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MATHEMATICAL MODELING AND NUMERICAL ANALYSIS OF NONSTATIONARY PLANE-PARALLEL FLOWS OF VISCOUS INCOMPRESSIBLE FLUID BY R-FUNCTIONS AND GALERKIN METHOD

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Abstract. This paper is dedicated to nonstationary plane-parallel flows of viscous incompressible fluid in finite simply connected domains. Theorem of the solution uniqueness is presented. The method of successive approximation, the Galerkin method and the R-functions method are used to obtain the numerical solution, which was tested on the problem with known solution.

Key words: nonstationary flow, incompressible fluid, stream function, method of successive approximation, R-functions method, Galerkin method.

INTRODUCTION

It is known that nonstationary plane-parallel flows computations are used for mathematical modeling in hydrodynamics, aerodynamics, heat-power engineering, biomedicine and etc. That's why such problems are relevant nowadays [2–6, 25, 29].

These problems are mainly solved using the finite difference and finite element methods [1,7–9, 11,12,24,30]. They are easy to program, but new grid generation and boundary simplification are required every time a transition to a new area is made. The R-functions method developed by the academician of the Ukrainian Academy of Sciences V.L. Rvachev is free of these issues [14,21–23, 26]. This method allows us to consider the geometry of the problem accurately.

The aim of this work is the mathematical simulation of nonstationary plane-parallel flows of viscous incompressible in finite simply connected domains by means of the R-functions method, the Galerkin method and the method of successive approximation.

PROBLEM STATEMENT

Let's consider simply connected area Ω bounded by piecewise smooth bound $\partial\Omega$. Also consider the stream function $\psi(x, y, t)$ connected with the vector $\mathbf{v} = (v_x, v_y)$ of fluid velocity by the equations below:

$$v_x = \frac{\partial\psi}{\partial y},\ v_y = -\frac{\partial\psi}{\partial x}.$$

The mathematical model using stream function and dimensionless variables in area Ω takes the following form [16–18]:

$$-\frac{\partial\Delta\psi}{\partial t} + \nu\,\Delta^2\psi = \frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}, \quad (1)$$

where: x and y are dimensionless coordinates, $t > 0$ – dimensionless time, ν – kinematic coef ficient of viscosity, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ – Laplace operator.

Based on the statement of $\mathbf{v}|_{\partial\Omega}$ and $\mathbf{v}|_{t=0}$ we can complete the equation (1) with boundary and initial conditions:

$$\psi|_{\partial\Omega} = f_0(s,t), \tag{2}$$

$$\left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{\partial\Omega} = g_0(s,t), \quad s\in\partial\Omega,\ t\ge 0, \tag{3}$$

$$\psi|_{t=0} = \psi_0(x,y), \quad (x,y)\in\bar{\Omega}, \tag{4}$$

where: $\frac{\partial f_0}{\partial s}$, g_0 – some distributions of the velocity normal and tangential components, $\mathbf{n}$ – outer normal vector to the boundary.

SOLUTION METHOD

The Galerkin method, the R-functions method and the method of successive approximation are used for the initial-boundary problem (1) – (4) solving.

Let's consider an area Ω in space $\mathbb{R}^2$ with a piecewise smooth bound $\partial\Omega$. It is required to construct a function $\omega(x,y)$ that would be positive inside Ω, negative outside of Ω, equal to zero at $\partial\Omega$ and $\frac{\partial\omega}{\partial\mathbf{n}} = -1$. The equation $\omega(x,y)=0$ determines an implicit form of the locus for the points that belong to the boundary $\partial\Omega$ of the region Ω.

The works [13,27,28] showed that the following bundle of functions satisfies the boundary conditions (2), (3):

$$\psi = f - \omega(D_1 f + g) + \omega^2\Phi,$$

where: $f = ECf_0$, $g = ECg_0$ – extensions of f_0 and g_0 to Ω respectively, $\Phi = \Phi(x,y,t)$ – unknown structure component,

$$D_1 v = (\nabla\omega, \nabla v) = \frac{\partial\omega}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial\omega}{\partial y}\frac{\partial v}{\partial y}.$$

Let

$$J(u,v) = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}.$$

Let u_0 is the solution of the following problem:

$$\frac{\partial(-\Delta u_0)}{\partial t} + \nu\Delta^2 u_0 = 0,$$

$$u_0|_{\partial\Omega} = f_0(s,t), \quad \left.\frac{\partial u_0}{\partial\mathbf{n}}\right|_{\partial\Omega} = g_0(s,t),$$

$$u_0|_{t=0} = \psi_0(x,y).$$

Let's make a change in the problem (1) – (4):

$$\psi = u_0 + u,$$

where u – new unknown function. The solution of the problem for u_0 can be obtained using algorithm for the linear problem [3].

In order to achieve this, the initial-boundary problem (1) – (4) can be written as:

$$\frac{\partial(-\Delta u)}{\partial t} + \nu\Delta^2 u = J(\Delta(u_0+u), u_0+u), \tag{5}$$

$$u|_{\partial\Omega} = 0, \quad \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0, \tag{6}$$

$$u|_{t=0} = 0. \tag{7}$$

Let's consider operators A, B and J with their domains and energy norms respectively:

$$Au = \Delta^2 u, \quad Bu = -\Delta u,$$

$$J = J(\Delta(u_0+u), u_0+u),$$

$$D_A = \left\{ u \,\middle|\, u\in C^4(\Omega)\cap C^1(\bar{\Omega}), u|_{\partial\Omega} = \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0 \right\},$$

$$D_B = \left\{ u \,\middle|\, u\in C^2(\Omega)\cap C^1(\bar{\Omega}), u|_{\partial\Omega} = \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0 \right\},$$

$$D_J = \left\{ u \,\middle|\, u\in C^3(\Omega)\cap C^1(\bar{\Omega}), u|_{\partial\Omega} = \left.\frac{\partial u}{\partial\mathbf{n}}\right|_{\partial\Omega} = 0 \right\},$$

$$\|u\|_A^2 = \iint_\Omega (\Delta u)^2\,dxdy, \quad \|u\|_B^2 = \iint_\Omega |\nabla u|^2\,dxdy.$$

Thus, (1) – (4) can be written in the operator form:

$$\frac{d}{dt}Bu + \nu Au = Ju, \ (x,y)\in\Omega,\ t>0, \tag{8}$$

$$u|_{t=0} = 0. \tag{9}$$

Let's denote the classical solution of the problem (8), (9) as $u(t)$, i.e. for any $t\ge 0$ $u(t)\in D_A$ and $u(t)$ is continuously differentiable and satisfies (8) and (9).

Also let us assume $v(t)$ denotes the smooth function in $\bar{\Omega}\times[0,+\infty)$, which satisfies the boundary conditions (6) and at some value $T>0$ $v(T)=0$. Multiply (8) in $L_2(\Omega)$ by the arbitrary function $v(t)$ and integrate it from 0 to T:

$$-\int_0^T \left[u, \frac{\partial v}{\partial t}\right]_B dt + \nu\int_0^T [u,v]_A\,dt =$$

$$= [u_0, v(0)]_B + \int_0^T (Ju, v)_{L_2(\Omega)}\,dt. \tag{10}$$

Last equation is assumed to be a generalized (weak) solution of (8), (9).

Let's denote:

$$W_T = \{u \mid u \in L_2(0,T;H_A),$$

$$u' \in L_2(0,T;L_2(\Omega)),\ u(T)=0\},$$

as some set of functions.

Function $u(t)$ is called a generalized (weak) solution of (8), (9) if the following:

a) $u(t) \in L_2(0,T;\overset{\circ}{W}{}_2^2(\Omega))$,

b) for any $v(t) \in W_T$ the equation (10) is true.

Consider the method of successive approximation to solve the problem (8), (9) (therefore, problem the (1) – (4)). Assume that an initial approximation $u^{(0)}$ is set. Then one can find next the $(k+1)$ approximation using known the k approximation as a linear problem solution:

$$\frac{\partial(-\Delta u^{(k+1)})}{\partial t} + \nu\Delta^2 u^{(k+1)} =$$

$$= J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) \text{ in } \Omega,\ t>0, \qquad (11)$$

$$u^{(k+1)}\Big|_{\partial\Omega}=0,\quad \left.\frac{\partial u^{(k+1)}}{\partial \mathbf{n}}\right|_{\partial\Omega}=0, \qquad (12)$$

$$u^{(k+1)}\Big|_{t=0}=0,\quad k=0,1,2,\ldots \qquad (13)$$

The variational formulation of the (11) – (13) can be written as follows:

$$\frac{1}{2}\frac{d}{dt}|u^{(k+1)}|_B^2 + \nu|u^{(k+1)}|_A^2 =$$

$$= (J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}), u^{(k+1)})_{L_2(\Omega)}, \qquad (14)$$

$$\left\|u^{(k+1)}\right\|_{L_2(\Omega)}^2 = 0,\quad t=0. \qquad (15)$$

Let's integrate (14) from 0 to t and using some equalities and inequalities listed below [15]:

$$(J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}), u^{(k+1)})_{L_2(\Omega)} =$$

$$= (J(u_0+u^{(k)}, u^{(k+1)}), \Delta(u_0+u^{(k)}))_{L_2(\Omega)},$$

$$|(u,v)_H| \le \|u\|_H \|v\|_H,$$

$$\left|(J(u,v),\Delta u)_{L_2(\Omega)}\right| \le$$

$$\le c_0 \|\Delta v\|_{L_2(\Omega)} \|\nabla u\|_{L_2(\Omega)} \|\Delta u\|_{L_2(\Omega)},$$

$$u,v \in W_2^2(\Omega);$$

$$\|\nabla u\|_{L_4(\Omega)}^2 \le c\|\nabla u\|_{L_2(\Omega)} \|\Delta u\|_{L_2(\Omega)},$$

$$u \in \overset{\circ}{W}{}_2^1(\Omega) \cap W_2^2(\Omega),$$

we are able to estimate (14) as follows:

$$\left\|u^{(k+1)}(t)\right\|_{L_2(\Omega)}^2 + \int_0^t |u^{(k+1)}|_A^2 d\tau \le \frac{c_1}{\nu}T +$$

$$+\frac{c_2}{\nu}\left(\operatorname*{ess\,sup}_{0\le t\le T}\left\|u^{(k)}\right\|_{L_2(\Omega)}^2 + \int_0^T |u^{(k)}|_A^2\, d\tau\right)^2, \qquad (16)$$

where: c_1 and c_2 are known constants, which depend only on the area geometry.

Therefore, we can say that the boundedness of our solution is proved in the space:

$$V = L_\infty(0,T;L_2(\Omega)) \cap L_2(0,T;H_A).$$

Further, let's prove the iterative (11) – (13) convergence. Consider differences $\delta u^{(k+1)} = u^{(k+1)} - u^{(k)}$, which satisfy the following equation and the boundary and initial conditions:

$$\frac{\partial(-\Delta\delta u^{(k+1)})}{\partial t} + \nu\Delta^2\delta u^{(k+1)} =$$

$$= J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) -$$

$$-J(\Delta(u_0+u^{(k-1)}), u_0+u^{(k-1)}), \qquad (17)$$

$$\delta u^{(k+1)}\Big|_{\partial\Omega}=0,\quad \left.\frac{\partial \delta u^{(k+1)}}{\partial \mathbf{n}}\right|_{\partial\Omega}=0, \qquad (18)$$

$$\delta u^{(k+1)}\Big|_{t=0}=0. \qquad (19)$$

The variational formulation of the (17) – (19) can be written as follows:

$$[\delta u'^{(k+1)}, v]_B + \nu[\delta u^{(k+1)}, v]_A =$$

$$= (J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) -$$

$$-J(\Delta(u_0+u^{(k-1)}), u_0+u^{(k-1)}), v)_{L_2(\Omega)}, \qquad (20)$$

$$(u^{(k+1)}, v)_{L_2(\Omega)} = 0,\ t=0.$$

Let's integrate (20) from 0 to t and substitute $u^{(k+1)}$ instead of v:

$$\frac{1}{2}|\delta u^{(k+1)}(t)|_B^2 + \nu\int_0^t |\delta u^{(k+1)}|_A^2 d\tau =$$

$$= \int_0^t (J(\Delta(u_0+u^{(k)}), u_0+u^{(k)}) -$$

$$-J(\Delta(u_0+u^{(k-1)}), u_0+u^{(k-1)}), \delta u^{(k+1)})_{L_2(\Omega)} d\tau,$$

$$\left\|\delta u^{(k+1)}\right\|_{L_2(\Omega)}^2 = 0.$$

One can estimate the last equation using the previous equalities and inequalities and the next ones:

$$J(u_1,v_1) - J(u_2,v_2) =$$

$$= J(u_2, v_1-v_2) + J(u_1-u_2, v_2),$$

$$\left|(J(u,v),w)_{L_2(\Omega)}\right| \le$$

$$\le c_1 \|\Delta u\|_{L_2(\Omega)} \|\Delta v\|_{L_2(\Omega)} \|w\|_{L_2(\Omega)},$$

$$u, v \in \overset{\circ}{W}{}_2^2(\Omega), \quad w \in L_2(\Omega).$$

Therefore:

$$\operatorname*{ess\,sup}_{0\le t\le T} \left\|\delta u^{(k+1)}\right\|^2_{L_2(\Omega)} + \int_0^T | \delta u^{(k+1)} |_A^2 d\tau \le$$

$$\le \frac{c_3}{\nu} \left(\operatorname*{ess\,sup}_{0\le t\le T} \left\|\delta u^{(k)}\right\|^2_{L_2(\Omega)} + \int_0^T | \delta u^{(k)} |_A^2 \, d\tau \right),$$

where $t \in (0;T]$.

Hence we can say that the boundedness of $\delta u^{(k+1)}$ is proved in the metric space V.

Therefore, if $\frac{c_3}{\nu} \le \alpha < 1$ then:

$$\left\|\delta u^{(k+1)}\right\|_V \le \alpha \left\|\delta u^{(k)}\right\|_V \le \ldots \le \alpha^k \left\|\delta u^*\right\|_V,$$

i.e. the limit below exists:

$$\lim_{k\to\infty} u^{(k)} = u^*.$$

One can prove the following theorem.

Theorem. Let function $u_0 \in L_2(0,T;L_2(\Omega))$. Therefore the variational problem (14), (15) has a unique solution:

$$u \in L_\infty(0,T;L_2(\Omega)) \cap L_2(0,T;H_A).$$

COMPUTATION SCHEME

According to the R-functions method the solution structure of (11) – (13) is:

$$u^{(k+1)}(x,y,t) = \omega^2(x,y)\Phi^{(k+1)}(x,y,t).$$

Next, let's approximate an undefined component:

$$\Phi^{(k+1)}(x,y,t) \approx \Phi_N^{(k+1)}(x,y,t) =$$

$$= \sum_{j=1}^{N} c_j^{(k+1)}(t)\tau_j(x,y),$$

where: $\{\tau_j\}$ – some complete system of functions in the space $L_2(\Omega)$ (trigonometric or algebraic polynomial, B-splines and etc.). Then an approximation for $u^{(k+1)}(x,y,t)$ has the following form:

$$u_N^{(k+1)}(x,y,t) = \sum_{j=1}^{N} c_j^{(k+1)}(t)\varphi_j(x,y),$$

where: $\varphi_j = \omega^2\tau_j$.

According to the Galerkin method [19] for the nonstationary problems one can find functions $c_j^{(k+1)}(t)$, $j = 1,\ldots,N$, using the following ordinary differential equation system:

$$\left(\frac{d}{dt} B u_N^{(k+1)} + \nu A u_N^{(k+1)} - \right.$$

$$\left. - C(\varphi + u_N^{(k)}) - F, \varphi_j \right)_{L_2(\Omega)} = 0,$$

$$(u_N^{(k+1)}\big|_{t=0} - u_0, \varphi_j)_{L_2(\Omega)} = 0, \; j = 1,2,\ldots,N,$$

or in expended form:

$$\sum_{j=1}^{N} \dot{c}_j^{(k+1)}(t)[\varphi_j,\varphi_i]_B + \nu \sum_{j=1}^{N} c_j^{(k+1)}(t)[\varphi_j,\varphi_i]_A =$$

$$= \left(C(\varphi + u^{(k)}) + F, \varphi_i \right)_{L_2(\Omega)}, \qquad (21)$$

$$\sum_{j=1}^{N} c_j^{(k+1)}(0)(\varphi_j,\varphi_i)_{L_2(\Omega)} =$$

$$= (u_0,\varphi_i)_{L_2(\Omega)}, \; i = 1,2,\ldots,N, \qquad (22)$$

where the dot denotes the time derivative.

Let's consider the matrices and vectors:

$$\Xi = \left\|[\varphi_j,\varphi_i]_B\right\|_{i,j=\overline{1,N}}, \quad \Upsilon = \left\|[\varphi_j,\varphi_i]_A\right\|_{i,j=\overline{1,N}},$$

$$\Gamma = \left\|(\varphi_j,\varphi_i)_{L_2(\Omega)}\right\|_{i,j=\overline{1,N}},$$

$$\xi(t) = \left\|\left(C(\varphi + u^{(k)}) + F, \varphi_i \right)_{L_2(\Omega)}\right\|_{i=\overline{1,N}},$$

$$\gamma = \left\|(u_0,\varphi_i)_{L_2(\Omega)}\right\|_{i=\overline{1,N}}.$$

We note that matrices Ξ, Υ, Γ are symmetric and invertable.

Denote:

$$c^{(k+1)}(t) = (c_1^{(k+1)}(t),\ldots,c_N^{(k+1)}(t)),$$

$$\dot{c}^{(k+1)}(t) = (\dot{c}_1^{(k+1)}(t),\ldots,\dot{c}_N^{(k+1)}(t)),$$

therefore, a Cauchy problem (21), (22) can be written as:

$$\Xi \dot{c}^{(k+1)}(t) + \nu \Upsilon c^{(k+1)}(t) = \xi(t), \qquad (23)$$

$$\Gamma c(0) = \gamma. \qquad (24)$$

We can use the Runge–Kutta method to solve (23), (24).

NUMERICAL RESULTS

Problem 1. Let's consider a test problem [21] to validate the proposed method. It consists of the equation (1) and boundary and initial conditions listed below:

$$\psi\big|_{\partial\Omega} = f_0(s,t) = \begin{cases} e^{-2\pi^2 t}\cos\pi y, x=0, \\ e^{-2\pi^2 t}\cos\pi x, y=0, \\ 0,\ x=\frac{1}{2}, y=\frac{1}{2}, \end{cases}$$

$$\frac{\partial\psi}{\partial\mathbf{n}}\bigg|_{\partial\Omega} = g_0(s,t) = \begin{cases} -\pi e^{-2\pi^2 t}\cos\pi x, y=\frac{1}{2}, \\ -\pi e^{-2\pi^2 t}\cos\pi y, x=\frac{1}{2}, \\ 0,\ x=0 \text{ or } y=0, \end{cases}$$

$$\psi\big|_{t=0} = \psi_0(x,y) = \cos\pi x\cos\pi y .$$

Assume that $\nu = 1$, Ω – square $0 < x < \frac{1}{2}$, $0 < y < \frac{1}{2}$, $t \in [0,1]$.

Function $\omega(x,y)$ have the below form:

$$\omega(x,y) = x(1-2x) \wedge_0 y(1-2y),$$

where $\wedge_0$ - R-conjunction:

$$u \wedge_0 v = u + v - \sqrt{u^2+v^2} .$$

Functions $f(x,y,t) = ECf_0(s,t)$ and $g(x,y,t) = ECg_0(s,t)$ are set as follows:

$$f(x,y,t) = \\ = \frac{e^{-2\pi^2 t}\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right)(y\cos\pi y + x\cos\pi x)}{y\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + x\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + xy},$$

$$g(x,y,t) = -\pi e^{-2\pi^2 t}xy \times \\ \times \frac{e^{-2\pi^2 t}\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right)(y\cos\pi y + x\cos\pi x)}{y\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + x\left(\frac{1}{2}-x\right)\left(\frac{1}{2}-y\right) + xy}.$$

The exact solution of this problem is:

$$\psi(x,y,t) = e^{-2\pi^2 t}\cos\pi x\cos\pi y .$$

We used the Runge–Kutta method to solve (23), (24) and B-splines [10] as τ. The Gauss formula with 16 knots was used for evaluation of integrals in the Galerkin method.

Now let's have a look at the results of this numerical experiment.

The differences between the exact and approximated solution in 3D are presented below on Fig. 1 and Fig. 2. The difference reduces with time.

The stream lines and stream function in 3D are given in figures 3 and 4. They are similar to the exact solution.

The error norm in $L_2(\Omega)$ is shown on Fig. 5 with dependency from time. Fig. 5 shows method convergence.

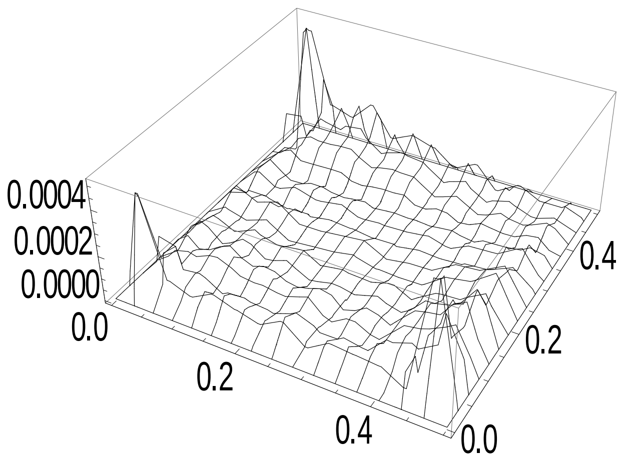

Fig. 1. The difference between the exact and approximated solution, $t = 0.1$

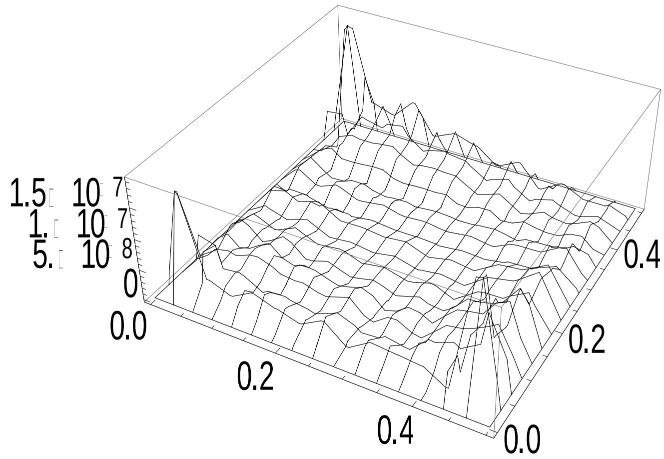

Fig. 2. The difference between the exact and approximated solution, $t = 0.5$

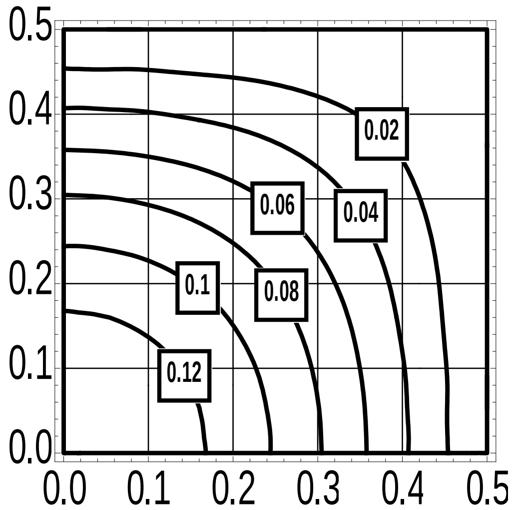

Fig. 3. The stream lines, $t = 0.1$

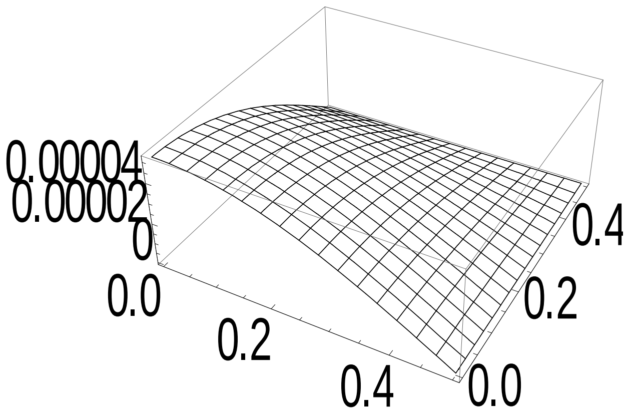

Fig. 4. The stream function, $t = 0.1$

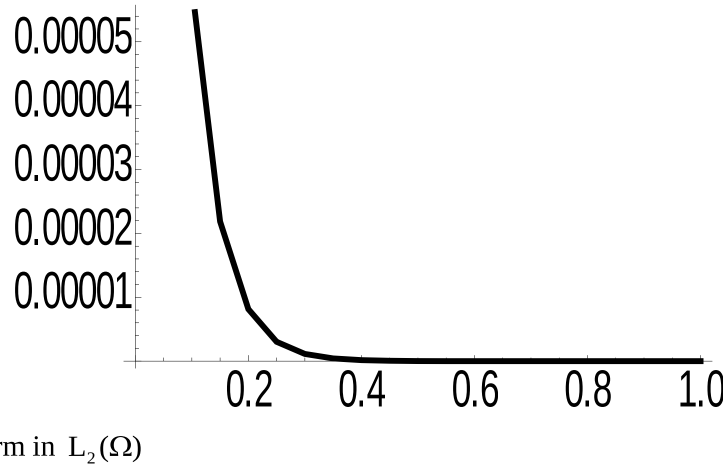

Fig. 5. The error norm in $L_2(\Omega)$

P r o b l e m 2. Let's consider the equation (1) and next boundary and initial conditions:

$$\psi\big|_{\partial\Omega} = 0,$$

$$\left.\frac{\partial\psi}{\partial\mathbf{n}}\right|_{\partial\Omega} = \begin{cases} e^{-t} - 1, & y = 1, \\ 0, & x = 0,\ y = 0,\ x = 1, \end{cases}$$

$$\psi\big|_{t=0} = 0,$$

where: $\Omega = \{(x, y) \,|\, 0 < x < 1,\ 0 < y < 1\}$, $\nu = 1$, $t \in (0;5]$. Function $g(x, y, t) = ECg_0(s, t)$ is set as follows:

$$g(x,y,t)=\frac{\dfrac{e^{-t}-1}{\omega_1(x,y)}}{\dfrac{1}{\omega_1(x,y)}+\dfrac{1}{\omega_2(x,y)}}=$$

$$=\frac{(e^{-t}-1)(y-4(x-0,5)^2)}{y-4(x-0,5)^2+\sqrt{64(x-0,5)^2+1}},$$

where:

$$\omega_1(x,y)=1-y,$$

$$\omega_2(x,y)=\frac{y-4(x-0,5)^2}{\sqrt{64(x-0,5)^2+1}}.$$

Therefore, the problem structure is

$$\psi(x,y,t)=$$
$$=-\omega(x,y)\frac{(e^{-t}-1)(y-4(x-0,5)^2)}{y-4(x-0,5)^2+\sqrt{64(x-0,5)^2+1}}+$$
$$+\omega^2(x,y)\Phi(x,y,t).$$

We also used the Runge–Kutta method to solve (23), (24) and B-splines as τ. The Gauss formula with 16 knots was used for evaluation of integrals in the Galerkin method.

The stream lines and stream function in 3D are given in figures 6, 7. The vorticity lines and vorticity function in 3D are given in figures 8, 9.

Fig. 6 – 9 showed that the achieved numerical results are consistent with other results [7].

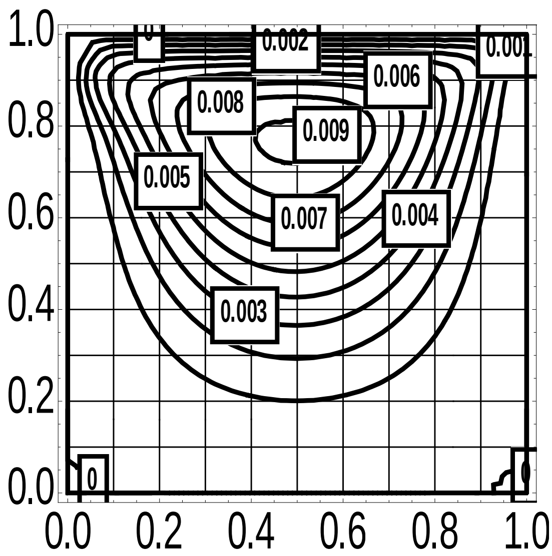

Fig. 6. The stream lines, $t=0.1$

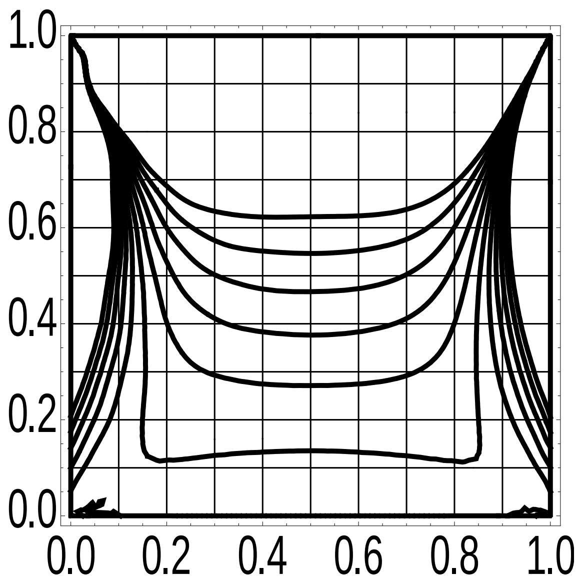

Fig. 8. The vorticity lines, $t=0.1$

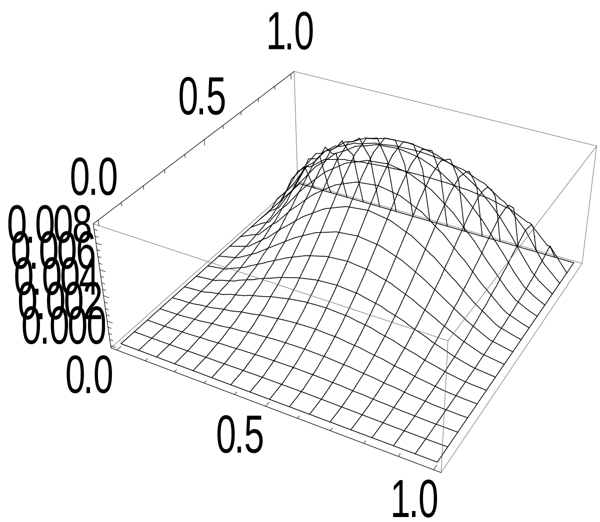

Fig. 7. The stream function, $t=0.1$

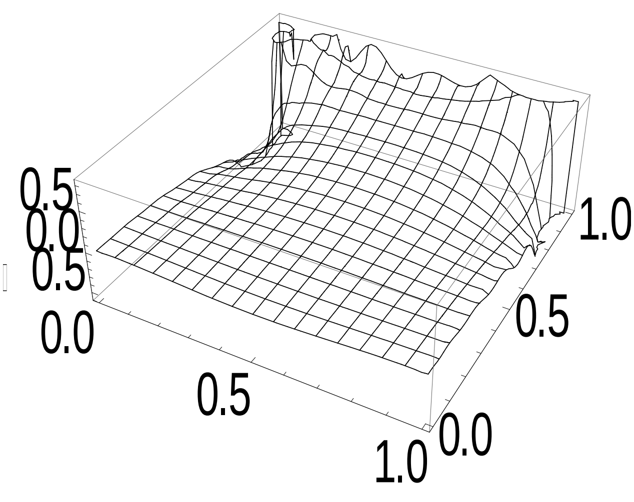

Fig. 9 The vorticity function, $t=0.1$

CONCLUSIONS

The nonstationary plane-parallel flow of viscous incompressible fluid is investigated. The algorithm for solving the problem based on the R-functions method and the Galerkin method is used. The solution structures of unknown function were built by means of the R-functions method, and the Galerkin method was used for the approximate undefined components. Thus, the stream function was represented in an analytical way.

The advantage of the suggested algorithm is that it does not have to be modified for different geometries of the regions being reviewed, which illustrates the scientific innovation of the results obtained. As a result, the approximate solution for such streams investigation problems is obtained in the non-classic geometry field.

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