Convergence Analysis of the Hierarchical Routing Method

Olena Nevzorova, Ahmad K. Hasan, Olena A. Korshets

Abstract – **In this paper the analysis of convergence of the two-level hierarchical routing method with coordination is given. The convergence of routing method provides with an agreed transmission of packer flows through the channels. In order to avoid link overload the two-level traffic management method based on the coordination principle can be used.**

Keywords – **Coordination method, Iteration, Hierarchical Routing, Flow.**

I. INTRODUCTION

Hierarchical structure of telecommunication network is dictated by the necessity of reliability improvement, quality of service and overall system performance. For minimization overloads and packet loss in network the routing problems are decomposed into a number of smaller tasks. However, this solution causes other problems, for example growth of overhead traffic in network, which in turn leads to delays. It is equally important the coordination and compatibility of problems solved at different levels.

II. MODEL OF HIERARCHICAL ROUTING

In order to coordinate solving of routing let us define as control variable $x_{ij}^{k_r}$, where $x_{ij}^{k_r}$ is routing variable, equals intensity of the packet's flow k_r in link (i,j); k_r is packet flow, that is arriving through r^{th} boundary network router, intensity of traffic k_r is λ^{k_r}; ϕ_{ij} is link capacity. In order to prevent packet loss at the router and in network in general it is necessary to meet the following constraints [1],[2]

$$\begin{cases} \sum\limits_{j:(i,j)\in E} x_{ij}^{k_r} - \sum\limits_{j:(j,i)\in E} x_{ji}^{k_r} = 1, \text{ if } i^{th} \text{ is router-sender;} \\ \sum\limits_{j:(i,j)\in E} x_{ij}^{k_r} - \sum\limits_{j:(j,i)\in E} x_{ji}^{k_r} = 0, \text{ if } i^{th} \text{ is transit router;} \\ \sum\limits_{j:(i,j)\in E} x_{ij}^{k_r} - \sum\limits_{j:(j,i)\in E} x_{ji}^{k_r} = -1, \text{ if } i^{th} \text{ is router-receiver;} \end{cases} \quad (1)$$

The system of Eq. (1) must be satisfied for each packet flow. In addition, in order to prevent overload of communication channels it is important to fulfill the conditions

$$\sum_{r\in M_r} \sum_{k_r\in K} \lambda^{k_r} \cdot x_{ij}^{k_r} \le \phi_{ij}. \quad (2)$$

Each router determines the route of transmission of packet's flow without information about another source calculation when taking account the decentralized calculation of routing variables [1]. Thus, the conditions Eq. (2) can be written as (in explicitly form)

$$\sum_{k_r\in K_r} \lambda^{k_r} \cdot x_{ij}^{k_r} \le \phi_{ij} - \sum_{\substack{s\in M \\ s\ne r}} \sum_{k_s\in K_s} \lambda^{k_s} \cdot x_{ij}^{k_s}. \quad (3)$$

It should be imposed boundaries to the route variables

$$0 \le x_{ij}^{k_r} \le 1. \quad (4)$$

The Eqs. (1)-(4) can be represented in vector-matrix form

$$A_r \cdot \vec{x}_r = \vec{a}_r, \quad (5)$$

$$B_r \cdot \vec{x}_r \le \sum_{\substack{s\in M \\ s\ne r}} C_{rs}\vec{x}_s. \quad (6)$$

As the criterion of the optimality for calculation of the vector of variables $\vec{x}_r$ the minimum of next objective function can be chosen:

$$\min F$$

$$\text{where} \quad F = \sum_{r\in M_r} \vec{x}_r^t H_r \vec{x}_r, \quad (7)$$

where coordinates of the H_r are metrics of the communication channels, $[]^t$ - transpose function of the vector (matrix).

Then, taking the unconditioned extremum problem, it is necessary to maximize the Lagrangian by the Lagrange multipliers:

$$\min_x F = \max_\mu L,$$

$$L = \sum_{r\in M_r} \vec{x}_r^t H_r \vec{x}_r + \sum_{r\in M_r} \mu_r^t (B_r \cdot \vec{x}_r - \sum_{\substack{s\in M \\ s\ne r}} C_{rs}\vec{x}_s) \quad (8)$$

III. METHOD OF HIERARCHICAL ROUTING WITH COORDINATION

To solve the optimization problem which was formulated, it is using coordination method [3], [4]. The Lagrangian (8) can be represented as:

$$L = \sum_{r\in M_r} \vec{x}_r^t H_r \vec{x}_r + \sum_{r\in M_r} \mu_r^t (B_r \cdot \vec{x}_r) - \sum_{r\in M_r} \mu_r^t \sum_{\substack{s\in M \\ s\ne r}} C_{rs}\vec{x}_s. \quad (9)$$

Olena Nevzorova, Ahmad K. Hasan, Olena A. Korshets – Kharkiv National University of Radioelectronics, Kharkiv, 61164, UKRAINE, E-mail: elena.nevzorova@rambler.ru

Supposing that the values μ_r are fixed, the Eq. (9) takes the form

$$L = \sum_{r \in M_r} L_r \,, \qquad (10)$$

where

$$L_r = \vec{x}_r^t H_r \vec{x}_r + \mu_r^t (B_r \cdot \vec{x}_r) - \sum_{\substack{s \in M \\ s \neq r}} \mu_s^t C_{rs} \vec{x}_s .$$

Thus, the function (10) takes the separable form. The general routing problem was decomposed into a number of routing tasks. The task of maximizing the Eq. (8) defines the lower level of calculations. It is carried out a modification of the vector of Lagrange multipliers on the upper level, whose main task is to coordinate the solutions obtained from the lower level to prevent overload of the transmission network paths Eq. (2). Gradient procedure for modification of the vector of Lagrange multipliers is

$$\mu_r(\alpha+1) = \mu_r(\alpha) + \nabla \mu_r \,, \qquad (11)$$

where $\nabla \mu_r$ is the gradient of the function calculated from the results of calculation routing tasks in each router-sender which was obtained at the upper level

$$\nabla \mu_r(x) \Big|_{x = x^*} = B_r \cdot \vec{x}_r^{\,*} - \sum_{\substack{s \in M \\ s \neq r}} C_{rs} \vec{x}_s^{\,*} . \qquad (12)$$

Visually the computational structure of two-level hierarchical method of coordination routing was shown in Fig. 1.

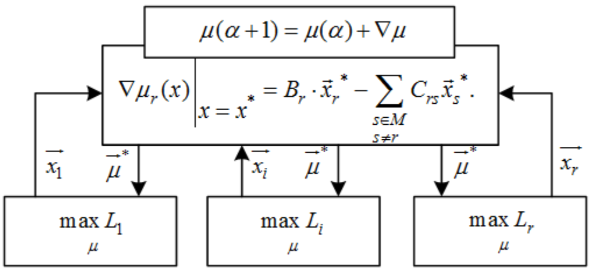

Fig.1. Computational structure of two-level hierarchical method of coordination routing

IV. Providing of Analysis of Convergence

When two or more routers are transmitting the packets simultaneously the overload of the link can occur. In order to avoid link overload the two-level traffic management method based on the coordination principle can be used. This principle is based on the possibility to convert the original minimization problem into the simpler maximization problem and the solution to this problem by using a two-level iterative computational structure. There is a calculation of routing variables on the lower level, and on the upper level solutions obtained at the lower level are coordinating to prevent overload of communication channels. The coordinator increases the value of the vector of Lagrange multiplier on the upper level, thus "penalizing" nodes, to reduce the transmission of the packets through the loaded link. The number of the coordinator actions to modify of the vector of Lagrange multiplier (number of iterations) depends on the intensity of the packet sources. Thus, it is the topicality to reduce the number of iterations.

Within the investigation the number of iterations which depends on the intensities of transmitted packet flows was analyzed (Table 1). As a numerical results show the number of coordinator actions increases with the increasing the value of traffic intensity.

TABLE 1
THE DEPENDENCE OF THE ITERATION NUMBER ON INTENSITIES OF TRANSMITTED PACKET STREAMS

№ of calcu-lat-ing	The intensity of first packet flow, 1/s	The intensity of se-cond packet flow, 1/s	The number of overloaded channels	The number of itera-tions
1	20	20	-	1
2	40	20	-	1
3	60	20	-	1
4	80	20	-	1
5	100	20	-	1
6	120	20	-	1
7	140	20	-	1
8	160	20	-	1
9	180	20	-	1
10	200	20	1	3
	…			
57	140	180	7	7
58	140	200	7	8
	…			
92	200	40	3	3
93	200	60	3	3
94	200	80	3	3
95	200	100	3	3
96	200	120	6	6
97	200	140	7	8
98	200	160	7	9
99	200	180	7	12
100	200	200	7	13

The convergence of routing method provides with an agreed transmission of packer flows through the channels. Thus, Table 1 shows that the hierarchical routing method at different intensities is observing at different number of iterations.

It is observed only one iteration if the packet streams are transmitted with small intensity. For example, for intensities which amount doesn't exceed 200 1/s. The maximum of iteration number is observed at intensities equal 200 1/s from each source.

Visually dependent of the iteration number on intensities of transmitted packet streams are shown in Fig.2.

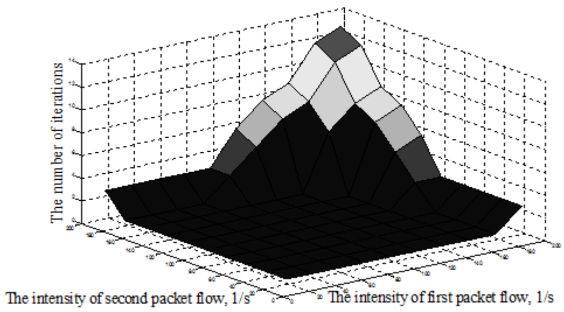

Fig.2. The dependence of the iteration number on intensities of transmitted packet flows

The resolution, which has already coordinated, of intensity distribution of packets stream through the communication channels with intensities equal 200 1/s from each source was shown in Fig.3.

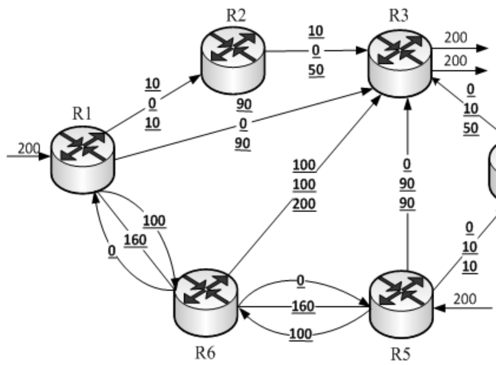

Fig.3. The resolution which has coordinated of distribution flows through the links with intensities equal 200 1/s from each source

On arcs which are representing communications channels (Fig.3) there are (from top to bottom) the intensity of the packets flow from the first source (1/s), the intensity of packets stream from second source (1/s), the bandwidth of communication channel bandwidth (1/s). Fig. 3 shows the solution already at 13 iterations (Table 1).

Each source which is transmitting packets stream distributing it through the communication channels does not know about the calculation of the second source, so there is an overload on 7 links (Table 1). The number of iterations equal 13 are observe due to the fact that sources cannot fit in with the simultaneous transmission of packets. Therefore, they overload links. The coordinators action for a modification of the vector of Lagrange multipliers is a result of the overload. Fig.3 has shown solution, which have already coordinated after thirteenth iterations.

V. Conclusion

The article describes a two-level routing model represented by Eqs. (1)-(8). It was used the coordination method by Eqs. (9)-(12). The procedure for calculating routing variables becomes a two-level. On the lower level it is calculation the routing table, and on the top level there is coordinating decisions which obtained from the lower level, in order to avoid overloading communications channels. Number of coordination actions (number of iterations) depends on the intensity of the packets flow (Fig. 2). There isn't overload under low intensities and therefore the number of iterations is equal to 1, i.e. the coordinator does nothing. The increasing intensity leads to growth of the number of iterations. There is the number of iterations equal to 13 under maximal intensity of transmitted packet flows. And this value is large enough. This number of iterations increases the overhead traffic, since routers have to recalculate the routes, taking into account imposed coordinator "penalizing" to prevent congestion in the communication links. This in turn leads to delays in the network.

Thus, the relevance is the reducing the number of iterations. This problem can be solved by changing weights (metrics) of communication links.

The advantage of two-level routing model basing on the coordination method is the simplicity of the calculation tasks. Well as separation of calculations of routing variables for two levels are using to facilitate network scalability. Disadvantage is a significant number of iterations for the modification of the vector Lagrange multiplier (coordinator action on the upper level), which may lead to delays in the network.

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