A Mathematical Model of Multipath QoS-based Routing in Multiservice Networks

Olexandr Lemeshko, Olga Drobot

Abstract – **In this paper, an approach to the mathematical modeling of multipath QoS-based routing in multiservice networks is considered. The Gallager's model was assumed as a basis of offered model with introduction of additional restrictions on quality of service. Tensor interpretation of model has allowed to receive in an analytical kind of a condition of maintenance of quality of service simultaneously on high-speed and time parameters QoS.**

Keywords – **Multiservice Network, Quality of Service, QoS-based Routing, Mathematical Model, Delay, Jitter.**

I. INTRODUCTION

Modern telecommunication networks develop aside creations of multiservice next generation networks (NGN). The requirement for timely delivery of digitized audio-visual information, multimedia applications raises new challenges for the NGN. Quality of Service (QoS) routing is an important component of such networks and has received considerable attention over the past decade. The goal of routing solutions is two-fold: satisfying the QoS requirements for every admitted connection and achieving the global efficiency in resource utilization. Shortest path routing can lead to unbalanced traffic distribution-links on frequently used shortest paths become increasingly congested, while other links are underloaded [1]. The multipath routing is proposed as an alternative to single shortest path routing to distribute load and alleviate congestion in the network. In other words, multipath routing uses multiple "good" paths instead of a single "best" path for routing. The base of any routing protocol is made with adequate mathematical model of a network. The Gallager's model of network has been chosen as base model [2].

II. BASIC MODEL OF NETWORK

Let the nodes of an m-node network be represented by the integers 1, 2, … m, and let a link from node i to node k be represented by (i, j). Let $L = (i, j)$: a link goes from i to k be the set of links. In order to discuss traffic flow, we distinguish link (i,k) from (k, j), but assume that if one exists the other does also [2]. Let $r_i(j) \geq 0$ be the expected traffic, in bits/s, entering the network at node i and destined for node j. We assume that this input traffic forms an ergodic process such as, for example, a Poisson process of message arrivals with a geometric distribution on message lengths. Let $t_i(j)$ be the total expected traffic (or node flow) at node i destined for node j. Thus $t_i(j)$ includes both $r_i(j)$ and the traffic from other nodes that is routed through i for destination j.

Finally let $\phi_{i,k}(j)$ be the fraction of the node flow $t_i(j)$ that is routed over link (i,k). We take $\phi_{i,k}(j) = 0$ for $(i,k) \notin L$ (i.e., no traffic is routed over nonexistent links). We also take $\phi_{i,k}(j) = 0$ for $i = j$ (i.e., traffic which has reached its destination is not sent back into the network). Since the node flow $t_i(j)$ at node i is the sum of the input traffic and the traffic routed to i from other nodes,

$$t_i(j) = r_i(j) + \sum_l t_l(j)\phi_{li}(j), \quad \forall i, j. \tag{1}$$

Equation (1) implicitly expresses the conservation of flow at each node; the expected traffic into a node for a given destination is equal to the expected traffic out of the node for that destination. Now let $f_{i,k}$ be the expected traffic, in bits/s, on link (i,k) (with $f_{i,k} = 0$ if $(i,k) \notin L$). Since $t_i(j)\phi_{i,k}(j)$ is the traffic destined for j on (i,k), we have

$$f_{i,k} = \sum_j t_i(j)\phi_{i,k}(j). \tag{2}$$

In what follows we refer to the set of expected inputs $r_i(j)$ as the input set r; the set of expected total node flows $t_i(j)$ as the node flow set t, the set of fractions $\phi_{i,k}(j)$ as the routing variable set ϕ, and the set of expected link traffics $f_{i,k}(j)$ as the link flow set f. We have seen for an arbitrary strategy of routing (subject to the existence of the expectations $t_i(j)$ and the conservation of flow) that r, t, ϕ and f all have meaning and satisfy (1) and (2) [2].

A routing variable set ϕ for an m-node network with links L is a set of nonnegative numbers $\phi_{i,k}(j)$ $1 \leq i,k,j \leq m$, satisfying the following conditions:

1) $\phi_{i,k}(j) = 0$ if $(i,k) \notin L$ or if $i = j$.

2) $\sum_k \phi_{i,k}(j) = 1$.

3) For each i, j $(i \neq j)$ there is a routing path from i to j, which means there is a sequence of nodes i, k, l, …, h, j such that $\phi_{i,k}(j) > 0$, $\phi_{k,l}(j) > 0$, …, $\phi_{h,j}(j) > 0$.

Calculation of routing variables is carried out by minimization of cost function, for example

$$D = \sum_{(i,k)} D_{i,k}(f_{i,k}), \tag{3}$$

where $D_{i,k}$ - cost of loading of link (i,k) by flow $f_{i,k}$.

The resulted model has been advanced by the formulation of conditions finding loop-free paths [3].

Olexandr Lemeshko – Kharkiv National University of Radioelectronics, UKRAINE, E-mail: avlem@ukr.net

Olga Drobot – Kharkiv Air Force University, Klochkovskaya Str., 228, Kharkiv, 61165, UKRAINE.

III. TENSOR MODELLING OF NETWORK

Mathematical model of a network (1-3) we shall present in tensor's variant for conclusion of QoS-constraints. One of main advantages of the network tensor analysis is the possibility of considerable simplification of solving problems in their initial statement at the expense of changing an aspect of subject consideration. In the tensor interpretation, changing coordinate systems (CS) for considering the tensor – a model of this subject – corresponds to this. Herewith dimensionality and order of a tensor depends on both configuration of the network under simulation and the number of parameters of user traffics being taken into consideration.

The structure of orthogonal network determines anisotropic space formed by network branches. Its dimensionality numerically equals the number of branches. Totality of independent paths in the network – closed (circuits) and open (nodal couples) ones – form a coordinate systems [4, 5], so these paths we will call as basic or coordinate. Conversion of the network structure with preservation of the number of branches or transition from one totality of independent circuits or nodal couples to another is interpreted as CS conversion.

In inserted space, a mathematical model of network can be presented as a mixed-dimensional geometric object

$$G = G^{(1)} \otimes \ldots \otimes G^{(z)} \otimes \ldots \otimes G^{(Z)}. \quad (4)$$

Whereas in terms of the tensor analysis of networks G is named as a multitensor. Multitensor components (4), in their turn, are defined in accordance with the following expression

$$G^{(z)} = \Lambda^{(z)}_{(1)} \otimes \ldots \otimes \Lambda^{(z)}_{(k)} \otimes \ldots \otimes \Lambda^{(z)}_{(K_{(z)})} \otimes \otimes T^{(z)}_{(1)} \otimes \ldots$$

$$\ldots \otimes T^{(z)}_{(k)} \otimes \ldots \otimes T^{(z)}_{(K_{(z)})} \otimes \Sigma^{(z)}_{(1)} \otimes \ldots \otimes \Sigma^{(z)}_{(k)} \otimes \ldots \otimes \Sigma^{(z)}_{(K_{(z)})}, \quad (5)$$

where $\otimes$ – is the sign of the tensor multiplication, $K_{(z)}$ – is the total number of information traffics circulating between z-th nodal couple of network. Expression (5) sets a mixed tensor $G^{(z)}$ of $3K_{(z)}$ order, $2K_{(z)}$ times covariant and $K_{(z)}$ times contravariant, whose components

$$\Lambda^{(z)}_{(k)} = \begin{bmatrix} \lambda^{1^{(z)}_k} \\ \lambda^{2^{(z)}_k} \\ \vdots \\ \lambda^{n^{(z)}_k} \end{bmatrix}^t, \quad T^{(z)}_{(k)} = \begin{bmatrix} \tau_{1^{(z)}_k} \\ \tau_{2^{(z)}_k} \\ \vdots \\ \tau_{n^{(z)}_k} \end{bmatrix}^t \text{ and } \Sigma^{(z)}_{(k)} = \begin{bmatrix} \sigma_{1^{(z)}_k} \\ \sigma_{2^{(z)}_k} \\ \vdots \\ \sigma_{n^{(z)}_k} \end{bmatrix}^t$$

represent one-order contravariant and covariant tensors, respectively, $[\,]^t$ is a transposition symbol. Coordinates $\lambda^{i^{(z)}_k}$ characterize a value of the information flow in load (flow) of network coordinate path by the k-th traffic flowing between the z-th nodal couple. This value is measured in traffic units, for example, in packets per second (s^{-1}). Coordinates $\tau_{i^{(z)}_k}$ and $\sigma_{i^{(z)}_k}$ reflect values of time delays and jitter (in s) of the same traffic in network basic paths.

For practice, characteristic is the case, when traffics of the z-th couple can be serviced by means of predetermined multitude of branches $n^{(z)}$ ($n^{(z)} \le n$), which determine the z-th $n^{(z)}$-dimensional subspace in the initial n-dimensional space. Network with a structure corresponding to the z-th subspace of the tensor $G^{(z)}$ consideration will be called as z-network. Thus, every tensor $G^{(z)}$ is given in its z-th subspace formed by corresponding branches of z-network, whereas indices i, j in expression (5), independently of one another, can take on all the values from 1 to $n^{(z)}$.

A functional side of the network under modelling can be reflected by a set of invariant equations of network behaviour [4]

$$\Lambda^{(z)}_{(k)} = L^{(z)}_{(k)} T^{(z)}_{(k)}, \; \Lambda^{(z)}_{(k)} = Y^{(z)}_{(k)} \Sigma^{(z)}_{(k)} \; (z = \overline{1,Z};\; k = \overline{1,K_{(z)}}), \quad (6)$$

where $L^{(z)}_{(k)}$ and $Y^{(z)}_{(k)}$ - is a twice contravariant tensors of the z-th subspace. In addition to this, tensor $L^{(z)}_{(k)}$ - contravariant metric tensor.

In practice, the choice of the necessary CS means search in frames of taken invariants for network with the simplest structure, which in terms of conceptual apparatus of the network tensor analysis is named as primitive [5]. Usually as primitive there is being chosen a network consisting of separate open branches [4, 5], what corresponds to a choice of the orthogonal CS of tensor (5) consideration, in which a matrix of the diagonal structure is a projection of the metric tensor.

Formulas (6) in CS of networks branches will be transformed to a kind

$$\Lambda^{(z)}_{(k)b} = L^{(z)}_{(k)b} T^{(z)}_{(k)b} \text{ and } \Lambda^{(z)}_{(k)b} = Y^{(z)}_{(k)b} \Sigma^{(z)}_{(k)b} \; (z = \overline{1,Z}; k = \overline{1,K_{(z)}}), \quad (7)$$

where $\Lambda^{(z)}_{(k)b}$, $T^{(z)}_{(k)b}$, $\Sigma^{(z)}_{(k)b}$, $L^{(z)}_{(k)b}$ and $Y^{(z)}_{(k)b}$ are projections of tensors $\Lambda^{(z)}_{(k)}$, $T^{(z)}_{(k)}$, $\Sigma^{(z)}_{(k)}$, $L^{(z)}_{(k)}$ and $Y^{(z)}_{(k)}$ in network branch CS.

An example, if any link of network is modelled as system with query such as $M/M/1$, then coordinates of matrixes $L^{(z)}_{(k)b}$ and $Y^{(z)}_{(k)b}$ (without indexes $^{(z)}_{k}$) -

$$l^{i,i}_b = \lambda^i_b(\mu_i - \lambda^i_b) \text{ and } y^{i,i}_b = \lambda^i_b(\mu_i - \lambda^i_b)^2 \; (i, j = \overline{1, n^{(z)}}).$$

In turn, the real network structure corresponds to CS of independent contours and nodal couples (CNC), wherein a system of invariant equations (6) takes a view

$$\Lambda^{(z)}_{(k)c.n.c} = L^{(z)}_{(k)c.n.c} T^{(z)}_{(k)c.n.c} \text{ and } \Lambda^{(z)}_{(k)c.n.c} = Y^{(z)}_{(k)c.n.c} \Sigma^{(z)}_{(k)c.n.c} \quad (8)$$

where $\Lambda^{(z)}_{(k)c.n.c}$, $T^{(z)}_{(k)c.n.c}$, $\Sigma^{(z)}_{(k)c.n.c}$, $L^{(z)}_{(k)c.n.c}$ and $Y^{(z)}_{(k)c.n.c}$ are projections of tensors $\Lambda^{(z)}_{(k)}$, $T^{(z)}_{(k)}$, $\Sigma^{(z)}_{(k)}$, $L^{(z)}_{(k)}$ and $Y^{(z)}_{(k)}$ in CNC coordinate system ($z = \overline{1,Z}$, $k = \overline{1,K_{(z)}}$).

Character of tensors entering expressions (6, 7, 8) is confirmed by laws of their coordinate conversion when passing from one CS to another [4]

$$\Lambda^{(z)}_{(k)b} = C_{(z)} \Lambda^{(z)}_{(k)c.n.c}; T^{(z)}_{(k)b} = A_{(z)} T^{(z)}_{(k)c.n.c}; \Sigma^{(z)}_{(k)b} = A_{(z)} \Sigma^{(z)}_{(k)c.n.c};$$

$$L^{(z)}_{(k)b} = C_{(z)} L^{(z)}_{(k)c.n.c} C^t_{(z)}; \; Y^{(z)}_{(k)b} = C_{(z)} Y^{(z)}_{(k)c.n.c} C^t_{(z)},$$

where matrices of coordinate conversion $C_{(z)}$ and $A_{(z)}$ are bound by the orthogonality condition $C^t_{(z)} A_{(z)} = I_{(z)}$, whereas $I_{(z)}$ is a $n^{(z)} \times n^{(z)}$ unit matrix.

IV. Formulation of QoS-Constraints

For the formulation of QoS-constraints in the frames of an orthogonal tensor model, it is convenient to present every equation of set (6) in a compound-tensor view [5]

$$\begin{bmatrix} \Lambda^{(z)}_{(k)c} \\ ---- \\ \Lambda^{(z)}_{(k)n.c} \end{bmatrix} = \begin{bmatrix} {}^{\langle 1\rangle}L^{(z)}_{(k)c.n.c} & | & {}^{\langle 2\rangle}L^{(z)}_{(k)c.n.c} \\ ------ & | & ------ \\ {}^{\langle 3\rangle}L^{(z)}_{(k)c.n.c} & | & {}^{\langle 4\rangle}L^{(z)}_{(k)c.n.c} \end{bmatrix} \begin{bmatrix} T^{(z)}_{(k)c} \\ ---- \\ T^{(z)}_{(k)n.c} \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} \Lambda^{(z)}_{(k)c} \\ ---- \\ \Lambda^{(z)}_{(k)n.c} \end{bmatrix} = \begin{bmatrix} {}^{\langle 1\rangle}Y^{(z)}_{(k)c.n.c} & | & {}^{\langle 2\rangle}Y^{(z)}_{(k)c.n.c} \\ ------ & | & ------ \\ {}^{\langle 3\rangle}Y^{(z)}_{(k)c.n.c} & | & {}^{\langle 4\rangle}Y^{(z)}_{(k)c.n.c} \end{bmatrix} \begin{bmatrix} \Sigma^{(z)}_{(k)c} \\ ---- \\ \Sigma^{(z)}_{(k)n.c} \end{bmatrix}, \quad (9)$$

where $\Lambda^{(z)}_{(k)c}$, $T^{(z)}_{(k)c}$ and $\Sigma^{(z)}_{(k)c}$ are contour summands, $\Lambda^{(z)}_{(k)n.c}$, $T^{(z)}_{(k)n.c}$ and $\Sigma^{(z)}_{(k)n.c}$ are nodal summands of vectors $\Lambda^{(z)}_{(k)c.n.c}$, $T^{(z)}_{(k)c.n.c}$ and $\Sigma^{(z)}_{(k)c.n.c}$, respectively, that is

$$\begin{bmatrix} \Lambda^{(z)}_{(k)c} \\ ---- \\ \Lambda^{(z)}_{(k)n.c} \end{bmatrix} = \Lambda^{(z)}_{(k)c.n.c}; \quad \begin{bmatrix} T^{(z)}_{(k)c} \\ ---- \\ T^{(z)}_{(k)n.c} \end{bmatrix} = T^{(z)}_{(k)c.n.c}; \quad \begin{bmatrix} \Sigma^{(z)}_{(k)c} \\ ---- \\ \Sigma^{(z)}_{(k)n.c} \end{bmatrix} = \Sigma^{(z)}_{(k)c.n.c},$$

$$\begin{bmatrix} {}^{\langle 1\rangle}L^{(z)}_{(k)c.n.c} & | & {}^{\langle 2\rangle}L^{(z)}_{(k)c.n.c} \\ ------ & | & ------ \\ {}^{\langle 3\rangle}L^{(z)}_{(k)c.n.c} & | & {}^{\langle 4\rangle}L^{(z)}_{(k)c.n.c} \end{bmatrix} = L^{(z)}_{(k)c.n.c};$$

$$\begin{bmatrix} {}^{\langle 1\rangle}Y^{(z)}_{(k)c.n.c} & | & {}^{\langle 2\rangle}Y^{(z)}_{(k)c.n.c} \\ ------ & | & ------ \\ {}^{\langle 3\rangle}Y^{(z)}_{(k)c.n.c} & | & {}^{\langle 4\rangle}Y^{(z)}_{(k)c.n.c} \end{bmatrix} = Y^{(z)}_{(k)c.n.c}.$$

Dimensionality of summands $\Lambda^{(z)}_{(k)c}$, $T^{(z)}_{(k)c}$ and $\Sigma^{(z)}_{(k)c}$ is equal to $w^{(z)}$, what corresponds to the number of independent contours in the network. Dimensionality of summands $\Lambda^{(z)}_{(k)n.c}$, $T^{(z)}_{(k)n.c}$ and $\Sigma^{(z)}_{(k)n.c}$ corresponding to a quantity of independent nodal couples in the z-network is equal to $s^{(z)}$.

The coordinates of vectors $T^{(z)}_{(k)c}$ and $\Sigma^{(z)}_{(k)c}$ are know and equal to zero, that to provide calculation of loop-free paths. Part of coordinates of vectors

$$T^{(z)}_{(k)n.c} = \begin{bmatrix} T^{(z)\langle 1\rangle}_{(k)n.c} \\ ---- \\ T^{(z)\langle 2\rangle}_{(k)n.c} \end{bmatrix} \text{ and } \Sigma^{(z)}_{(k)n.c} = \begin{bmatrix} \Sigma^{(z)\langle 1\rangle}_{(k)n.c} \\ ---- \\ \Sigma^{(z)\langle 2\rangle}_{(k)n.c} \end{bmatrix}$$

are unknown ($T^{(z)\langle 2\rangle}_{(k)n.c}$ and $\Sigma^{(z)\langle 2\rangle}_{(k)n.c}$), and a part are known ($T^{(z)\langle 1\rangle}_{(k)n.c}$, $\Sigma^{(z)\langle 1\rangle}_{(k)n.c}$). The coordinates of vectors $T^{(z)\langle 1\rangle}_{(k)n.c}$, $\Sigma^{(z)\langle 1\rangle}_{(k)n.c}$ are define a required delay and jitter. The coordinates of vectors $\Lambda^{(z)}_{(k)c}$ are unknown. The coordinates of vectors

$$\Lambda^{(z)}_{(k)n.c} = \begin{bmatrix} \Lambda^{(z)\langle 1\rangle}_{(k)n.c} \\ ---- \\ \Lambda^{(z)\langle 2\rangle}_{(k)n.c} \end{bmatrix}$$

are know. The coordinates of vectors $\Lambda^{(z)\langle 2\rangle}_{(k)n.c}$ are equal to zero, the coordinates of vectors $\Lambda^{(z)\langle 1\rangle}_{(k)n.c}$ are define a required value of k-th traffic circulating between z-th nodal couple.

From expressions (9) we have (indexes ${}^{(z)}_{(k)}$ are omitted) [4]

$$\Lambda^{\langle 1\rangle}_{n.c} \le \left(L^{\langle 4,1\rangle}_{c.n.c} - L^{\langle 4,2\rangle}_{c.n.c} \left[L^{\langle 4,4\rangle}_{c.n.c} \right]^{-1} L^{\langle 4,3\rangle}_{c.n.c} \right) T^{\langle 1\rangle}_{n.c}; \quad (10)$$

$$\Lambda^{\langle 1\rangle}_{n.c} \le \left(Y^{\langle 4,1\rangle}_{c.n.c} - Y^{\langle 4,2\rangle}_{c.n.c} \left[Y^{\langle 4,4\rangle}_{c.n.c} \right]^{-1} Y^{\langle 4,3\rangle}_{c.n.c} \right) \Sigma^{\langle 1\rangle}_{n.c}, \quad (11)$$

where

$$\left\| \begin{matrix} L^{\langle 4,1\rangle}_{c.n.c} & | & L^{\langle 4,2\rangle}_{c.n.c} \\ --- & + & --- \\ L^{\langle 4,3\rangle}_{c.n.c} & | & L^{\langle 4,4\rangle}_{c.n.c} \end{matrix} \right\| = L^{\langle 4\rangle}_{c.n.c}, \quad \left\| \begin{matrix} Y^{\langle 4,1\rangle}_{c.n.c} & | & Y^{\langle 4,2\rangle}_{c.n.c} \\ --- & + & --- \\ Y^{\langle 4,3\rangle}_{c.n.c} & | & Y^{\langle 4,4\rangle}_{c.n.c} \end{matrix} \right\| = Y^{\langle 4\rangle}_{c.n.c}.$$

The performance of conditions (10) and (11) guarantees the set parameters of quality of service - intensity of the traffic ($\Lambda^{(z)\langle 1\rangle}_{(k)n.c}$), an average delay ($T^{(z)\langle 1\rangle}_{(k)n.c}$) and jitter ($\Sigma^{(z)\langle 1\rangle}_{(k)n.c}$).

V. Conclusion

In the paper, a mathematical model of multipath QoS-based routing in multiservice networks is offered. This model develops the Gallager's flow-based model of routing with introduction of additional constraints on quality of service. The QoS-constraints are obtained with the use of opportunities of the mathematical apparatus of tensor analysis of networks. The average delay, jitter and intensity of the traffic acted in model as parameters of quality of service.

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