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Electromagnetic Wave Scattering with Special Spatial Lattices of Magnetodielectric Spheres*

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ABSTRACT: A problem solution of electromagnetic wave scattering with special compound lattices of resonance spheres has been considered, their spatial distribution has been subjected to a geometric progression. Expressions for the scattered fields have been obtained.

INTRODUCTION

Of special interest are the lattices, which electromagnetic interaction between two scattering elements as well as the elements themselves possesses resonance features. Such characteristics belong to lattices built of resonant magnetodielectric spheres.

Scattering features of lattices are determined with their topological structure and character features of scattering elements. Once the topological structure is specified it is possible to obtain lattices with necessary anisotropic characteristics. One of the ways to specify the topological structure is to subject it to different number structures. Such lattices differ from the periodic ones, so let us term them as the special lattices.

Let us consider a number structure determined with a geometric progression presented as a number table (see the Table) as an example.

Table

$\lvert t\rvert$ \ $\lvert s\rvert$	0	1	2	3	4	5	6	7
0	1	1	1	1	1	1	1	1
1	1	2^1	3^1	4^1	5^1	6^1	7^1	8^1
2	1	2^2	3^2	4^2	5^2	6^2	7^2	8^2
3	1	2^3	3^3	4^3	5^3	6^3	7^3	8^3
4	1	2^4	3^4	4^4	5^4	6^4	7^4	8^4
5	1	2^5	3^5	4^5	5^5	6^5	7^5	8^5
6	1	2^6	3^6	4^6	5^6	6^6	7^6	8^6
7	1	2^7	3^7	4^7	5^7	6^7	7^7	8^7

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For the presented number structure, the common term of the geometric progression $a_n = a_1 q_0^{n-1}$ can be written as: $a_{|t|+1} = (|s|+1)^{|t|}$ for the case when $a_1 = 1$, $q_0 = |s|+1$, $n = |t|+1$, where $|s| = 0,1,2,3,\ldots$; $|t| = 0,1,2,3,\ldots$.

For every value $|s|$ there is its own column of the number table. For $|s| = 0$ there is a sequence of numbers consisting of ones.

The aim of the article is to describe solution of a problem of electromagnetic wave scattering with the special compound spatial lattices of small homogeneous resonant magnetodielectric spheres, which spatial distribution is subjected to the geometric progression. In this problem, a length of a scattered wave is comparable with lattice constants, which makes it possible to examine an impact of the lattice structure resonances of electromagnetic spheres coupling on inner resonances of the lattice spheres and on their fine structure. The proposed solution makes it possible to study features of domains of an abnormal lattice dispersion.

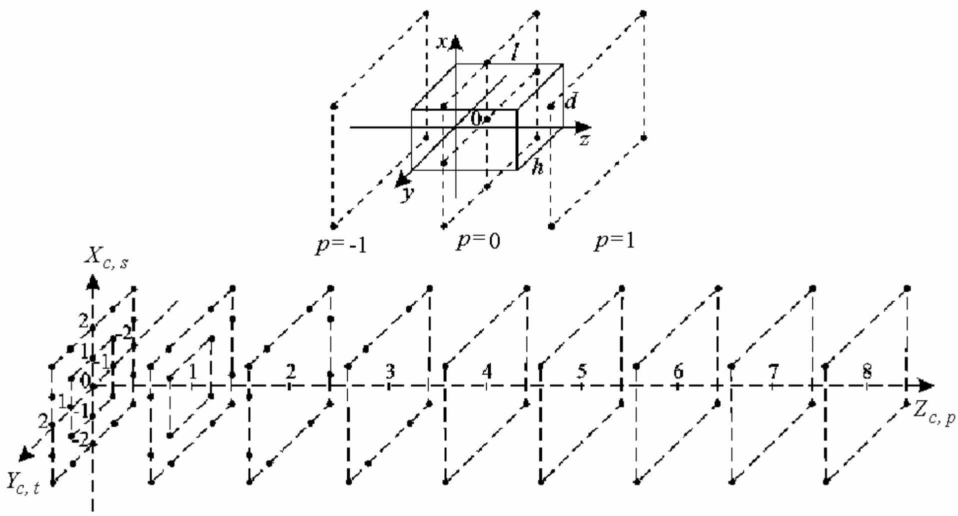

FIGURE 1. Spatial lattice of nodes

STATEMENT AND SOLUTION OF THE PROBLEM

Let us consider a special complicated spatial lattice consisting of C special sublattices $c\left(c \in C\right)$. These special sublattices c are generated by a coordinate presentation, which has the form in Cartesian coordinate system

$$
\begin{aligned}
&x_{c,s} = \left[s - 0.5\left\{ (-1)^s - 1 \right\} \right] d - (-1)^{s-1} x_{c,s=0} \\
&\left(s = 0, \pm 1, \pm 2, ... \right), \\
&y_{c,t} = \left[t - 0.5\left\{ (-1)^t - 1 \right\} \right] h - (-1)^{t-1} y_{c,t=0} \\
&\left(t = 0, \pm 1, \pm 2, ... \right), \\
&z_{c,p} = \left[p - 0.5\left\{ (-1)^p - 1 \right\} \right] l - (-1)^{p-1} z_{c,p=0} \\
&\left(p = 0, \pm 1, \pm 2, ..., \pm \left[\left(|s| + 1 \right)^{|t|} - 1 \right] \right),
\end{aligned}
\tag{1}
$$

Here, values d , h , l are determined with the conditions $x = 0$, $x = d$; $y = 0$, $y = h$; $z = 0$, $z = l$; and $x_{c,s=0}$, $y_{c,t=0}$, $z_{c,p=0}$ are the coordinates of the node that generates a special sublattice c and is located inside the domain (see Fig. 1).

$$
\begin{aligned}
&0 \le x_{c,s=0} \le d, \\
&0 \le y_{c,t=0} \le h, \\
&0 \le z_{c,p=0} \le l.
\end{aligned}
\tag{2}
$$

The term “special” will be further dropped in name of lattices and sublattices for brevity sake. Coordinates $x_{c,s}$, $y_{c,t}$, $z_{c,p}$ determine location of nodes of sublattice c outside the of domain (2) and are the functions of coordinates $x_{c,s=0}$, $y_{c,t=0}$, $z_{c,p=0}$. Time dependence can be introduced into the coordinate presentation (1) if coordinates $x_{c,s=0}$, $y_{c,t=0}$, $z_{c,p=0}$ are assumed to be certain time functions. For every node of the spatial sublattice c (1) there is a well-ordered number triple $u = c(p, s, t)$; the marked node of the lattice will be denoted as $u' = c'(p', s', t')$ and the node inside the lattice (2) will be denoted as $u = c(p = 0, s = 0, t = 0)$. Specifying the maximum values for the numbers (p, t, s) in (1) it is possible to consider finite and infinite lattices.

The required type of unit cell of the lattice (primitive, body-centered, face-centered etc.) is formed of C nodes inside domain (2), which is repeated by coordinate presentation (1) outside the domain (2) in the form of a spatial lattice of a certain form.

Figure 1 shows spatial distribution of lattice nodes when the generating lattice node is located in the center of domain (2) for the cases: $p = 0, \pm 1, s = 0, \pm 1, t = 0, \pm 1$ and $p = 0, 2, 3, 4, 5, 6, 7, 8,$

$s = 0, \pm 1, \pm 2,\ t = 0, \pm 1, \pm 2$. Distribution of nodes along axis z is subjected to the number table (Fig. 2). For every number triple $(p = 0, s, t)$ of a plane $x_{c,s}$, $y_{c,t}$, $z_{c,p=0}$ there is a certain number from the table; for example, for point (p = 0, s = –3, t = –3), there is number 4^3, for point (p = 0, s = –3, t = –3), there is number 6^4 and for point (p = 0, s = 4, t = 3), there is number 5^3. These numbers determine the number of nodes along axis z for the considered point $(p = 0, s, t)$ (Fig. 2). The relation between the number triple $(p = 0, s, t)$ of plane $x_{c,s}$, $y_{c,t}$, $z_{c,p=0}$ and number of Table (Fig. 2) is characterized with function

$$\left(|s| + 1\right)^{|t|}.$$

Hence the sequence of numbers p concerned with node coordinates along axis z has the form

$$0, \quad \pm 1, \quad \pm 2, \quad ..., \quad \pm\left(\left(|s| + 1\right)^{|t|} - 1\right),$$

where $|s|, |t| = 0, 1, 2, 3, \ldots$.

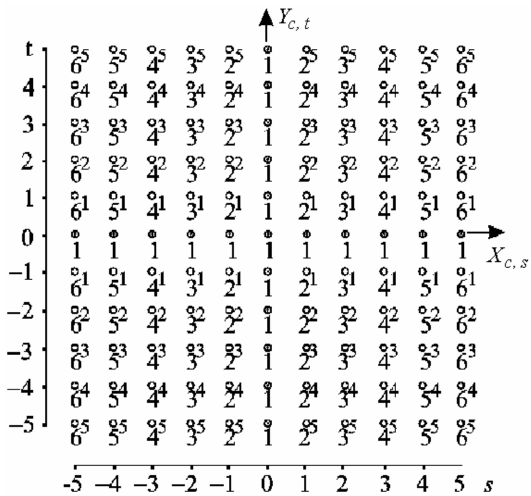

FIGURE 2. Number Table in plane $X_{c,s}, Y_{c,t}$

If coordinates of nodes in domain (2) are changed, the locations of nodes outside this domain will also shift in a certain way, cells will be rebuilt, and spatial lattice configuration will be formed.

The distance between nodes is determined as follows:

$$r_{c'(p',s',t'),c(p,s,t)} = \sqrt{\left(x_{c',s'} - x_{c,s}\right)^2 + \left(y_{c',t'} - y_{c,t}\right)^2 - \left(z_{c',p'} - z_{c,p}\right)^2} \; . \quad (3)$$

If one generating lattice node is located in the center of domain (2), using (1) we obtain a plain lattice with the nodes distributed along axis z and subjected to a natural number sequence for the case ($p,s,t=1$) (see Fig. 3).

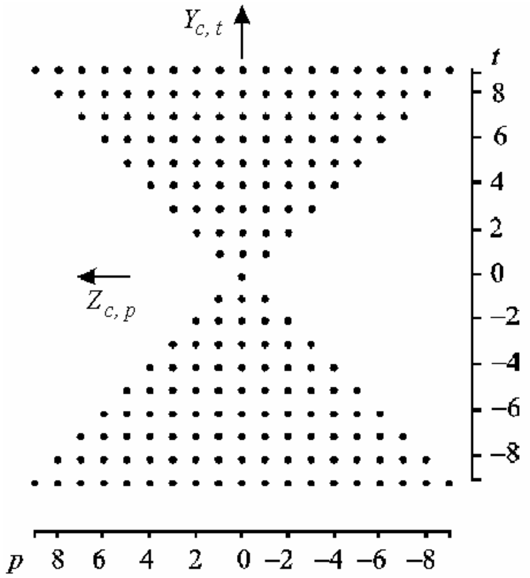

FIGURE 3. Plain lattice of nodes ($p,s,t=1$)

When $(p=0, s=0, t)$, a linear lattice with nodes distributed along axis y arises. When $(p, s=1, t)$, a plain lattice with nodes distributed along axis z and subjected to a natural number sequence $1, 2^1, 2^2, 2^3, 2^4, 2^5, \ldots$ is formed (Fig. 4).

Having performed corresponding sections of nodes (1) it is possible to obtain reconfigurable plain lattices of different forms.

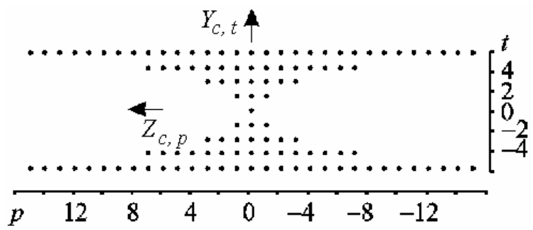

FIGURE 4. Plain lattice of nodes $(p, s = 1, t)$.

If an elementary cell is formed in domain (2), so instead of the single nodes the cells from domain (2) will be located in lattices (Figs. 3 and 4).

The centers of spheres with permittivity and permeability $\varepsilon_{c(p,s,t)}$, $\mu_{c(p,s,t)}$ and radiuses $a_{c(p,s,t)}$ subsequently referred to as ε_c, μ_c, a_c are placed in lattice nodes (1). Lattice spheres are located in a free space. Let us consider that condition $a_c/\lambda << 1$ is satisfied outside the sphere, but a resonant case $a_c/\lambda_g \sim 1$ is possible inside the sphere; here, λ is the wavelength outside the spheres, and λ_g is the wavelength inside the sphere [1]. Let us solve the problem in two steps using integral equations [2]. At the first step, let us determine an internal field of the scattering spheres, and at the second stage, let us determine the field scattered with spatial lattice of the spheres. The fields are presented in form $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{i\omega t}$, $\vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{i\omega t}$.

The scattered field is determined using the known internal field of scatterers in terms of the electric $\vec{\Pi}^{\mathrm{E}}$ and magnetic $\vec{\Pi}^{\mathrm{M}}$ Hertz potentials:

$$\vec{E}_{sc} = \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\Pi}^E - ik\mu_0\left[\nabla, \vec{\Pi}^M\right],$$
$$\vec{H}_{sc} = \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\Pi}^M - ik\varepsilon_0\left[\nabla, \vec{\Pi}^E\right]. \quad (4)$$

The Hertz potentials of the scattered field have the form:

$$\vec{\Pi}^E = \frac{1}{4\pi}\int_V \left(\frac{\varepsilon}{\varepsilon_0} - 1\right)\vec{E}^0(\vec{r}')\, f\left(\left|\vec{r} - \vec{r}'\right|\right) dV,$$
$$\vec{\Pi}^M = \frac{1}{4\pi}\int_V \left(\frac{\mu}{\mu_0} - 1\right)\vec{H}^0(\vec{r}')\, f\left(\left|\vec{r} - \vec{r}'\right|\right) dV, \quad (5)$$

where $\vec{E}^0(\vec{r}')$, $\vec{H}^0(\vec{r}')$ are the internal fields of scatterers; V is the volume of a scatterer; ε_0, μ_0 are the permittivity and magnetic conductivity of filling the space outside the spheres; function $f(|\vec{r}-\vec{r}'|)$ is the solution of equation

$$\Delta f(|\vec{r}-\vec{r}'|)+k^2\varepsilon_0\mu_0 f(|\vec{r}-\vec{r}'|)=-4\pi\delta(|\vec{r}-\vec{r}'|),$$

that satisfies the condition of radiation at infinity and has the form

$$f(|\vec{r}-\vec{r}'|)=\frac{e^{-ki\sqrt{\varepsilon_0\mu_0}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}. \tag{6}$$

It is possible to prove that for the points outside the sphere $(r > r')$, the integral of the Green function over the sphere volume has the form

$$W(\vec{r})=\int_V \frac{e^{-ki\sqrt{\varepsilon_0\mu_0}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dV=\frac{4\pi}{k_1^3}(\sin k_1a-k_1a\cos k_1a)\frac{e^{-ik_1r}}{r} \tag{7}$$

for the free space (6). Here, $k_1=k\sqrt{\varepsilon_0\mu_0}$, $k=2\pi/\lambda$, r is the distance from the center of the sphere to the points outside it.

The internal field of a small homogeneous magnetodielectric sphere with the center in $c'(p',s',t')$ is obtained from the system of inhomogeneous equations, which can be built upon integral equations [2] and results of work [3]. The inhomogeneous equations included in this system, have the following form

$$
\begin{aligned}
&\vec{E}_{0c'(p',s',t')}(\vec{r}',t)=\left(\frac{(\varepsilon_{c'eff}+2\varepsilon_0)+\theta_{1c'}^2\varepsilon_{c'eff}+i\theta_{1c'}(\varepsilon_{c'eff}+2\varepsilon_0)}{3\varepsilon_0 e^{i\theta_{1c'}}}\vec{E}^0_{c'(p',s',t')}(\vec{r}',t)-\right.\\
&-\sum_{\substack{p\\c'(p,s,t)\neq c'(p',s',t')}}\sum_s\sum_t\left\{(\nabla\nabla+k^2\varepsilon_0\mu_0)\frac{1}{4\pi}\left(\frac{\varepsilon_{c'eff}}{\varepsilon_0}-1\right)W^E_{c'(p,s,t)}(\vec{r})\vec{E}^0_{c'(p,s,t)}(\vec{r}',t)-\right.\\
&\left.\left.-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^M_{c'(p,s,t)}(\vec{r})\vec{H}^0_{c'(p,s,t)}(\vec{r}',t)\right]\right\}\right)-\\
&-\sum_{\substack{c=1\\(c\neq c')}}^{C}\left(\sum_p\sum_s\sum_t\left\{(\nabla\nabla+k^2\varepsilon_0\mu_0)\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^E_{c(p,s,t)}(\vec{r})\vec{E}^0_{c(p,s,t)}(\vec{r}',t)-\right.\right.\\
&\left.\left.-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^M_{c(p,s,t)}(\vec{r})\vec{H}^0_{c(p,s,t)}(\vec{r}',t)\right]\right\}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\vec{H}_{0c'(p',s',t')}(\vec{r}',t)=\left(\frac{(\mu_{c'eff}+2\mu_0)+\theta_{1c'}^2\mu_{c'eff}+i\theta_{1c'}(\mu_{c'eff}+2\mu_0)}{3\mu_0 e^{i\theta_{1c'}}}\vec{H}^0_{c'(p',s',t')}(\vec{r}',t)-\right.\\
&-\sum_{\substack{p\\c'(p,s,t)\neq c'(p',s',t')}}\sum_s\sum_t\left\{(\nabla\nabla+k^2\varepsilon_0\mu_0)\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^E_{c'(p,s,t)}(\vec{r})\vec{H}^0_{c'(p,s,t)}(\vec{r}',t)+\right.\\
&\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{c'eff}}{\varepsilon_0}-1\right)W^E_{c'(p,s,t)}(\vec{r})\vec{E}^0_{c'(p,s,t)}(\vec{r}',t)\right]\right\}\right)-\qquad(8)\\
&-\sum_{\substack{c=1\\(c\neq c')}}^{C}\left(\sum_p\sum_s\sum_t\left\{(\nabla\nabla+k^2\varepsilon_0\mu_0)\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^M_{c(p,s,t)}(\vec{r})\vec{H}^0_{c(p,s,t)}(\vec{r}',t)+\right.\right.\\
&\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^E_{c(p,s,t)}(\vec{r})\vec{E}^0_{c(p,s,t)}(\vec{r}',t)\right]\right\}\right)
\end{aligned}
$$

for an arbitrary separeted sphere. Here $\vec{E}_{0c'(p',s',t')}(\vec{r}',t)$, $\vec{H}_{0c'(p',s',t')}(\vec{r}',t)$ and $\vec{E}^0_{c'(p',s',t')}(\vec{r}',t)$, $\vec{H}^0_{c'(p',s',t')}(\vec{r}',t)$ are the fields of an incident wave and internal fields of the sphere respectively; $\vec{E}^0_{c(p,s,t)}(\vec{r}',t)$, $\vec{H}^0_{c(p,s,t)}(\vec{r}',t)$ are the internal fields of the other spheres; $\theta_{1c'}^2=k^2a_{c'}^2\varepsilon_0\mu_0$.

Values $W^E_{c(p,s,t)}(\vec{r}')$, $W^M_{c(p,s,t)}(\vec{r}')$ have the form

$$W^{E}_{c(p,s,t)}\left(\vec{r}'\right)=\frac{4\pi}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\frac{e^{-ik_1 r_{c'(p',s',t'),c(p,s,t)}}}{r_{c'(p',s',t'),c(p,s,t)}},$$

$$W^{M}_{c(p,s,t)}\left(\vec{r}'\right)=\frac{4\pi}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\frac{e^{-ik_1 r_{c'(p',s',t'),c(p,s,t)}}}{r_{c'(p',s',t'),c(p,s,t)}},$$

and values $\varepsilon_{\text{ceff}}$, μ_{ceff} can be written as [1,3,4]

$$\varepsilon_{ceff}=\varepsilon_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right),$$

$$\mu_{ceff}=\mu_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right), \tag{9}$$

where

$$F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=\frac{2\left(\sin ka_c\sqrt{\varepsilon_c\mu_c}-ka_c\sqrt{\varepsilon_c\mu_c}\cos ka_c\sqrt{\varepsilon_c\mu_c}\right)}{\left(k^2a_c^2\varepsilon_c\mu_c-1\right)\sin ka_c\sqrt{\varepsilon_c\mu_c}+ka_c\sqrt{\varepsilon_c\mu_c}\cos ka_c\sqrt{\varepsilon_c\mu_c}}.$$

The first summands in the right parts of equations (8) are concerned with the internal field of the sphere without regard for an impact of all other spheres; the rest of summands take into account impact on the sphere-scatterer with its center in $c'\left(p',s',t'\right)$ of all other spheres. The basic matrix of this system of equations (8) contains information about specific features of electromagnetic interaction between spheres of considered type of the lattices.

The system of equations (8) is an algebraic system consisting of $2N=2\sum_{c=1}^{C}N_c$ vector inhomogeneous equations, where N is the general number of lattice spheres, and N_c is the number of spheres of the sublattice c. For the separated sphere, the solution of this system has the form

$$\vec{E}^{0}_{c'(p',s',t')}\left(\vec{r}',t\right)=\frac{1}{\Delta^{EM}}\sum_{c=1}^{C}\left(\sum_{u}\left[\hat{g}_{u}^{Eu'}\vec{E}_{0c(p,s,t)}\left(\vec{r}',t\right)+\hat{\beta}_{u}^{Eu'}\vec{H}_{0c(p,s,t)}\left(\vec{r}',t\right)\right]\right),$$

$$\vec{H}^{0}_{c'(p',s',t')}\left(\vec{r}',t\right)=\frac{1}{\Delta^{EM}}\sum_{c=1}^{C}\left(\sum_{u}\left[\hat{\beta}_{u}^{Mu'}\vec{H}_{0c(p,s,t)}\left(\vec{r}',t\right)+\hat{g}_{u}^{Mu'}\vec{E}_{0c(p,s,t)}\left(\vec{r}',t\right)\right]\right), \qquad (10)$$

$$\hat{g}_{u}^{Eu'}=\begin{bmatrix} g_{xxu}^{Eu'} & g_{xyu}^{Eu'} & g_{xzu}^{Eu'} \\ g_{yxu}^{Eu'} & g_{yyu}^{Eu'} & g_{yzu}^{Eu'} \\ g_{zxu}^{Eu'} & g_{zyu}^{Eu'} & g_{zzu}^{Eu'} \end{bmatrix};$$

$$\hat{\beta}_{u}^{Eu'}=\begin{bmatrix} \beta_{xxu}^{Eu'} & \beta_{xyu}^{Eu'} & \beta_{xzu}^{Eu'} \\ \beta_{yxu}^{Eu'} & \beta_{yyu}^{Eu'} & \beta_{yzu}^{Eu'} \\ \beta_{zxu}^{Eu'} & \beta_{zyu}^{Eu'} & \beta_{zzu}^{Eu'} \end{bmatrix};$$

$$\hat{g}_{u}^{Mu'}=\begin{bmatrix} g_{xxu}^{Mu'} & g_{xyu}^{Mu'} & g_{xzu}^{Mu'} \\ g_{yxu}^{Mu'} & g_{yyu}^{Mu'} & g_{yzu}^{Mu'} \\ g_{zxu}^{Mu'} & g_{zyu}^{Mu'} & g_{zzu}^{Mu'} \end{bmatrix};$$

$$\hat{\beta}_{u}^{Mu'}=\begin{bmatrix} \beta_{xxu}^{Mu'} & \beta_{xyu}^{Mu'} & \beta_{xzu}^{Mu'} \\ \beta_{yxu}^{Mu'} & \beta_{yyu}^{Mu'} & \beta_{yzu}^{Mu'} \\ \beta_{zxu}^{Mu'} & \beta_{zyu}^{Mu'} & \beta_{zzu}^{Mu'} \end{bmatrix}.$$

Here Δ^{EM} is the determinant of the basic matrix of the equation system (8).

The component of internal electric field of the sphere (10) is presented as

$$E^{0}_{xu'}\left(\vec{r}',t\right)=\frac{1}{\Delta^{EM}}\sum_{c=1}^{C}\left(\sum_{u}\left[g_{xxu}^{Eu'}\vec{E}_{0xu}\left(\vec{r}',t\right)+g_{xyu}^{Eu'}\vec{E}_{0yu}\left(\vec{r}',t\right)+\right.\right.$$
$$\left.\left.+g_{xzu}^{Eu'}\vec{E}_{0zu}\left(\vec{r}',t\right)+\beta_{xxu}^{Eu'}H_{0xu}\left(\vec{r}',t\right)+\beta_{xyu}^{Eu'}H_{0yu}\left(\vec{r}',t\right)+\beta_{xzu}^{Eu'}H_{0zu}\left(\vec{r}',t\right)\right]\right).$$

The rest of components of internal fields of the sphere are similarly obtained from (10).

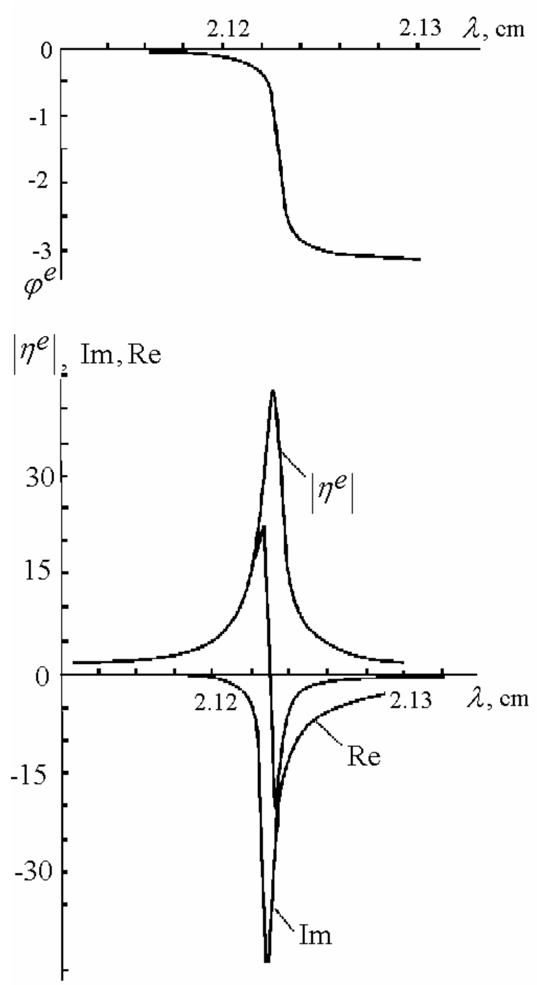

FIGURE 5. The first internal resonance of an electric type sphere

If electromagnetic interaction between lattice spheres is neglected, expressions (10) take the form

$$\vec{E}^0_{c(p,s,t)}(\vec{r}',t) = \vec{E}_{0c(p,s,t)}(\vec{r}',t)\frac{3\varepsilon_0 e^{i\theta_{1c}}}{(\varepsilon_{ceff}+2\varepsilon_0)+\theta_{1c}^2\varepsilon_{ceff}+i\theta_{1c}(\varepsilon_{ceff}+2\varepsilon_0)},$$

$$\vec{H}^0_{c(p,s,t)}(\vec{r}',t) = \vec{H}_{0c(p,s,t)}(\vec{r}',t)\frac{3\mu_0 e^{i\theta_{1c}}}{(\mu_{ceff}+2\mu_0)+\theta_{1c}^2\mu_{ceff}+i\theta_{1c}(\mu_{ceff}+2\mu_0)} \quad (10a)$$

for the internal field of the arbitrary sphere.

Figure 5 shows module $\left|\eta^{\mathrm{E}}\right|$ and argument φ^{E} of real and imaginary parts of the expression for the internal electric field of sphere (10a) as a function of incident wave length λ in the domain of the first internal resonance of electric type sphere for the case when $a_c = 0.1145$ cm, $\varepsilon_{\mathrm{c}} = 174$ and loss tangent is $\tan\delta_\varepsilon = 0,\ \mu = \mu_0 = \varepsilon_0 = 1$.

Taking into account (10) the Hertz potentials (5) of the field scattered with the lattice spheres can be presented as a superposition of the Hertz potentials of single lattice spheres:

$$\vec{\Pi}^E(\vec{r},t) = \sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t \frac{1}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\times \right.$$
$$\left. \times\left(\frac{\varepsilon_{ceff}}{\varepsilon_0} - 1\right)\vec{E}^0_{c(p,s,t)}(\vec{r},t)\frac{e^{-ik_1 r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right],$$

$$\vec{\Pi}^M(\vec{r},t) = \sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t \frac{1}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\times \right. \tag{11}$$
$$\left. \times\left(\frac{\mu_{ceff}}{\mu_0} - 1\right)\vec{H}^0_{c(p,s,t)}(\vec{r},t)\frac{e^{-ik_1 r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right],$$

$$r_{c(p,s,t)} = \sqrt{\left(x - x_{c,x}\right)^2 + \left(y - y_{c,t}\right)^2 + \left(z - z_{c,p}\right)^2},$$

where (x, y, z) are coordinates of the observation point of the scattered field outside the lattice spheres and $\left(x_{c,s}, y_{c,t}, z_{c,p}\right)$ are the center coordinates of the scattering lattice sphere (1). Then taking into account (11) we obtain desired field scattered with the lattice spheres from (4):

$$\vec{E}_{sc}=\sum_{c=1}^{C}\left\{\sum_{p}\sum_{s}\sum_{t}\frac{1}{k_1^3}\left(\sin k_1 a_c-k_1 a_c\cos k_1 a_c\right)\left[\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)\hat{L}_c\vec{E}^0_{c(p,s,t)}\left(\vec{r}'\right)-\right.\right.$$
$$\left.\left.-ik\mu_0\left(\frac{\mu_{ceff}}{\mu_0}-1\right)\left(-1\right)\hat{P}_c\vec{H}^0_{c(p,s,t)}\left(\vec{r}'\right)\right]e^{i\left(\omega t-k_1 r_{c(p,s,t)}\right)}\right\},$$
(12)
$$\vec{H}_{sc}=\sum_{c=1}^{C}\left\{\sum_{p}\sum_{s}\sum_{t}\frac{1}{k_1^3}\left(\sin k_1 a_c-k_1 a_c\cos k_1 a_c\right)\left[\left(\frac{\mu_{ceff}}{\mu_0}-1\right)\left(-1\right)\hat{L}_c\vec{H}^0_{c(p,s,t)}\left(\vec{r}'\right)+\right.\right.$$
$$\left.\left.+ik\varepsilon_0\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)\hat{P}_c\vec{E}^0_{c(p,s,t)}\left(\vec{r}'\right)\right]e^{i\left(\omega t-k_1 r_{c(p,s,t)}\right)}\right\},$$

where $\hat{L}_c$ and $\hat{P}_c$ are the functional matrixes of the form

$$\hat{L}_c=\begin{bmatrix}\Psi_{xxc} & \Psi_{xyc} & \Psi_{xzc}\\ \Psi_{yxc} & \Psi_{yyc} & \Psi_{yzc}\\ \Psi_{zxc} & \Psi_{zyc} & \Psi_{zzc}\end{bmatrix},$$

$$\hat{P}_c=\begin{bmatrix}0 & \Psi_{zc} & \Psi^0_{yc}\\ \Psi^0_{zc} & 0 & \Psi_{xc}\\ \Psi_{yc} & \Psi^0_{xc} & 0\end{bmatrix}.$$

Values included in functional matrixes (12) have the form

$$\Psi_{xxc}=\frac{1}{r_{c(p,s,t)}}k^2\varepsilon_0\mu_0+\frac{3\left(x-x_{c,s}\right)^2-r^2_{c(p,s,t)}}{r^5_{c(p,s,t)}}-\frac{k_1^2\left(x-x_{c,s}\right)^2}{r^3_{c(p,s,t)}}+$$
$$+ik_1\frac{3\left(x-x_{c,s}\right)^2-r^2_{c(p,s,t)}}{r^4_{c(p,s,t)}},$$

$$\Psi_{xyc} = \Psi_{yxc} \frac{3\left(x - x_{c,s}\right) - \left(y - y_{c,t}\right)}{r^5_{c(p,s,t)}} - k_1^2 \frac{\left(x - x_{c,s}\right)\left(y - y_{c,t}\right)}{r^3_{c(p,s,t)}} + \\ + ik_1 \frac{3\left(x - x_{c,s}\right)\left(y - y_{c,t}\right)}{r^4_{c(p,s,t)}},$$

$$\Psi_{xzc} = \Psi_{zxc} \frac{3\left(x - x_{c,s}\right) - \left(z - z_{c,p}\right)}{r^5_{c(p,s,t)}} - k_1^2 \frac{\left(x - x_{c,s}\right)\left(z - z_{c,p}\right)}{r^3_{c(p,s,t)}} + \\ + ik_1 \frac{3\left(x - x_{c,s}\right)\left(z - z_{c,p}\right)}{r^4_{c(p,s,t)}},$$

$$\Psi_{xyc} = \Psi_{zyc} \frac{3\left(y - y_{c,t}\right) - \left(z - z_{c,p}\right)}{r^5_{c(p,s,t)}} - k_1^2 \frac{\left(y - y_{c,t}\right)\left(z - z_{c,p}\right)}{r^3_{c(p,s,t)}} + \\ + ik_1 \frac{3\left(y - y_{c,t}\right)\left(z - z_{c,p}\right)}{r^4_{c(p,s,t)}},$$

$$\Psi_{xc} \frac{\left(x - x_{c,s}\right)}{r^3_{c(p,s,t)}} + ik_1 \frac{\left(x - x_{c,s}\right)}{r^2_{c(p,s,t)}},$$

$$\Psi^0_{xc} = -\Psi_{xc}, \quad \Psi^0_{yc} = -\Psi_{yc}, \quad \Psi^0_{zc} = -\Psi_{zc}.$$

Components ψ_{yyc}, ψ_{zzc} and ψ_{yc}, ψ_{zc} of matrixes $\hat{L}_c$ and $\hat{P}_c$ can be obtained from notation of components ψ_{xxc} and ψ_{xc}, substituting correspondently x, $x_{c,s}$ for y, $y_{c,t}$ and $z, z_{c,p}$ there.

Let us present the field at an arbitrary space point located outside the spheres as

$$\vec{E}\left(\vec{r},t\right) = \vec{E}_0\left(\vec{r},t\right) + \vec{E}_{sc}\left(\vec{r},t\right),$$

where $\vec{E}_0\left(\vec{r},t\right)$ is an unperturbed field of an incident wave.

For the particular case $\left(c = 1, p = 0, s, t = 0\right)$ of similar spheres (Fig. 2) when the wave

$$\vec{E}_{0x}(z,t)=\vec{E}_0 e^{i(\omega t-k_1 z)},\quad \vec{H}_{0y}(z,t)=\vec{H}_0 e^{i(\omega t-k_1 z)}$$

is scattered and electromagnetic interaction between the spheres can be neglected, the component of the scattered field $E_{x\,\mathrm{sc}}(r,t)$ has the form

$$E_{xsc}(\vec{r},t)=\frac{3}{k_1^3}(\sin k_1 a-k_1 a\cos k_1 a)\Bigg[k_1^2\frac{(\varepsilon_{eff}-\varepsilon_0)e^{i\theta_1}}{(\varepsilon_{eff}+2\varepsilon_0)+\theta_1^2\varepsilon_{eff}+i\theta_1(\varepsilon_{eff}+2\varepsilon_0)}\times$$
$$\times E_0\sum_{-s}^{s}\left(\frac{1}{r_s}+\left|-\frac{(x-x_s)^2}{r_s^3}\right|\right)e^{-ik_1 r_s}-k_1 k\mu_0\frac{(\mu_{eff}-\mu_0)e^{i\theta_1}}{(\mu_{eff}+2\mu_0)+\theta_1^2\mu_{eff}+i\theta_1(\mu_{eff}+2\mu_0)}\times$$
$$\times H_0\sum_{-s}^{s}z\frac{e^{-ik_1 r_s}}{r_s^2}\Bigg]e^{i\omega r},$$

for the far field zone $\left(r_{c(p,s,t)}=r_s, x_{c,s}=x_s\right)$, and

$$E_{xsc}(\vec{r},t)=\frac{3}{k_1^3}(\sin k_1 a-k_1 a\cos k_1 a)\Bigg[k_1^2\frac{(\varepsilon_{eff}-\varepsilon_0)e^{i\theta_1}}{(\varepsilon_{eff}+2\varepsilon_0)+\theta_1^2\varepsilon_{eff}+i\theta_1(\varepsilon_{eff}+2\varepsilon_0)}\times$$
$$\times E_0\sum_{-s}^{s}\left|3(x-x_s)^2-r_s^2\right|\left(\frac{1}{r_s^5}+ik_1\frac{1}{r_s^4}\right)+$$
$$+ik\mu_0\frac{(\mu_{eff}-\mu_0)e^{i\theta_1}}{(\mu_{eff}+2\mu_0)+\theta_1^2\mu_{eff}+i\theta_1(\mu_{eff}+2\mu_0)}H_0\sum_{-s}^{s}\frac{z}{r_s^3}\Bigg]e^{i\omega r}$$

for the near field zone $\left(e^{-ik_1 r_s}\approx 1\right)$.

The resonant conditions are determined from the determinant of an equation system (8). When permittivity and permeability ε_c and μ_c of similar lattice spheres are real and $a_c/\lambda_g\sim 1$, the resonant conditions can be obtained from the equation

$$\det\mathrm{Re}\left\|\alpha_{sj}\right\|=0, \qquad (13)$$

solved with respect to function $F\left(\theta_c\right)$ (9). Here, $\left\|\alpha_{sj}\right\|$ is the basic matrix of equations (8) [3]. If electromagnetic interaction of the spheres is neglected in equation (13) and it is solved with respect to function $F\left(\theta_c\right)$, the condition for internal resonances of an electric type sphere with the center in node $c(p,s,t)$ can be presented as:

$$F\left(\theta_c\right)=-\frac{2\varepsilon_0\left(\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right)}{\varepsilon_c\left[\left(1+\theta_{1c}^2\right)\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right]},$$

where

$$\theta_c=ka_c\sqrt{\varepsilon_c\mu_c},\quad \theta_{1c}=ka_c\sqrt{\varepsilon_c\mu_c}\ .$$

SUMMARY

We pioneered consideration of the electromagnetic wave scattering with special spatial lattices of magnetodielectric spheres, which topological structure is assigned with a structure of numbers determined with a geometric progression. Expressions for internal and scattered fields are obtained; they describe the structural resonances of electromagnetic interaction between spheres of lattices and internal resonances of spheres as well as impact of those resonances on one another. The solution obtained for the studied type of lattices with an anisotropic topological structure specified with a geometric progression may be useful when developing devices for control of radiation field of electromagnetic sources and for creating composite materials with high dispersion using the domains of abnormal lattice dispersion.

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