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MICROWAVE ELECTRONICS

TERAHERTZ BAND DOUBLE-FREQUENCY DIFFRACTION RADIATION OSCILLATOR WITH INCLINED FOCUSING FIELD

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It is considered an oscillator-frequency multiplier on the Smith-Purcell effect, in which two modes of beam-wave interaction – the surface wave mode and the diffraction radiation mode – are realized simultaneously. The vector of magnetic focusing field induction is directed at the angle to the slow-wave system surface like in a klynotron. Oscillations in the diffraction radiation mode are excited on the third harmonic of the surface wave mode, which performs a distributed parametric modulation of the electron beam along the entire interaction space. Using the non-linear multidimensional theory, the peculiarities of the influence of a focusing field inclination upon the output characteristics of the device are investigated. Increasing of the focusing field inclined angle results in improvement of the interaction efficiency and increasing of the output signal amplitude on the third harmonic of the surface wave mode in the terahertz frequency band.

KEY WORDS: *terahertz band, double-frequency oscillator, diffraction radiation, parametric modulation, inclined focusing field, multidimensional theory*

1. INTRODUCTION

Development of radiation sources in the short-wave domain of the submillimeter wave band (terahertz frequency band) is referred to top-priority trends in development of the present-day vacuum electronics. Free electron lasers [1,2] are applied for obtaining of high power in this domain of the electromagnetic waves spectrum, low levels of the power are attained by applying backward wave oscillators (BWO), semiconductor oscillators and specialized laser systems [3-5]. Alongside with the above, the sources of medium-power radiation possessing the set spectral characteristics and capable of performing electronic tuning of frequency, are required for most of the practical applications. A special place here is occupied by resonance BWO with the inclined focusing field – klynotrons [6], and the diffraction radiation oscillators (DRO), which are also known as orotrons [7,8]. Other options of the same type of devices are represented by developed in Japan ledatrons and laddertrons [9,10]. It is known, that in devices with distributed interaction the coupling resistance is decreased as the frequency is increased. That leads to the diminution of an interaction efficiency. In klynotrons due to application of the inclined focusing field a partial compensation of this basic physical mechanism can by attained. High-Q electromagnetic systems of DRO (open resonators) allow obtaining narrowband signals in the millimeter and submillimeter wavelength bands, however, increasing of the frequency is often accompanied in this case with decreasing of the output power due to localization of the electromagnetic field of synchronous with the electron bunch slow waves in the vicinity of the slow-wave system surface. Using of inclined focusing for the electron beam in DRO results in the efficiency enhancement at all stages of development of the oscillatory process and represents one of the promising techniques for improvement of operational characteristics in a terahertz frequency band [11,12].

Simultaneous existence of the surface and volume wave modes for slow-wave systems of such devices at various frequencies [8] is an important feature of power exchange between the electron beam and the electromagnetic field in resonance oscillators with long-term interaction (ledatron, orotron, DRO). Excitation of oscillations in the former mode is realized on one of the backward slow waves, so it can be determined as the surface wave mode. The mode of volume (fast) waves is directly related to the diffraction radiation (Smith-Purcell radiation [13]) originated at motion of the electron beam in the vicinity of the periodic structure surface. Therefore, it is usually defined as the diffraction radiation mode. From the dispersion characteristics of the comb slow-wave systems it follows that those two separated modes can exist on different frequencies. In particular cases, the diffraction radiation mode exists on one of the higher frequency harmonics of the surface wave mode. This phenomenon is confirmed by both the data of multidimensional numerical simulation [14,15], and by experimental results obtained for the orotron [16]. Thus, there exists a possibility of simultaneous realization of excitation of the oscillations at the frequency of the surface wave mode with the correspondent electron beam modulation and multiplication of the said frequency at excitation of the volume waves with the electron beam, which is modulated by the surface wave field. This frequency multiplication technique with the purpose of obtaining terahertz band signals has an important advantage if compared to other techniques because of the fact that in this case the modulation of the electron beam occurs at a comparatively low frequency of the surface wave mode and, therefore, a relatively high value of the coupling resistance is secured. In this respect it is originated an actual problem of searching for ancillary opportunities for the efficiency enhancement in the two-mode beam-wave system and correspondent increasing of the output power of the terahertz band signals. Application of the inclined focusing field is one of the probable means for solving of the above problem because the approach like that provides for improvement of operational characteristics in both the surface and the volume wave modes.

Main particularities of beam-wave interaction in a combined device with the inclined focusing field in the case when the diffraction radiation mode is realized on the third harmonic of the surface wave mode are investigated in this paper on the basis of the non-linear multidimensional theory.

2. BASIC EQUATIONS

It is considered the oscillator-frequency multiplier on the basis of a standard open-resonator DRO. On the lower mirror there is a reflecting diffraction grating performing the function of a slow-wave system in the surface wave mode on the first harmonic of the BWO oscillations and on the n-th time harmonic in the diffraction radiation mode. The sheet beam, which is focused by a constant magnetic field, is directed over the grating surface. The induction vector of the focusing magnetic field $\vec{B}$ forms an angle χ with the plane of the slow-wave system. Block diagram of the device and the correspondent system of coordinates are provided in Fig. 1. It has to be noted that this technique matches an idealized case of an infinitely high value of the focusing fields induction, when the electrons of the beam move rectilinearly and in parallel to the vector $\vec{B}$. Under the actual conditions the trajectories of electrons are described by far more complicated curve shapes that must be considered at building of multidimensional simulations of the beam-wave interaction.

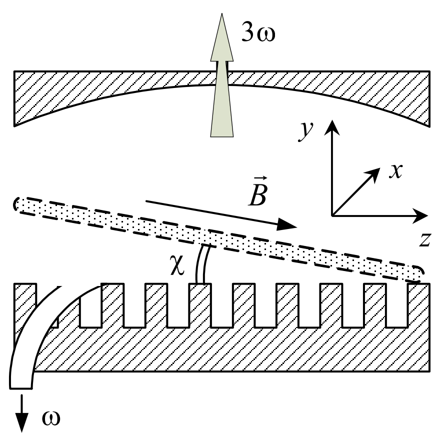

FIG. 1: Block diagram of the double-frequency oscillator

It is assumed that excitation of oscillations in the surface wave mode is performed at the base frequency ω. Usually, output of that signal is executed directly from the slow-wave system, as it is demonstrated in Fig. 1. Excitation of oscillations in the diffraction radiation mode occurs on the third harmonic of the base frequency that is secured by selection of the electromagnetic system parameters and the initial velocity of the electron beam. The device for output of the above signal at the volume wave is positioned on the upper mirror of the open resonator. Therefore, this system, in its essence, is a double-frequency oscillator.

To obtain a basic self-consistent system of equations for the beam-wave interaction, we apply standard for these types of devices assumptions on a sufficiently high Q-factor of the electromagnetic system and the opportunity of applying a single-wave approximation for each of the modes. A non-relativistic option of execution of the device is considered.

The equation of motion is represented by the Lorentz equation for a charged particle in the electromagnetic field:

$$\frac{d\vec{v}}{dt} = -\frac{|e|}{m}\left\{\vec{E} + \left[\vec{v}, \vec{B}\right]\right\}. \tag{1}$$

Here $\vec{v}$ is the electron velocity vector; e and m are the charge and the mass of the electron, respectively; $\vec{E}$ is the vector of the electric field intensity, which field includes the vortex high-frequency component and the potential component describing the spatial charge field. Within the limits of the 3D electron motion simulation the vector equation (1) is represented in the form of a system of three scalar equations with respect to the velocity coordinate components. To describe the electric $\vec{E}$ and magnetic $\vec{B}$ fields there are applied the longitudinal (axis Oz) and transverse (axis Oy) coordinate components with respect to the slow-wave system plane:

$$\vec{E} = \left(0, E_y, E_z\right); \ \vec{B} = \left(0, B_y, B_z\right). \tag{2}$$

Due to application of a non-relativistic approximation, it is considered only the static part of the magnetic field, which is used for focusing of the electron beam. The high-frequency component of the magnetic field is disregarded. For the inclined magnetostatic field the coordinate components of the induction vector $\vec{B}$ are represented as follows:

$$B_z = B_0 \cos\chi, \qquad B_y = B_0 \sin\chi, \tag{3}$$

where B_0 is the absolute value of the focusing field induction. The range between the lower edge of the electron beam at the input of the interaction space and the slow-wave system surface also plays an important role in this case. The charged particles trajectory shapes and, correspondingly, integral power and frequency characteristics of the beam-wave interaction for each of the modes are dependent upon that parameter.

The coordinate components of high-frequency electric intensity are determined from the solution to Helmholtz equation. For the surface wave field:

$$\tilde{E}_y = i\tilde{E}_z\,, \qquad \tilde{E}_z = C(t)f(z)\psi(y)\cos\left(\omega t - \beta z\right), \tag{4}$$

where $C(t)$ is the slowly variable in time complex field amplitude, which is determined at solving of the problem; $f(z)$ and $\psi(y)$ are the longitudinal and transversal amplitude enveloping curves of the high-frequency field; $\beta = \frac{\omega}{v}$ is the slow wave propagation constant; v is the slow wave phase velocity. It should be noted that at recording of the field in the volume waves mode we have to use one of higher harmonics of the basic frequency ω.

In the surface wave mode (BWO mode) the longitudinal enveloping curve of the field is described with the help of cosine dependence:

$$f_1(z) = \sqrt{2}\cos\left(\frac{\pi}{2}\frac{z}{L}\right), \tag{5}$$

where L is the length of the interaction space. In the diffraction radiation mode the shape of the enveloping curve is determined by the mode field structure of the open resonator and characterized by the Gaussian coordinate dependence:

$$f_2(z) = \left(w\sqrt{\frac{\pi}{2}}\right)^{-\frac{1}{2}} \exp\left[-\frac{\left(z - 0.5L\right)^2}{w^2}\right], \tag{6}$$

where w is the Gaussian beam field "spot" radius on the lower mirror of the open resonator (on the slow-wave system surface).

An abridged equation describing the oscillatory process dynamics in the high-Q electromagnetic system with a modulated electron beam is used as the equation of the electromagnetic field excitation. For the case of a two-component electromagnetic field the equation of excitation at the base frequency has the following form:

$$\frac{dC(t)}{dt} - i\left(\omega - \omega_s\right)C(t) = \frac{|I_0|}{2\pi N\Delta}\int\limits_0^L\int\limits_\Delta\int\limits_0^{2\pi}\left[E_z^* + \frac{dy}{dz}E_y^*\right]e^{i\omega t}\,d\omega t_0\,dy_0\,dz\,, \tag{7}$$

where $\omega_s = \omega_s' - i\frac{\omega_s'}{2Q}$ is the complex circular frequency of the resonator operation mode; Q is the loaded Q-factor of the resonator on that mode; N is the operation mode norm; Δ is the primary thickness of the electron beam; I_0 is the beam current constant component. Zero indices near integration variables in the expression (7) mean that integration is performed at the input of the interaction space. It should be noted that consideration of ohmic losses results in the necessity of correcting the expression for the real part of eigen frequency of the resonator operation mode [17,18]. Then, the expression for the complex eigen frequency must be put down as follows:

$$\omega_s = \omega_s' \left(1 - \frac{1+i}{2Q_s} \right).$$

The excitation equation (7) is obtained for the volume resonator; however, it is also used for investigation of excitation with the help of electron beams of high-Q open resonators where the continuous wave spectrum radiated from an open resonator can be neglected. Besides, it is noted in [19] that correlations of the theory of excitation for open and volume resonators formally match at the correspondent redistribution of the oscillations norm.

The equations (1) and (7) comprise the self-consistent system of beam-wave interaction equations put down using the Euler variables. The approach using the Lagrange variables is more suitable for investigation of the non-linear processes because it allows eliminating ambiguities in determining of the electron trajectory characteristics in the case when some charged particles overtake others. After transition to non-dimensional variables with respect to the basic frequency of the surface wave mode we obtain the following system of scalar equations of motion:

$$\frac{d^2\theta}{d\xi^2} = \left(1 + \frac{1}{\Phi}\frac{d\theta}{d\xi}\right)^3 \begin{Bmatrix} \frac{\Phi}{2}\left[F_1 f_1(\xi)\psi_1(Y)\cos(\alpha) + F_2 f_2(\xi)\psi_2(Y)\cos(n\alpha+\gamma)\right] + \\ E_{qz} - \Phi\beta_c^2 v_x \sin\chi \end{Bmatrix},$$

$$\frac{d^2 Y}{d\xi^2} = \left(1 + \frac{1}{\Phi}\frac{d\theta}{d\xi}\right)^2 \begin{Bmatrix} \begin{pmatrix} -\frac{\Phi}{4}\left[F_1 f_1(\xi)\psi_1(Y)\sin(\alpha) + F_2 f_2(\xi)\psi_2(Y)\sin(n\alpha+\gamma)\right] + \\ \frac{dY}{2d\xi}\left[F_1 f_1(\xi)\psi_1(Y)\cos(\alpha) + F_2 f_2(\xi)\psi_2(Y)\cos(n\alpha+\gamma)\right] \end{pmatrix} - \\ -\beta_c^2 v_x \left(\frac{\Phi}{2}\cos\chi + \frac{dY}{d\xi}\sin\chi\right) + \beta_q^2\left(Y - Y_b\right) \end{Bmatrix}, \quad (8)$$

$$v_x = \frac{2}{\Phi}\left(Y - Y_0\right)\cos\chi - \xi\sin\chi,$$

$$\theta = \omega t - \Phi\xi - \varphi, \quad \alpha = \theta + \Phi b\xi + \varphi.$$

Here $\Phi = \beta_e L$ is the static angle of electron transit through the interaction space with the length L; F_1 and F_2 are the amplitudes of BWO and DRO signals, which are linked to the complex amplitude via the correlation $C = F\exp(-i\gamma)$; $\beta_e = \frac{\omega}{v_0}$ is the electron wave number; $\xi = \frac{z}{L}$ and $Y = \frac{y}{H}$ are the normalized upon a correspondent scale longitudinal and transversal coordinates; n is the number of the base frequency harmonic (in this case $n = 3$); $b = 1 - \frac{v_0}{v}$ is the relative desynchronization between the primary longitudinal velocity of electrons v_0 and the phase velocity synchronous with the wave beam v; γ is the phase difference between BWO and DRO oscillations; $\beta_c = \Phi\frac{\omega_c}{\omega}$ is the cyclotron transit angle; $\omega_c = \frac{|e|B_0}{m}$ is the cyclotron frequency; $\beta_q = \Phi\frac{\omega_q}{\omega}$ is the plasma transit angle; $\omega_q = \sqrt{\frac{\rho|e|}{\varepsilon_0 m}}$ is the plasma frequency; ρ is the charge density; ε_0 is the dielectric constant; E_{qz} is the longitudinal dynamic component of the spatial charge field; Y_b is the transversal coordinate of the middle of the beam normalized upon the transversal scale. The transversal amplitude enveloping curves $\psi_1(Y)$ and $\psi_2(Y)$ are characterizing exponential decreasing of field amplitude of the slow waves at their moving away from the slow-wave system surface:

$$\psi_1(Y) = \exp(-kY), \quad \psi_2(Y) = \exp(-nkY),$$

where $k = \Phi(1-b)\frac{H}{L}$ is the normalized transversal wave number.

In the used model the longitudinal dynamic E_{qz} and the transversal static components of the spatial charge field are considered that is stipulated by a small value of the transversal dimension Δ of the sheet beam compared with the longitudinal dimension, which is approximately equal to the interaction space length.

The formally self-consistent system of equations contains two excitation equations: with respect to the BWO amplitude on the first harmonic of the base frequency ω and with respect to the DRO amplitude on one of higher harmonics $n\omega$. The BWO signal amplitude F_1 is determined from the solution to the excitation equation, in which the diffraction radiation mode signal is neglected. At an approach like that, retroaction of the signal on the third harmonic upon modulation of the electron beam by the surface wave field with the base frequency ω is considered to be rather weak. In this case the signal of the surface wave mode is actually an external parametric influence upon the oscillatory system of the diffraction radiation mode. There occurs a distributed upon the interaction space parametric modulation of the beam by the high-frequency BWO signal at the base frequency under availability of the third harmonic of the diffraction radiation. Then, using the definition of the average transconductance, which is well-known in the theory of oscillations, the system of excitation equations with respect to the amplitude and phase of oscillations in the diffraction radiation mode can be represented as follows:

$$\frac{dF_2}{d\tau} = F_2\left[GS_1\left(F_1, F_2, \gamma\right) - 1\right], \tag{9}$$

$$\frac{d\gamma}{d\tau} = -GS_2(F_1, F_2, \gamma) - \delta\omega\,, \tag{10}$$

where $\tau = \frac{\omega_s'}{2Q}t$ is the non-dimensional time; S_1 and S_2 are the real and imaginary parts of the average transconductance; $G = \frac{2|I_0|Q\,L^2}{\omega_s' N U_0}$ is the interaction efficiency parameter, which is actually characterizing the value of positive feedback in the self-sustained oscillation system; $\delta\omega = \frac{\left(\omega - \omega_s'\right)}{\omega_s'}2Q$ is the relative frequency mismatch. Using the accepted designations the expression for the complex average transconductance is put down as follows:

$$S(F_1, F_2, \gamma) = \frac{1}{2\pi F_2 \Delta}\int_0^1 f_2(\xi)\int_{Y_1}^{Y_2}\psi_2(Y)\int_0^{2\pi}\left(1 - i\frac{2}{\Phi}\frac{dY}{d\xi}\right)\exp[i(n\alpha + \gamma)]d\varphi dY_0 d\xi\,. \tag{11}$$

Here integration upon the transversal coordinate Y is performed within the boundaries of the electron beam at the input to the interaction space. If there occurs settling of a part of the electron beam upon the electromagnetic system surface, then it is performed correction of the integration limits Y_1 and Y_2 according to variation of geometrical dimensions of the beam.

The system of equations (8)-(10) has to be supplemented by the basic conditions determining characteristics of the electron beam at the input to the interaction space. It is assumed that premodulation of the beam upon the velocity and density is absent at the input to the system. Moreover the vector of initial velocity of electrons possesses only one coordinate component directed along the axis Oz. Then, the basic conditions for equations of motion acquire the following representation:

$$\theta\big|_{\xi=0} = \left.\frac{d\theta}{d\xi}\right|_{\xi=0} = \left.\frac{dY}{d\xi}\right|_{\xi=0} = 0\,, \qquad Y\big|_{\xi=0} = Y_0\,. \tag{12}$$

Here Y_0 is the transversal coordinate of the electron at the input to the interaction space. At calculation of trajectories of the charged particles it is used a layered model of the electron beam, in which it is assumed that the basic transversal coordinate of all the electrons is the same for each partial layer.

It is worth noting that in the real conditions the angular distribution of initial velocities of the electrons is determined mostly by lens effects of the electron gun anode hole. In this case there have to be considered static pulsations of the electron beam boundaries. For that purpose we have to use the following basic condition:

$$\left.\frac{\partial Y}{\partial \xi}\right|_{\xi=0} = \Phi\left(\frac{Y_0 - h_0}{3d}\right), \tag{13}$$

where h_0 is the transversal coordinate of the anode hole center; d is the range between the cathode and anode in the electron gun. Here it is considered that the absolute value of the anode lens focusing range (the slit-shaped aperture) is equal to $1.5d$ [20].

3. ANALYSIS OF THE RESULTS

Numeric calculations are performed on the basis of the solution to the system of integral-differential equations (8)-(10) for different values of amplitude of the BWO signal, which is considered in this case as an external parametric influence upon the oscillatory system, as well as of the incline angle value of the vector of magnetostatic field induction. The value of induction B_0 is characterized by the normalized cyclotron frequency, the value of which is accepted as equal to $\frac{\omega_c}{\omega} = 0.4$. At the same time, the developed model allows investigating peculiarities of physical processes in the beam-wave system for arbitrary values of the focusing field induction.

Application of the inclined focusing field (the angle of incline is χ) results in settling of electrons upon the slow-wave system surface. On the one hand, this phenomenon decreases the interaction efficiency due to decreasing of the number of charged particles in the interaction space, and on the other hand, under certain conditions, it might be accompanied by the enhancement of the device efficiency due to phase grading of electrons [21]. Besides, high-frequency layering of the electron beam due to a spatial heterogeneity of the slow-wave system field and the finite value of the beam thickness Δ is an important factor of variation of the interaction efficiency. To consider the above-mentioned mechanisms stipulated by multidimensionality of the interaction space, it is applied a multi-beam model of the charged particles beam. Five partial beams are considered in this paper.

Figure 2 provides the results of solving of the basic system of equations in a stationary case when in the excitation equations (9) and (10) the left-hand sides are equal to zero. The provided curves represent the lines of equal values of average amplitude transconductance $S_1(F_1, F_2, \gamma) = G^{-1}$ and indicate positioning of the points with the coordinates equal to stationary values of amplitude and phase of the oscillations on the third harmonic of the base frequency. The calculations were performed for the following values of the system parameters: $\Phi b = -2.1$; $F_1 = 0.1$; $\chi = 0$. It is evident that increasing of the interaction efficiency parameter G results in expansion of the range of values of the phase γ, in which there exist stationary values of amplitude of the oscillations F_2. Moreover, it is observed an increase of the value F_2 with forming of two branches of the solution to the stationary system of equations.

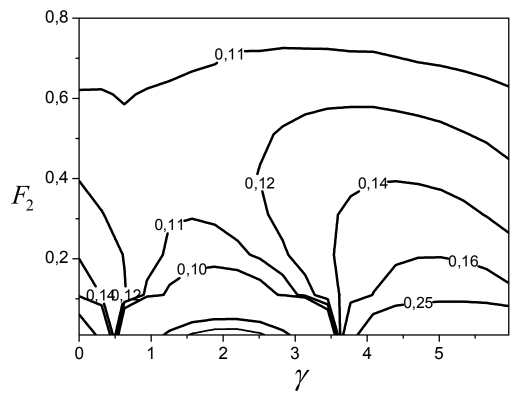

FIG. 2: Stationary values of amplitude and phase of the oscillations

The change in the focusing field angle inclination results in a substantial variation of distribution of stationary solutions on the plane (γ, F_2). This is illustrated in Fig. 3 where the calculation results are provided for different values of the angle χ (in angular minutes) and the amplitude F_1. The graphs match the values $F_1 = 0.1$ and 0.2 in Figs. 3(a) and 3(b) correspondingly ($G^{-1} = 0.25$). For a lower value of the BWO signal amplitude it is observed an essential increase of the amplitude of the diffraction radiation mode signal with increasing of the focusing field incline angle – variation of value of the angle χ from zero to $-20'$ is accompanied with increasing of the maximal value of F_2 by practically one order. Subsequent increasing of the angle inclination of focusing field results in decreasing of the interaction efficiency due to settling of a significant portion of the electron beam upon the slow-wave system surface. However, even for the value $\chi = -40'$, when settling of all the electrons of the beam is realized in the system, the output signal amplitude is exceeding the correspondent value for the case when $\chi = 0$ by several times.

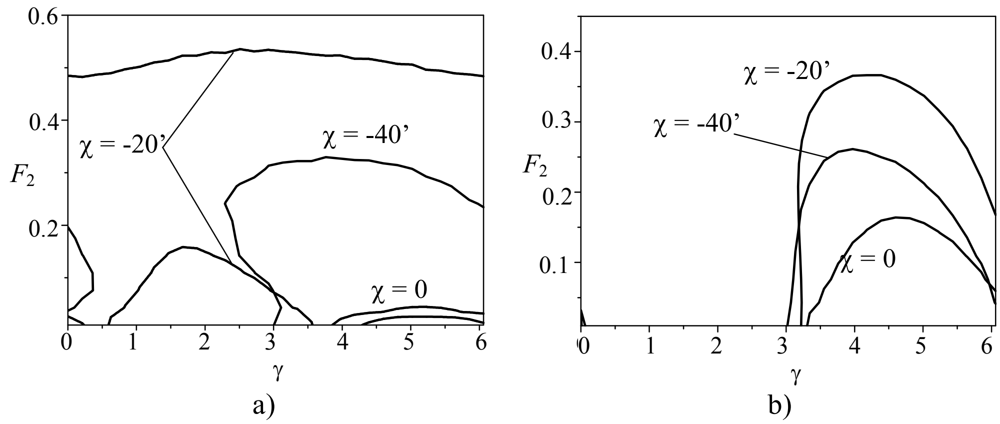

FIG. 3: Stationary values of amplitude and phase of the oscillations for different values of the focusing field inclination angle

Increasing of the value of the angle χ in Fig. 3(a) is also accompanied with variation of the operation mode of the externally influenced oscillatory system. For the case of a standard focusing ($\chi = 0$) the dependence $F_2(\gamma)$ is single-valued and the stationary solutions to the excitation equations exist within a limited range of values of the phase γ. This situation is typical for the regenerative amplification mode. For the value of $\chi = -20'$ it is realized the forced synchronization mode with two branches of solutions to the stationary excitation equations. In this case there are formed multiple-valued dependences $F_2(\gamma)$, and the stationary values of oscillation amplitudes in the diffraction radiation mode are observed for all of the possible values of the phase γ. Subsequent increasing of the focusing field incline angle χ results in realization of the forced synchronization mode with one branch of the stationary solutions, which is also characterized by multiple values of the amplitude-phase dependence.

A different situation is observed at increasing of the amplitude F_1 of the external signal (Fig. 3(b)). Here, for all of the considered values of the incline angle χ the stationary values of amplitude of the diffraction radiation mode oscillations exist within a limited range of phase values, which remains practically unchanged with the increase of χ. Moreover, all the dependences $F_2(\gamma)$ are single-valued. Therefore, in this case the oscillatory system is in the mode of regenerative amplification of the BWO signal for all of the considered values of the focusing field incline angle. It is also worth mentioning that in this case the inclined focusing does not provide for an essential efficiency enhancement on the third harmonic of the base frequency. This is explained by the fact that for the longitudinal amplitude distribution of the high-frequency field (5), which is typical for the surface wave mode, the inclined focusing is a less efficient means of enhancement of operational characteristics of the device as compared to the field enveloping curve (6) in the diffraction radiation mode [11,12]. In the backward surface wave mode the intensity maximum of the high-frequency field is formed in the vicinity of the gun-end of the slow-wave system, i.e., in the beginning of the interaction space. At the same time, the high-frequency current attains its maximum near the center of the slow-wave system or closely to the collector. Incline of trajectories of the electrons with respect to the slow-wave system surface provides for only a partial compensation of that spatial misalignment of the current and the field. In the diffraction radiation mode the field maximum of the open resonator operating mode is positioned in the center of the interaction space. At the expense of the focusing field incline we can provide for a spatial alignment of the high-frequency field and current maximums that stipulates an additional efficiency enhancement.

Stability of stationary values of the oscillation amplitude is very important for performing analysis of the non-autonomous oscillatory systems. This characteristic is significant for determining the widths of synchronization regions of the system, analysis of non-stationary processes, spectral characteristics etc. In this paper determining of stability of the obtained solutions is executed on the basis of application of the Routh-Hurwitz criterion that allows formulating the conditions for realization of both the amplitude and phase asymptotic stabilities [22] – in the system. This criterion provides for the following expressions:

$$G\left[\frac{\partial S_2}{\partial \gamma}-\frac{\partial S_1}{\partial F_2}\right]>GS_1-1, \tag{14}$$

$$GF_2\left[\frac{\partial S_1}{\partial \gamma}\frac{\partial S_2}{\partial F_2}-\frac{\partial S_1}{\partial F_2}\frac{\partial S_2}{\partial \gamma}\right]>\left(GS_1-1\right)\frac{\partial S_2}{\partial \gamma}. \tag{15}$$

The inequality (14) determines the amplitude stability condition, and (15) is the phase stability condition. The above expressions are of a rather general kind and allow performing analysis of solution on the entire plane $\left(\gamma, F_2\right)$. While performing investigation of stability of the stationary solutions to excitation equations under the conditions of (14) and (15), it is necessary to equal their right-hand sides to zero because the stationary value of the oscillation amplitude is found from the equation $GS_1-1=0$.

Figure 4 provides resonance characteristics of the non-autonomous oscillatory system for different values of the angle χ. The remaining parameters of the system are the same as for Fig. 2; $G^{-1}=0.22$. Solid curves correspond to stable values of the stationary oscillation amplitude, dashed curves – to the non-stable ones. In the case of a standard focusing ($\chi=0$) all the stationary values of DRO amplitude are stable and do not exceed the value $F_2=0.05$. Increasing of angle of inclination of the focusing field results in increasing of the amplitude F_2. For the value $\chi = -10'$ a practically fivefold increase of the maximal value of DRO amplitude is observed. In this case the system is in the mode of regenerative amplification of the BWO signal, same as for $\chi = 0$. A subsequent increase of the focusing field incline angle is accompanied, alongside with increasing of the amplitude F_2, with the transition to the forced synchronization mode and formation of the restricted synchronization region, within the limits of which the DRO signal frequency is exactly equal to the third harmonic of the basic frequency of external influence. The harmonic beating mode exists beyond the synchronization region. From the graphs in Fig. 4 it is evident that with increasing of the angle χ the synchronization region width decreases and it is formed a closed branch of the resonance characteristic. Besides, the shape of dependences $F_2(\delta\omega)$ in the forced synchronization mode is witnessing for the non-isochronism of the investigated non-linear oscillatory system.

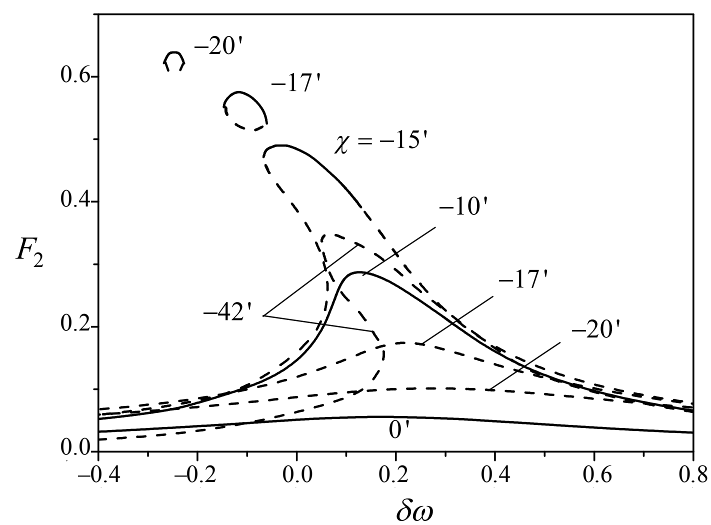

FIG. 4: Resonance characteristics for different values of the angle of inclination of focusing field

Maximal value of the DRO amplitude on the third harmonic of the base frequency is observed for the value $\chi = -20'$. In this case the synchronization region width is minimal and the system is maximally close to the autonomous mode. Increasing of the number of electrons settling on the slow-wave system surface results in decreasing of the amplitude F_2 and transition to the resonance characteristics with one branch of solutions to the stationary excitation equations. Figure 4 represents the characteristic for the value $\chi = -42'$, on which all the stationary values of the DRO oscillation amplitude are unstable.

Change in shape of the resonance curves with increasing of the focusing field incline angle to the value $\chi = -20'$ matches the peculiarities, which are typical for variation of power of the external influence upon a self-sustained oscillation system. Therefore, increasing of the angle χ is to a certain extent equivalent to decreasing of the BWO signal amplitude F_1. Thus, by means of varying focusing field incline angle we can adjust the extent of coupling between two modes of beam-wave interaction on surface and volume waves and provide for a significant efficiency enhancement of basic frequency multiplication.

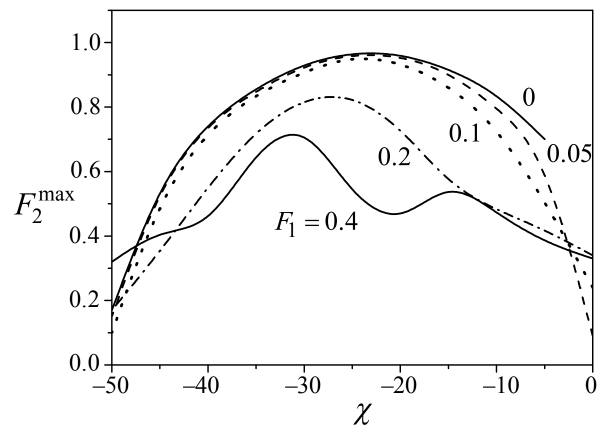

FIG. 5: Angular dependences of maximal signal amplitude in the diffraction radiation mode

Figure 5 illustrates the dependences of maximal amplitude of oscillations in the volume wave mode upon the focusing field incline angle for different values of amplitude of oscillations in the surface wave mode. The values of the incline angle are laid in angular minutes. Maximal value of the amplitude F_2 was selected upon the resonance characteristic for a fixed value of the interaction efficiency parameter ($G^{-1} = 0.15$). For all of the considered values of the amplitude F_1 the dependences $F_2^{max}(\chi)$ possess an extreme for a certain value of the angle χ, which corresponds to the maximal interaction efficiency. For relatively low values of the amplitude F_1 the angular dependences are non-essentially different from the case of a self-contained oscillation system ($F_1 = 0$). Increasing of the surface wave mode signal amplitude F_1 results in decreasing of signal amplitude on the third harmonic of the base frequency and in shifting of the angular dependences maximum towards increasing of absolute value of the incline angle. Besides, it is evident that with the increase of the amplitude F_1 the interaction efficiency becomes less critical to settling of the electron beam upon the slow-wave system surface. It is quite logical because in this case interaction with the field of the backward surface wave, the intensity of which is maximal in the beginning of the interaction space, is the mechanism determining the said phenomenon. In this case settling of electrons on the slow-wave system surface occurs primarily in the domain of low electromagnetic field intensity and exerts no significant influence upon the interaction efficiency.

4. CONCLUSIONS

Investigation of the influence of the inclined focusing upon the efficiency of power exchange between the sheet beam and high-frequency field on the third harmonic of the beam modulation frequency is performed within the framework of the non-linear multidimensional beam-wave interaction in the oscillator-frequency multiplier. In the surface wave mode like in the diffraction radiation mode propagation of electrons at an angle to the slow-wave system surface results in increasing of interaction efficiency at all stages of the oscillatory process primarily due to increasing of the coupling resistance.

Increasing of angle of inclination of the focusing field is accompanied by transition from the regenerative amplification mode to the forced synchronization of the DRO oscillatory system with an external parametric influence, which is stipulated by modulation of the electron beam with the backward surface wave signal. At increasing of power of the modulating signal, the regenerative amplification mode is realized for all of the considered values of the focusing field incline angle. Increased number of electrons settling upon the slow-wave system surface stipulates formation of the interaction efficiency maximum for a determined value of the incline angle and acts as the basic mechanism for decreasing of the output signal amplitude in the case of the inclined focusing.

Analysis of resonance characteristics demonstrated that under the conditions of a forced synchronization increasing of angle of inclination of the focusing field results in an increment of the signal amplitude on the third harmonic of the base frequency of the surface wave mode and narrowing of the synchronization region, within the limits of which stable values of the stationary amplitude of the diffraction radiation mode are affected. After attaining of the interaction efficiency maximum, subsequent incline of the beam with respect to the slow-wave system plane is accompanied by decreasing of the interaction efficiency and transition to single-valued resonance characteristics, for which all the values of stationary amplitude are unstable and correspond to the harmonic beatings.

Therefore, application of the inclined focusing field in the double-frequency oscillator-frequency multiplier allows increasing by several times the signal amplitude on the third harmonic of the basic frequency in a diffraction radiation mode and controlling spectral characteristics of the device at the expense of variation of a synchronization mode. Increasing of the signal power at higher harmonics of the surface wave mode in the double-frequency DRO is witnessing for the perspectives of application of inclined focusing of the electron beam for increasing of operation efficiency of electrovacuum sources of radiation in terahertz and subterahertz bands.

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