Dispersion Properties of a One-dimensional Anisotropic Magnetophotonic Crystal with a Gyrotropic Layer

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***Abstract*— In this work, we obtain the dispersion relation for magnetophotonic one-dimensional crystal with a gyrotropic plasma layer in the analytical form. The numerical analysis of the dispersion relation for waves in the crystal at different parameters of the effective permittivity of the plasma layer is carried out. We predict the propagation of bulk fast and slow waves in such structures. The transmission and forbidden bands in the dispersion diagram for the bulk waves are presented in the area for surface plasma waves.**

Keywords—magnetophotonic one-dimensional crystals; gyrotropic media; dispersion characteristics; transmission and forbidden bands

I. INTRODUCTION

Photonic crystals are widely used in different applications of modern science. The peculiarities of electromagnetic fields propagation in such structures are completely defined by the geometric sizes of the layers and by the frequency dependence of their material parameters. The properties of isotropic photonic crystals are well studied on the basis of obtained analytical dispersion equations both for TE and TM waves [1, 2]. One of the most promising applications is related to the magnetophotonic crystal with the gyrotropy of one of the structure layers. For such structures, there is no analytical dispersion equation that could allow effective studying of their basic properties. Due to the gyrotropy, the material parameters are tensor quantities, that complexify significantly their analysis. Depending on the direction of the applied DC magnetic field, different phenomena can be observed in such gyromagnetic media, such as the Faraday effect, magnetic birefringence, rotation of the polarization plane, nonreciprocal effects for the direct and reverse waves, existence of the surface waves.

Here we study the a one-dimensional magnetophotonic crystal in the presence of the external transvers DC magnetic field $\vec{H}_0 = \vec{z}_0 H_0$. One of the layers in each period of the crystal is a semiconductor which permittivity is a tensor, while the other layer is a magnetodielectric.

II. MAIN PART

The permittivity of the gyrotropic plasma in the semiconducting layer with the thickness a is a tensor $\vec{\varepsilon}$ of the standard type [3] (the magnetic field $\vec{H}_0$ is directed along the Oz axis).

$$\vec{\varepsilon} = \begin{pmatrix} \varepsilon_g & -i\varepsilon_a & 0 \\ i\varepsilon_a & \varepsilon_g & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

We study the waves with H_z polarization. The Helmholtz equation for the chosen polarization (TM waves) in two-dimensional approximation $\left(\frac{\partial}{\partial z} = 0\right)$ takes the following form:

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k^2 \varepsilon_\perp \mu_g H_z = 0, \tag{2}$$

where $\varepsilon_\perp = \varepsilon_g \left(1 - \frac{\varepsilon_a^2}{\varepsilon_g^2}\right)$ is effective permittivity of plasma, μ_g is the permeability of plasma. The magnetodielectric isotropic layer of thickness b has constant permittivity and

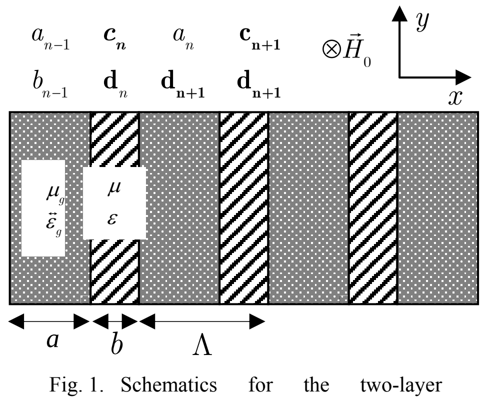

Fig. 1. Schematics for the two-layer magnetophotonic crystal.

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permeability parameters $\varepsilon,\ \mu$ (Fig. 1).

Solution of the Helmholtz equation (2) for two layers of the periodical structure can be presented in the following form:

$$H_z^1(x,y)=\left(a_n e^{i\xi_1(x-n\Lambda)}+b_n e^{-i\xi_1(x-n\Lambda)}\right)e^{i\beta y}, b<x-n\Lambda<\Lambda$$

$$E_y^1(x,y)=\frac{1}{k\varepsilon_\perp}\left(\begin{array}{l} a_n\left(\xi_1+i\dfrac{\varepsilon_a}{\varepsilon_g}\beta\right)e^{i\xi_1(x-n\Lambda)}- \\ -b_n\left(\xi_1-i\dfrac{\varepsilon_a}{\varepsilon_g}\beta\right)e^{-i\xi_1(x-n\Lambda)}\end{array}\right)e^{i\beta y},$$

$$H_z^2(x,y)=\left(c_n e^{i\xi_2(x-n\Lambda)}+d_n e^{-i\xi_2(x-n\Lambda)}\right)e^{i\beta y},\quad 0<x-n\Lambda<b$$

$$E_y^2(x,y)=\frac{\xi_2}{k\varepsilon}\left(a_n e^{i\xi_2(x-n\Lambda)}-b_n e^{-i\xi_2(x-n\Lambda)}\right)e^{i\beta y}.$$

Here n is the cell number, $\xi_2=\sqrt{k^2\varepsilon\mu-\beta^2}$ and $\xi_1=\sqrt{k^2\varepsilon_\perp\mu_g-\beta^2}$ are the transverse wave numbers along the Ox axis, β is the longitudinal wave number along the layers, a_n, b_n, c_n, d_n are the unknown amplitudes of waves in the layers.

Using boundary conditions for the tangential components of the electromagnetic field on the interfaces of layers and the Bloch-Floquet theorem for periodic systems, we obtain the characteristic equation:

$$e^{-iK\Lambda}=\frac{1}{2}(A+D)\pm\left\{\left[\frac{1}{2}(A+D)\right]^2-1\right\}^{\frac{1}{2}}, \qquad (3)$$

where

$$A=\left\{\cos\xi_2 b-\frac{i\varepsilon}{2\varepsilon_\perp}\left[\frac{\xi_1}{\xi_2}+\frac{\xi_2}{\xi_1}\frac{\varepsilon_\perp^2}{\varepsilon^2}+\frac{\beta^2}{\xi_1\xi_2}\frac{\varepsilon_a^2}{\varepsilon_g^2}\right]\sin\xi_2 b\right\}e^{-i\xi_1 a},$$

$$D=\left\{\cos\xi_2 b+\frac{i\varepsilon}{2\varepsilon_\perp}\left[\frac{\xi_1}{\xi_2}+\frac{\xi_2}{\xi_1}\frac{\varepsilon_\perp^2}{\varepsilon^2}+\frac{\beta^2}{\xi_1\xi_2}\frac{\varepsilon_a^2}{\varepsilon_g^2}\right]\sin\xi_2 b\right\}e^{i\xi_1 a}.$$

Using Eq. (3) we can analytically derive the equation for the Bloch wave number $K(k,\beta)$

$$K(k,\beta)=\frac{1}{\Lambda}\arccos\frac{1}{2}(A+D)=$$
$$=\frac{1}{\Lambda}\arccos\left\{\begin{array}{l}\cos\xi_1 a\cos\xi_2 b- \\ -\dfrac{\varepsilon}{2\varepsilon_\perp}\left[\dfrac{\xi_1}{\xi_2}+\dfrac{\xi_2}{\xi_1}\dfrac{\varepsilon_\perp^2}{\varepsilon^2}+\dfrac{\beta^2}{\xi_1\xi_2}\dfrac{\varepsilon_a^2}{\varepsilon_g^2}\right]\sin\xi_1 a\sin\xi_2 b\end{array}\right\} \qquad (4)$$

It worth noticing that, for photonic crystal with two dielectric layers in each period, i.e., for $\varepsilon_a=0,\ \varepsilon_\perp=\varepsilon,\ \mu=1$, Eqs. (3) and (4) reduce to well known formulas [1, 2].

III. Analysis of the Results

As an example, Figs. 2 and 3 show the dispersion diagrams calculated using Eqs. (3) and (4). The diagrams are plotted as a result of numerical computations of the projection of $K(k,\beta)$ onto the surface k,β in the three-dimensional space of wave numbers. In the figures, the shaded regions represent the transmission bands, while the unshaded regions show the forbidden bands. To study the peculiarities of waves propagation in such structures, one should distinguish two cases for different values of the magnetic field H_0 (ε_a parameter), when $\varepsilon_\perp>0$ and $\varepsilon_\perp<0$. The analysis shows that the number of transmission and forbidden bands depends on the permittivity of each layer. There are two types of dispersion, which correspond to fast and slow (in comparison to the speed of light in the medium) waves. The fast waves exist in the regions $k^2\varepsilon\mu>\beta^2$ and $k^2\varepsilon_\perp\mu_g>\beta^2$ (in Figs. 2 and 3, these regions are confined by the lines $\beta=k\sqrt{\varepsilon\mu}$ and $\beta=k\sqrt{\varepsilon_\perp\mu_g}$) for corresponding layer, while slow waves are in the region where these inequalities have the opposite sign. Different combinations of these regimes are possible. The gyrotropy in one of the layers allows one to control the band width, its disposition, and the number of bands in the given frequency range by changing the value of the DC magnetic field. If $\varepsilon_\perp<0$, the delayed surface wave exists, that propagates along the interface between two neighboring layers of crystal. We emphasize the very interesting feature of the considering waves. Namely, from the solution of the dispersion equation (4) and the spatial distribution of fields, we conclude that the direct ($\beta=+Re|\beta|$) and reverse ($\beta=-Re|\beta|$) waves, which propagate along the layers (along Oy axis), have equal modulus of the speed, but different transverse structure of the fields (along the Ox axis). Some numerical computations of the dispersion are presented in Fig. 2 and Fig. 3.

The dashed line $k=\beta$ in the Fig. 2 corresponds to the condition, when the phase velocity of the wave is equal to the speed of light in the free space. The region where the fast bulk waves exist is above this line, while the region where the slow waves exist is below this line. Fig. 2 and Fig. 3 are plotted for

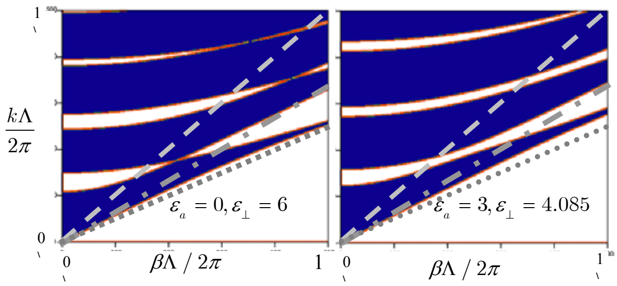

Fig. 2. Transmission (shaded) and forbidden (unshaded) bands in the plane (k, β) for $\varepsilon_\perp>0$

the following parameters: $a = b = 0.5$, $\mu = \mu_g = 1$, $\varepsilon = 2$, $\varepsilon_g = 6$, dash-dotted and dotted lines are defined by the relations $\beta = k\sqrt{2}$ and $\beta = k\sqrt{6}$, respectively. Fig. 2 corresponds to the positive values of the effective permittivity, $\varepsilon_\perp > 0$, of the plasma layer, while Fig. 3 corresponds to the negative values, $\varepsilon_\perp < 0$. The first panel in Fig. 2 corresponds to the absence of the gyrotropy in the layers ($\varepsilon_a = 0$). The lines $k\sqrt{2} = \beta$ and $k\sqrt{6} = \beta$ define the phase velocities of the waves in two dielectric layers with $\varepsilon = 2$ and $\varepsilon_g = 6$. Both bulk and surface waves can be excited in the dielectric layers of the periodical structure. This case is described in detail in the works [1, 2]. For the crystal with the plasma layer (panel 2, $\varepsilon_a = 3$), the effective permittivity of the plasma layer $\varepsilon_\perp$ decreases when increasing β, which leads to the shrinking of the transmission bands with the simultaneous enlarging of the forbidden ones.

The situation is different for the negative values of the effective permittivity $\varepsilon_\perp < 0$ (Fig. 3). Fig. 3 shows two fragments, which correspond to two different values of the effective permittivity ε_a of the gyrotropic layer. In the first case, $\varepsilon_a = 6.6$. There is only one transmission band. In the different regions of the transmission band, divided by the line $k = \beta$, the fast and slow bulk waves can exist. In the region

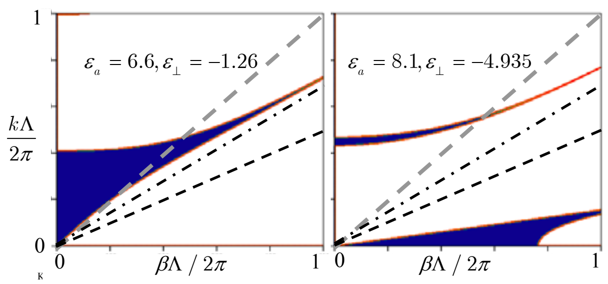

Fig. 3. Transmission (shaded) and forbidden (unshaded) bands in the plane (k, β) for $\varepsilon_\perp < 0$

where the slow waves exist, their phase velocities are greater than the phase velocity $v = \beta / \left(k\sqrt{2}\right)$ in the dielectric layer (see the dash-dotted line).

Increasing the DC magnetic field H_0 (increasing ε_a), the second transmission band appears. One of the transmission bands corresponds to the bulk waves (fast or slow), the other one corresponds to the surface slow waves (both the transmission bands are at $v < \beta / k\sqrt{6}$). The field distribution of surface waves has the maximal amplitude on the interface between the regions. The analysis of the spatial field distributions over the Ox axis (see Eq. (4)) for the direct, $\beta = +|\beta|$, and the reverse, $\beta = -|\beta|$, surface waves shows that their field structures are different. As is seen from Fig. 3, when increasing the parameter ε_a, the region where the surface plasma waves exist is shrinking with the simultaneous shrinking of the region where the bulk waves can propagate.

IV. Conclusion

In this work we are solved the problem for the eigenwaves in the two-layer infinite periodic structure, which consists of dielectric layers and gyrotropic plasma layers. We are obtained analytically the elements of the transmission matrix taking into account the anisotropy of the gyrotropic plasma layer. The analysis of the transmission and forbidden bands in the dispersion diagram is carried out for different material parameters of the layers taking into consideration the gyrotropy. We are shown that the transmission bands for the surface plasma waves can exist. In addition, we are defined the dependence of the band width on the gyrotropy parameter of the plasma layer.

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