characterized by a large number of ambiguities, a native speaker easily solves such questions, but a standard mathematical language cannot do this. However, generalized spaces will find their application in the study of the mechanisms of the language, since human language is a complex mechanism of transformation in space [5]. There are a lot of unexplored structures and processes in this area: a graphic representation of a word, phonetic speech recognition, etc. Perhaps all these problems will be able to find their solution in the near future using the theory of generalized spaces.

Interpretation of the generalized space as a Cartesian coordinate system. Consider the generalized space as follows: B (space carrier) is the set of all points of the plane; y (vector of space) is a point of the plane; $x_1$, $x_2$ (vector coordinates) - coordinates of a point of the plane; $A_1$, $A_2$ - abscissa and ordinate axes; $(x_1, x_2)$ = y coordinate representation of a point.

Obviously, in this interpretation, $y \neq (x_1, x_2)$, and $y = s\ (x_1, x_2)$ - that is, the mapping of space into its carrier. The space $A_1 \times A_2$ is the set of all pairs of the form $(x_1, x_2)$.

The predicate $S(x_1, x_2, ..., x_n, y)$ is called Cartesian on $A_1 \times A_2 \times \ldots \times A_n \times B$, if it has the following properties:

1) functionality: $\forall x_1 \in A_1\ \forall x_2 \in A_2 \ldots \forall x_n \in A_n\ \exists! y \in B\ S\ (x_1, x_2, ..., x_n, y)$, i.e. everywhere the definiteness and uniqueness of the predicate S,

2) inverse functionality: $\forall y \in B\ \exists! x_1 \in A_1 \exists! x_2 \in A_2 \ldots \exists!\ x_n \in A_n\ S\ (x_1, x_2, ..., x_n, y)$, i.e. subjectivity and infectivity of predicate S.

The space S defined by the predicate S is called Cartesian (bijective) if it is defined everywhere, uniquely, infectively and subjectively. Any Cartesian predicate defines some Cartesian space. In the Cartesian space, each vector y corresponds to a unique coordinate representation $(x_1, x_2, ..., x_n)$ and vice versa.

**Statement 2. On the isomorphism of Cartesian spaces**.

Let S and S' - Decartes predicates on $A_1 \times A_2 \times \ldots \times A_n \times B$ и $A_1' \times A_2' \times \ldots \times A_n' \times B'$. If there are bijections $\varphi_i$: $A_i \to A_i'$ (i = 1, …, n), then there is a bijection $\psi$: $B \to B'$, such that for any $x_1 \in A_1$, $x_2 \in A_2, \ldots, x_n \in A_n$, $y \in B$

$$S(x_1, x_2, \ldots, x_n, y) = S'(\varphi_1(x_1), \varphi_2(x_2), \ldots, \varphi_n(x_n), \psi(y)). \tag{12}$$

This property (12) means that when the coordinate axes of two Cartesian spaces differ only in the designations of their elements, then the spaces themselves and their carriers differ only in the designations of their vectors. The isomorphism property of Cartesian spaces is true only for Cartesian spaces. This is the reason for the widest dissemination of the concept of a Cartesian coordinate system (it is enough to specify a Cartesian space).

Any Cartesian predicate is injective, so it can be represented using projection predicates in the form:

$$S\ (x_1, x_2, \ldots, x_n, y) = G_1(y, x_1) \wedge G_2(y, x_2) \wedge \ldots \wedge G_n(y, x_n). \tag{13}$$

Then $g_1\ (y) = x_1$, $g_2\ (y) = x_2$ are projectors of the space. For the Cartesian space S, $g_1$, $g_2$ are functions.

Accompanying quasi-tolerance of the space $E_1(y_1, y_2)$, $E_2(y_1, y_2)$, given by expression (10), in the case of a Cartesian coordinate system, become accompanying equivalences, i.e. the following statement holds.

**Statement 3. On predicates accompanying Cartesian space**.
The predicates $E_1$, $E_2$, …, $E_n$, accompanying the Cartesian space $A=A_1\times A_2\times\ldots\times A_n$ over B, are equivalences on B. They have the property:

$$\forall y_1, y_2, \ldots, y_n \in B\ \exists! y\in B\ (E_1(y_1, y) \wedge E_2(y_2, y) \wedge \ldots \wedge E_n(y_n, y)). \qquad (14)$$

If we arbitrarily take one layer from each product corresponding to the equivalences $E_1$, $E_2$, …, $E_n$, then at the intersection of such layers we always get one element of the set B. Any n vectors define a single vector; it always exists (this is true only for the Cartesian coordinate system). Thus, the $E_1$ partition is a system of vertical lines, and $E_2$ is a system of horizontal lines. In the general case, the coordinate lines can be interrupted, occur, connect.

**Statement 4. On the definition of the Cartesian space by its accompanying equivalences**.
Any set of equivalences $E_1$, $E_2$, …, $E_n$ on B, possessing property (14), defines the only (up to a separate notation for the elements of $A_1$, $A_2$, …, $A_n$) the Cartesian space $A= A_1\times A_2\times\ldots\times A_n$ over B.

Statements 3 and 4 are a consequence of Statement 1.

In other words, if two Cartesian spaces $S(x_1, x_2, \ldots, x_n, y)$ on $A_1\times A_2\times\ldots\times A_n\times B$ and $S'(x_1', x_2', \ldots, x_n', y)$ on $A_1'\times A_2'\times\ldots\times A_n'\times B$ have the same accompanying equivalences $E_1$, $E_2$, …, $E_n$, then there exist bijections $\varphi_i$: $A_i\to A_i'$ ($i = 1, \ldots, n$), such that for any $x_1\in A_1$, $x_2\in A_2$, …, $x_n\in A_n$, $y\in B$

$$S(x_1, x_2, \ldots, x_n, y) = S'(\varphi_1(x_1), \varphi_2(x_2), \ldots, \varphi_n(x_n), y). \qquad (15)$$

From all of the above, it is obvious that the widely known concept of Cartesian space corresponds to the structure of a generalized space.

Quasi-Cartesian space. Consider the following application of a generalized space. A space is called quasi-Cartesian (Fig. 2) if it is unique, injective and surjective. Those, in contrast to Cartesian space, there is no condition everywhere for definiteness.

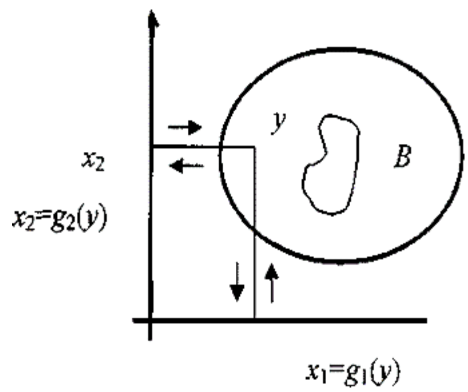


**Fig. 2.** Quasi-Cartesian space.

Predicate S on $A_1 \times A_2 \times \ldots \times A_n \times B$ is called quasi-Cartesian if it possesses the properties:

1) $\forall\, x_1 \in A_1\ \forall\, x_2 \in A_2 \ldots \forall\, x_n \in A_n\ \forall\, y, y' \in B\ (S(x_1, x_2, ..., x_n, y) \wedge S(x_1, x_2, ..., x_n, y') \supset D(y, y'))$, which means that the predicate S is unique in the variable y, but not everywhere definiteness,

2) inverse functionality: $\forall y \in B\ \exists! x_1 \in A_1 \exists! x_2 \in A_2 \ldots \exists!\ x_n \in A_n\ S(x_1, x_2, ..., x_n, y)$.

The property of isomorphism for quasi-Cartesian spaces is incorrect. But the accompanying predicates of the form (10) continue to work properly.

**Statement 5. On predicates accompanying quasi-Cartesian space.**
Predicates $E_1, E_2, \ldots, E_n$, accompanying the quasi-Cartesian space with the coordinate system $A = A_1 \times A_2 \times \ldots \times A_n$ over B, are equivalences on B. They have the property:

$$\forall y_1, y_2, \ldots, y_n, y', y'' \in B\ (E_1(y_1, y') \wedge E_2(y_2, y') \wedge \ldots \wedge E_n(y_n, y') \wedge E_1(y_1, y'') \wedge E_2(y_2, y'') \wedge \ldots \wedge E_n(y_n, y'') \supset D(y', y'')). \tag{16}$$

If we take one layer from the partition corresponding to the equivalences $E_1, E_2, \ldots, E_n$, then at the intersection of such layers we either get the empty set or only one element of the set B. This is the equivalent of the n-dimensionality of the space.

**Statement 6. On the assignment of a quasi-Cartesian space by its accompanying equivalences**.
Any set of equivalences $E_1, E_2, \ldots, E_n$ on B, possessing property (15), defines a unique (up to separate designations of elements of the sets $A_1, A_2, \ldots, A_n$) quasi-Cartesian space on the part $A_0 \subseteq A_n$ of the coordinate system $A = A_1 \times A_2 \times \ldots \times A_n$, determined by the condition $A_0\,(x_1, x_2, ..., x_n) = \exists y \in B\ \ S\ (x_1, x_2, ..., x_n, y)$.

The above property follows from Statement 1.

Inside the region $A_0$, the values of y (vectors) are one-to-one determined by their coordinate representations $(x_1, x_2)$. In other words, the quasi-Cartesian space can be defined by its coordinate grid.

# 3 Conclusions

The paper studies a formal approach to the description of n-ary relations and their arguments, in which relations are considered as mappings that carry out directional information processing. A distinctive feature of such spaces is that each vector corresponds not to a single (which is true for Cartesian space), but a multi-valued coordinate representation.

It is natural to apply generalized spaces when describing any information objects. Moreover, based on such a convenient theory, it will be possible to further study and develop the concept of relationship, which plays a fundamental role in the theory of intelligence. The developed interpretations can significantly increase the area of practical application of generalized spaces. They can be used to logically support the design of digital devices in artificial intelligence information systems, especially with a natural-language intelligent interface. Mathematical results of the work can be used in text processing systems (knowledge bases, expert systems, etc.).

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