A Matrix Electrodynamics: A Similarity to the Heisenberg's Mechanics?

Alexander Gritsunov, Anna Veryovkina
Department of Electronic Engineering
Kharkiv National University of Radio Electronics
14 Lenin Ave., Kharkiv, 61166, Ukraine. E-mail: gritsunov@gmail.com; gritsunov@list.ru

Abstract: *A matrix approach to solving the electrodynamic problems is described. This specificity consists in the treatment of an electrodynamic system (ES) as an oscillating system with a finite number of the degrees of freedom. The ES is considered as a set of spatially localized partial oscillators (oscillets). Matrices of unit mutual pseudoenergies and unit mutual energies of the oscillators are evaluated. The ES eigenvalues, eigenfunctions and excited potentials can be calculated then basing on the lumped element circuit matrix theory. The main advantage of such approach is substitution of the partial derivative differential equations with the linear algebra problems and the ordinary differential equations.*

Keywords: electrodynamic system; electromagnetic potential; eigenvalue problem; lumped element circuit; linear algebra.

Introduction

Many computational methods exist in the electrodynamics, nevertheless there are no perfect ones, which are fit in the enough degree for simulating, e.g., UWB electromagnetic potentials in numerical models of the microwave devices. The most popular FDTD/FDFD and FETD/FEFD methods are based on the lowest-order interpolation schemes, so those are rather "extensive". As a result, new numerical methods appear occasionally [1], and will appear in the future too. Possibly, this is profitable to seek ideas for new methods in other divisions of science.

E.g., two approaches to calculating the wave function Ψ are known in the quantum mechanics: the Schrödinger's wave mechanics and the Heisenberg's matrix mechanics [2]. The first is based on direct solving PDEs (Klein-Gordon or Schrödinger's equation). The second works with a vector space produced as ensemble of all possible solutions of the same PDEs. If this space is predefined, the matrix mechanics can be reduced to the linear algebra problems and ODEs, which are less computationally intensive comparing with the direct numerical integration of PDEs.

The D'Alembert equation for the electromagnetic potential is a particular case of the Klein-Gordon equation. Therefore, in the classic electrodynamics, two approaches to calculating spatio-temporal distributions of the potential also may exist. Let us call their as a "wave" and a "matrix" electrodynamics. Note that well-known expansion of the potential in eigenfunctions of an electrodynamic system (ES) is a special case of the matrix electrodynamics.

Theoretical Part

A full matrix theory of ES is described in [3]. That is based on spatially localized so-called partial functions of the ES (partial oscillators, oscillets). If the spectrum of the generic potential $\mathfrak{A}$ [4] of the ES is finite in the wavenumber domain, this potential can be written as a finite series:

$$\mathfrak{A}(t,x,y,z) = \mathbf{u}_{\mathrm{p}}(t)\mathfrak{A}_{\mathrm{p}}(x,y,z),$$

which is a generalization of the Shannon-Whittaker series. The vector of N partial functions $\mathfrak{A}_{\mathrm{p}}$ is a nontrivial solution of so-called intervalues problem for the ES:

$$\nabla^2\mathfrak{A}_{\mathrm{p}} + \left[k_{\mathrm{p}}^2\right]\mathfrak{A}_{\mathrm{p}} = 0, \qquad (1)$$

which spatially localizes all oscillets (intervalues matrix $\left[k_{\mathrm{p}}^2\right]$ and other designations are explained in [3] and [4]).

From the physical point of view, the oscillet is a "cloud" of the electromagnetic potential oscillating as a single whole, i.e. in the same phase. Therefore, the matrix theory of a lumped element circuit with N degrees of freedom can be applied to the ES treating one as a 1D, 2D or 3D "lattice" of the coupled partial oscillators.

Five fundamental features of the oscillets are given in [3]. Each solution of (1) has those features. An inverse supposition can be made: any function having all those features might be one of solutions of (1). Such assumption is of practical significance. If some of N partial functions of the ES are known a priori, the number of unknown ones in (1) reduces respectively. E.g., the oscillators allocated far from the ES boundaries are known a priori, as similar to the free-space oscillets. The oscillators lying close to a flat metal surface can be easy found too using "mirror reflections".

First, matrices of unit mutual pseudoenergies $\left[\tilde{W}_{\mathrm{p}}\right]$ and unit mutual energies $\left[W_{\mathrm{p}}\right]$ of the oscillets are evaluated [3]. Then, the intervalues matrix can be calculated as $\left[k_{\mathrm{p}}^2\right] = \varepsilon_0\mu_0\left[W_{\mathrm{p}}\right]\left[\tilde{W}_{\mathrm{p}}\right]^{-1}$. Diagonalizing $\left[k_{\mathrm{p}}^2\right]$ with a form-matrix: $\left[k_{\mathrm{e}}^2\right] = [F]\left[k_{\mathrm{p}}^2\right][F]^{-1}$ the eigenvalues matrix is obtained. Finally, vector of N eigenfunctions of ES is found as $\mathfrak{A}_{\mathrm{e}}(x,y,z) = [F]\mathfrak{A}_{\mathrm{p}}$. A matrix ODE for the instantaneous values vector $\mathbf{u}_{\mathrm{p}}$ (a substitute for the D'Alembert PDE in the matrix electrodynamics) is given in [3].

Numerical Results

As an example, the eigenvalue problem for 1D oscillating systems is solved using the matrix approach. Two systems both having 1024 units in the length and divided into 1024 intervals are simulated: (i) with the periodical boundary condition (BC), $N = 32$; and (ii) with the Neumann's BC, $N = 33$. A finite oscillet (Fig. 1) having 256 units in the length is used in both cases. The oscillet is synthesized using the truncated Gaussian normalization [3]. The amplitude spectrum of one in the basis of complex eigenfunctions of the closed-loop system is in Fig. 2. Two variants of the oscillet location are examined: (i) regular with the distance of 32 units; and (ii) stochastic with a random deviation of each oscillet position no more than ±16 units from the "regular" location of one. The Neumann's BC for the boundary oscillets are simulated by superposition of "mirror reflections" of the outside fragments with further normalization of the peak values of the obtained sums.

The results of simulations of the systems (i) and (ii) are given in Tables 1 and 2 respectively. Structures of both tables are similar. Column 1 is the eigenvalue numbers *m*. Column 2 shows the rigorous (analytically calculated) eigenvalues. Columns 3 and 4 contain numerically evaluated eigenvalues for the regular and the stochastic oscillet dispositions respectively with the described above technique.

As it can be seen from the tables, the regular location of the oscillets ensures almost perfect evaluation of the lowest roots. The stochastic position also gives good results for these values. The highest eigenvalues are evaluated less reliably. However, even in this case, the maximal relative error does not exceed 3…4 percent, i.e. is small enough.

Conclusion

The matrix electrodynamics is an equipollent alternative to the classical "wave" electrodynamics. This shows considerable promise for the simulations of electromagnetic potentials in ESs of various microwave devices.

References

1. Ala, G., A. Spagnuolo, F. Viola, "An Advanced Gridless Method for Electromagnetic Transient Simulation," in *Proc. Int. Symp. Electromagnetic Compat. (EMC Europe 2004)*, Eindhoven, 2004, pp. 54-59.
2. Wichmann, E. H., *Quantum Physics* (Berkeley Physics Course, vol. 4). New York: McGraw-Hill, 1971.
3. Gritsunov, A. V., "Expansion of Nonstationary Electromagnetic Potentials into Partial Functions of Electrodynamic System," *Radioelectronics and Comm. Systems*, vol. 49, no. 7, pp. 6-12, 2006.
4. Gritsunov, A. V., "Methods of Calculation of Nonstationary Nonharmonic Fields in Guiding Electrodynamic Structures," *J. Comm. Technol. and Electronics*, vol. 52, no. 6, pp. 601-616, 2007.

Table 1

1	2	3	4
0	+0.0000000	+0.0000000	+0.0000004
1	+0.0000376	+0.0000377	+0.0000377
2	+0.0000376	+0.0000377	+0.0000383
3	+0.0001506	+0.0001506	+0.0001507
4	+0.0001506	+0.0001506	+0.0001510
5	+0.0003388	+0.0003388	+0.0003389
6	+0.0003388	+0.0003388	+0.0003391
…	…	...	...
27	+0.0073793	+0.0073789	+0.0073946
28	+0.0073793	+0.0073789	+0.0074361
29	+0.0084711	+0.0085766	+0.0085294
30	+0.0084711	+0.0085766	+0.0087103
31	+0.0096383	+0.0096353	+0.0096803

Table 2

1	2	3	4
0	+0.0000000	+0.0000000	+0.0000003
1	+0.0000094	+0.0000094	+0.0000096
2	+0.0000376	+0.0000377	+0.0000380
3	+0.0000847	+0.0000849	+0.0000852
4	+0.0001506	+0.0001509	+0.0001509
5	+0.0002353	+0.0002358	+0.0002358
6	+0.0003388	+0.0003395	+0.0003396
…	…	...	...
28	+0.0073793	+0.0073894	+0.0073958
29	+0.0079158	+0.0079417	+0.0079668
30	+0.0084711	+0.0085696	+0.0085492
31	+0.0090453	+0.0092720	+0.0093444
32	+0.0096383	+0.0096507	+0.0096766

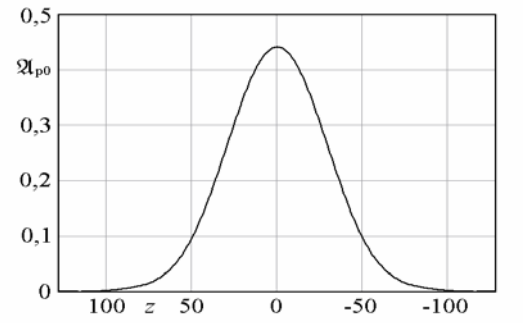

Figure 1

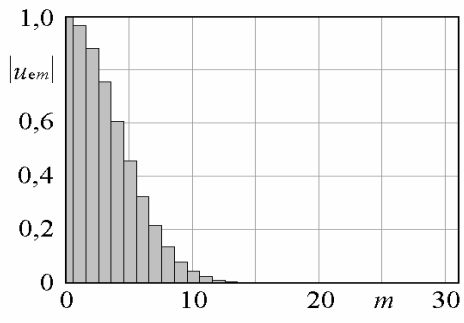

Figure 2