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# SYNTHESIS OF MOORE FSM WITH EXPANDED OF CODING SPACE FOR TELECOMMUNICATION SYSTEMS

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*В статті запропоновано метод оптимізації логічної схеми мікропрограмного автомату Мура, який орієнтований на використання замовних матриць. Метод заснований на використанні більшого ніж мінімальна кількість змінних в кодах внутрішніх станів автомату Мура.*

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*The proposed method is targeted on reduction of hardware amount in logic circuit of Moore finite-state machine implemented with customized matrices. The method is based on using more than minimal amount of variables in codes of FSM internal states.*

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*В статье предложен метод оптимизации логической схемы микропрограммного автомата Мура, который ориентирован на использование заказных матриц. Метод основан на использовании большего, чем минимальное количество переменных в кодах внутренних состояний автомата Мура.*

## Introduction

The model of Moore finite state machine (FSM) [1] is often used in telecommunication systems [2]. This model can be used as for representing interrelations and behavior of some parts of a system. Also it can be used for implementing control units used inside a system [3, 4]. The development of microelectronics has led to appearance of different programmable logic devices [3], which are used for implementing FSM logic circuit. But in the case of mass production of microelectronics products, they widely use so called customized circuits called ASIC (Application-Specified Integrated Circuits) [5]. In the case of ASIC, the logic circuits of FSM are designed, as a rule, using so called matrix structures. In these customized matrices, the principle of distributed logic is used [6]. This article discusses some problems of Moore FSM's implementing targets minimizing the area of ASIC used for a control unit of telecommunication system.

One of the important problems of FSM synthesis with matrix structures is the decrease in chip space occupied by FSM logic circuit. One of the ways to solve this problem is optimal coding of FSM internal states [2]. However, this approach does not allow optimization of the output signals generation part of FSM circuit.

In this work the optimization method is proposed. It is based on encoding of state codes using more than minimal amount of variables. These codes can be rearranged to optimize the matrix area for the part of the circuit implementing the system of microoperations. Such an approach allows reducing of hardware amount in FSM circuits and does not lead to speed loss. The reducing of hardware is connected with compressing of the FSM table of transitions. The fewer rows this table includes, the less amount of terms the im-

plemented systems of Boolean functions includes, too. In this article a control algorithm to be implemented is represented by a graph-scheme of algorithm (GSA) [1].

## I. The general aspects and basic idea of a proposed method

Let Moore FSM be represented by the structure table (ST) with columns [1]: $a_m$, $K(a_m)$, $a_s$, $K(a_s)$, $X_h$, $\Phi_h$, $h$. Here $a_m$ is an initial state of FSM; $K(a_m)$ is a code of state $a_m \in A$ having the capacity $R = \lceil \log_2 M \rceil$ (to code the states the internal variables from the set $T = \{T_1,..,T_R\}$ are used); $a_s$, $K(a_s)$, are a state of transition and its code respectively; $X_h$ is an input, which determines the transition $\langle a_m,\ a_s \rangle$, and it is equal to conjunction of some elements (or their complements) of a set of logic conditions $X = \{x_1,..,x_L\}$; $\Phi_h$ is a set of input memory functions for flip-flops of FSM memory, which are equal to 1 to change the content of the memory from $K(a_m)$ to $K(a_s)$, $\Phi_h \subseteq \Phi = \{\varphi_1,..,\varphi_R\}$; $h = 1,\ ...,\ H$ is a number of transition. In the column $a_m$ a set of microoperations $Y_q$ is written, these microoperations are generated in the state $a_m \in A$ ( $Y_q \subseteq Y = \{y_1,..,y_N\}$, $q = 1,...,Q$ ). This table is a basis to form the following systems of functions

$$\Phi = \Phi(T, X),\tag{1}$$

$$Y = Y(T).\tag{2}$$

These systems specify an FSM logic circuit. In the case of matrix implementation, systems (1)-(2) determine the model of Moore FSM $U_1$, shown in Fig.1.
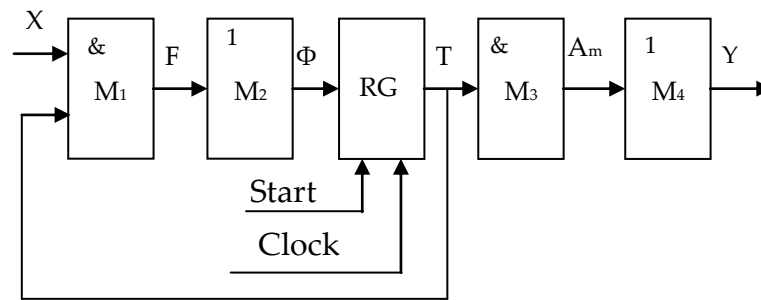


*Fig.* 1. Matrix circuit of Moore FSM $U_1$

In the FSM $U_1$, a conjunctive matrix $M_1$ implements the system of terms $F = \{F_1,...,F_H\}$; a disjunctive matrix $M_2$ implements system (1); a conjunctive matrix $M_3$ implements terms $A_m$ ($m = 1,...,M$), corresponding to FSM states; a disjunctive matrix $M_4$ implements system (2). The register RG keeps state codes; it is controlled by signals Start (clearing) and Clock (changing content depending on functions $\Phi$ ). The matrices $M_1$ and $M_2$ determine the block of input memory functions (BIM), whereas the matrices $M_3$ and $M_4$ determine the block of microoperations (BMO). The method of optimal state encoding [2] may be used for reducing the number of terms in system (1) up to $H_0$. Here the

symbol $H_0$ stands for the number of transitions for the equivalent Mealy FSM. An area of BMO may be decreased due to refined state encoding [2]. As an extreme solution, it permits to specify each microoperation $y_n \in Y$ by a single term of $M_3$ and the matrix $M_4$ is absent. Unfortunately, the mentioned methods cannot be applied simultaneously. It means that it is possible to decrease either a chip area for BIM, or for BMO. In this article we propose a method, which allows optimization for both blocks, BIM and BMO.

One of Moore FSM features is existence of pseudoequivalent states [2], which are the states with the same transitions by the effect of the same inputs. Such states correspond to the control algorithm operator vertices [1], outputs of which are connected with an input of the same vertex.

Let $\Pi_A = \{B_1,.., B_I\}$ be a partition of a set $A$ on classes of pseudoequivalent states. Let us code classes $B_i \in \Pi_A$ by binary codes $K(B_i)$ having $R_B$ bits, where

$$R_B = \lceil \log_2 I \rceil. \tag{3}$$

In the general case the following condition takes place:

$$R_B < R_A. \tag{4}$$

In this article we propose to represent the states by binary codes $K(a_m)$ in such a way that it allows represent the codes $K(B_i)$ of classes $B_i \in \Pi_A$ using variables $T_r \in T' \subset T$. It results in Moore FSM $U_2$. The structures of both $U_1$ and $U_2$ are the same. But in $U_2$ the block BIM implements functions

$$\Phi = \Phi(T', X). \tag{6}$$

Such an approach guarantees the reduction of the number of terms in system $\Phi$ up $H_0$. It leads to reducing area of matrix $M_1$, as well as the number of inputs for matrix $M_2$. Unfortunately, this approach does not permit terms reduction in system (2).

Let us represent the coding space as a Karnaugh map having $I_K$ columns, where

$$I_K = 2^{R_B}. \tag{7}$$

Let $|B_i| = m_i$ and $M_0 = \max(m_1,..., m_I)$, then the map should have

$$O_K = 2^{R_O} \tag{8}$$

rows, where

$$R_O = \lceil \log_2 M_O \rceil. \tag{9}$$

If condition (8) is true, then the states for any class $B_i \in \Pi_A$ are placed in the same column of the map and the code $K(B_i)$ is determined by the values of variables marking this column. If the following condition

$$R_B + R_O > R_A \tag{10}$$

takes place, then the expansion of coding space occurs.

The value of $R_0$ can be decreased if the following is condition true:

$$I_K > I. \tag{11}$$

In this case up to $(I_K - I)$ classes $B_i$ can be placed in adjacent cells of the map. Let us discuss the following example. Let the following partition $\Pi_A = \{B_1, ..., B_5\}$ be obtained for some FSM $S_1$: $B_1 = \{a_1\}$, $B_2 = \{a_2, a_3, a_4, a_5, a_6\}$, $B_3 = \{a_7, a_8, a_9, a_{10}\}$, $B_4 = \{a_{11}, a_{12}, a_{13}, a_{14}, a_{15}\}$ and $B_5 = \{a_{16}, a_{17}\}$. Now, we have $R_B = 3$, $M_O = 5$ and $R_o = 3$. Therefore, it is enough $R_B + R_O = 6$ variables to encode the states. But the map includes $I_K = 8$ columns and condition (11) is true. Obviously, those classes $B_i \in \Pi_A$ should be transformed having the maximum number of states. Let us represent the class $B_2$ as $B_2^1 = \{a_2, ..., a_5\}$ and $B_2^2 = \{a_6\}$, whereas the class $B_4$ as $B_4^1 = \{a_{11}, ..., a_{14}\}$ and $B_4^2 = \{a_{15}\}$. Now there are $M_O = 4$, $R_O = 2$ and variables $T_1, ..., T_5$ are used for state encoding (Fig. 2). As follows from Fig. 2, we have one code for each class $B_i \in \Pi_A$, namely: $K(B_1) = {*}00$, $K(B_2) = 0{*}1$, $K(B_3) = 010$, $K(B_4) = 11{*}$, $K(B_5) = 10{*}$. In this cased there is $R_A = 5$, so there is no code space expansion.

| $T_4T_5$ \ $T_1T_2T_3$ | 000 | 001 | 011 | 010 | 110 | 111 | 101 | 100 |
|---|---|---|---|---|---|---|---|---|
| 00 | $a_1$ | $a_2$ | $a_6$ | $a_7$ | $a_{11}$ | $a_{15}$ | $a_{16}$ | * |
| 01 | * | $a_3$ | * | $a_8$ | $a_{12}$ | * | $a_{17}$ | * |
| 11 | * | $a_4$ | * | $a_9$ | $a_{13}$ | * | * | * |
| 10 | * | $a_5$ | * | $a_{10}$ | $a_{14}$ | * | * | * |

*Fig.* 2. The codes of states for Moore FSM $S_1$

## II. Proposed design method

In this article we propose the following design method for Moore FSM $U_2$:

1. Construction of the marked GSA and the set of states $A$. There are no difficulties here, because the well-known approach [1] is used.

2. Construction of the classes of pseudoequivalent states. States $a_i, a_j \in A$ are called pseudoequivalent states if the vertices marked by them are connected with input of the same vertex of GSA $\Gamma$ [2]. The set $\Pi_A$ is formed in a trivial way using this definition.

3. Prime state encoding. This step is connected with finding values of $I_K$ and $O_K$. Here the possibility of decreasing $R_O$ under keeping $R_B$ is analyzed. This step is finished by placement of states $a_m \in B_i$ inside the columns of Karnaugh map having size $I_K * O_K$. It allows finding the set $T' \subset T$ and codes $K(B_i)$.

4. Secondary state encoding. This step is connected with optimizing system (2). Let us represent its equations as the following ones:

$$y_n = \overset{M}{\underset{m=1}{V}} C_{nm} A_m (n = 1,...,N).$$  (12)

In (12) the symbol $C_{nm}$ stands for the Boolean variable equal to 1 if the microoperation $y_n \in Y$ is formed in the state $a_m \in A$. States $a_m \in B_i$ are rearranged in the column $K(B_i)$ to decrease the number of terms in system (12). The "don't care" cells of the map are used for this optimization. Unfortunately, we have no an algorithm for solution of this problem.

5. Construction of transformed structure table. This table is constructed using the system of generalized formulae of transitions [2], which is the following one:

$$B_i \to \overset{H_0}{\underset{h=1}{V}} C_{ih} X_h a_s (i = 1,...,I).$$  (13)

In (13), the Boolean variable $C_{ih} = 1$, if the term $X_h a_s$ belongs to the system of transitions for the class $B_i \in \Pi_A$. This system is constructed using initial GSA $\Gamma$. The transformed structure table includes the columns $B_i$, $K(B_i)$, $a_S$, $K(a_S)$, $X_k$, $\Phi_k$, $h$. The rows of this table correspond to the terms $F_h \in F$:

$$F_h = (\overset{R_B}{\underset{r=1}{\wedge}} T_r^{l_{ri}}) * X_h (h = 1,...,H_0).$$  (14)

In (14), the first part is determined by the code $K(B_i)$ from the row $h$, whereas the value of $l_{ri} \in \{0,1,*\}$ is the value of code bit r; $T_r^0 = \overline{T_r}, T_r^1 = T_r, T_r^* = 1$, where $r = 1,...,R_B$.

These terms (14) belong to functions $D_r \in \Phi$, where:

$$D_r = \overset{H_0}{\underset{h=1}{V}} C_{rh} F_h (r = 1,...,R_B + R_O).$$  (15)

In (15) the Boolean variable $C_{rh} = 1$, if the row $h$ contains function $D_r \in \Phi$ ($h = 1,...,H_0$).

6. Implementation of FSM matrix circuit. Let us analyze the matrices from the circuit shown in Fig. 1. The matrix $M_1$ implements terms (14); it has $I_1 = 2(L + R_B)$ inputs and $O_1 = H_0$ outputs. The matrix $M_2$ implements functions (15); it has $I_2 = H_0$ inputs and $O_2 = R_B + R_O$. The matrix $M_3$ implements terms (12); it has $I_3$ inputs and $O_3$ outputs depending on outcome of the secondary encoding stage ($I_3 \le 2(R_B + R_O), O_3 \le M$). If some functions $y_n \in Y$ are represented by a single term, then they are formed directly by the matrix $M_3$. It decreases the number of inputs for $M_4$ from $N-1$ to 0. The lower bound corresponds to the situation when all microoperations are formed by the matrix $M_3$. In the general case the matrix $M_4$ has $I_4 \le M$ inputs and $O_4 \le N$ outputs.

## CONCLUSION

The proposed method of expansion of encoding space targets on decrease in the chip space occupied by the matrix circuit of Moore FSM. This approach guarantees the decrease

for the number of terms in the system of input memory functions of Moore FSM up to this value of the equivalent Mealy FSM. It allows decrease for the chip space used for implementing microoperations. But to make such a decrease, it is necessary to work out the corresponding method. It is reduced to change the cells of Karnaugh map occupied by states of FSM. Of course, there are many variants for this changing and an efficient method should do them in the right direction. This problem is very important for implementing control units of telecommunication systems.

To investigate the effectiveness of proposed method, we used around 50 test FSMs from the library [7]. We made both primary and secondary state encodings by hand. Next, we calculated the space of matrix implementations using both arbitrary state codes and our code approach. Our investigations show that the proposed approach allows the chip space decrease up to 78%. In 42% of tests, we witnessed the increase of FSM's performance. It can be explained by absence of the matrix $M_4$ due to successful secondary encoding, when each microoperation is represented by single term. The further direction of our research is development of the secondary encoding method. Next, we are going to check how this method can be applied for the cases when FSM logic circuits are implemented using some standard elements as PAL and PLA, as well as the programmable logic devices using these types of logic as its macrocells.

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