AN INITIAL AND BOUNDARY VALUE ELECTROMAGNETIC PROBLEM FOR PLASMA SPHERE

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The exact expressions for the transformed field caused by a plasma sphere after its instant formation are obtained as a solution of an initial and boundary value electromagnetic problem for the Maxwell's equations based on the longitudinal and the transverse spherical vector functions.

One of the possible applications of the plasmonic materials is to build antenna devices radiating and receiving electromagnetic energy at optical frequencies – a very important concept for the construction of a variety of probes. The high Q-factor, augmented sensitivity, and the potential directional characteristics offered by the optical nano-resonators make them important components for chemical- and bio-sensing applications also [1-3].

The interactions between light and a medium evolve in time in restricted space regions and ultrafast processes have a wide bandwidth, calling for a solution of the initial-boundary value problems. The sphere is one of the basic elements in many phenomena and devices. This paper is devoted to the investigation of the key processes in the interaction of an electromagnetic wave with spherical particles containing time-varying dispersive media which is represented by plasma which appears in the sphere region at some moment of time as a result of instant ionization. An unconditioned ratio of the sphere radius and the spatial scale of the electromagnetic field is assumed. The investigation takes into account the full vector nature of the electromagnetic field by using the expansions over the set of the orthogonal vector spherical functions.

A 3-D initial spherical symmetric problem for the Maxwell's equation when the medium permittivity changes in time inside a sphere is formulated in the form of the Volterra integral equation in time domain [4, 5]

$$\mathbf{E}(t,\mathbf{r}) = \mathbf{E}_0(t,\mathbf{r}) + \frac{1}{4\pi}\left(\nabla\nabla - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\int_0^t dt' \int_{V(t')} d\mathbf{r}'\, \hat{G}(t-t',\mathbf{r}-\mathbf{r}')\hat{V}_e(t')\mathbf{E}(t',\mathbf{r}') \qquad (1)$$

Here $\hat{G}$ is the dyadic Green's function in time domain and the environment operator $\hat{V}_e$ takes into account the medium properties including their changes in time.

Transformation of a plane wave $\mathbf{E}_0(t,\mathbf{r}) = \mathbf{e}_x E_0 e^{i\omega(t-z/v)}$ by instant ionization of the medium inside the sphere is considered. A solution to the problem constructed by using the Laplace transform and the spherical vector functions $\mathbf{M}_{mn}(pr,\Omega)$ and $\mathbf{N}_{mn}(pr,\Omega)$. The field inside the sphere has a form

$$\mathbf{E}(p,\mathbf{r}) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[A_{mn}(p)\mathbf{M}_{mn}(\frac{\omega r}{v},\Omega) + B_{mn}(p)\mathbf{M}_{mn}(\frac{-ipr}{v_1},\Omega)\right] + \\ +\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[\bar{A}_{mn}(p)\mathbf{N}_{mn}(\frac{\omega r}{v},\Omega) + \bar{B}_{mn}(p)\mathbf{N}_{mn}(\frac{-ipr}{v_1},\Omega)\right] \quad (2)$$

where v and v_1 are phase velocities outside and inside the sphere correspondingly. The field outside the sphere is obtained via integrating in (1).

It follows from the obtained equations that instead of one time harmonic dependence of the initial wave with the frequency ω the transformed field gets a whole spectrum $p_{nk} = i\omega_{nk} + \alpha_{nk}$ determined by the dispersion equation:

$$(v/v_1)\,j_{n+1}(-ipa/v_1)h_n^{(2)}(-ipa/v_1) - j_n(-ipa/v_1)h_{n1}^{(2)}(-ipa/v_1)] = 0 \quad (3)$$

The exterior field contains the waves of the same spectrum. The cross-section for the partial wave of the complex frequency is obtained

$$\sigma_{nk} = |a_n|^2 \frac{1+n^2(n+1)^2}{n(n+1/2)(n+1)} f(p_{nk})V_n(p_{nk}) f^*(p_{nk})V^*_n(p_{nk})u_n(p_{nk},r)w^*_n(p_{nk},r)e^{-2\alpha_{nk}t} \quad (4)$$

has the temporal dependence of a decaying character. Deexcitation of the different partial waves is going on with different rates which are determined by the real parts of the complex frequency $p_{nk} = i\omega_{nk} + \alpha_{nk}$.

Concluding, the solution to an initial and boundary value problem is obtained and the temporal analysis of the electromagnetic field transformed by instant ionization of a medium inside a sphere illuminated by a harmonic plane wave is carried out. The phenomena that are consequence of the finite size of the region are considered.

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