# Approaches to Functional Dependencies Representation for Industrial Automation Systems Mathematical Support

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***Abstract*:** The generalized approaches to representation of functional dependencies used in mathematical maintenance of the technological processes automation systems are proposed in this research. The principal idea of these approaches is in using the functional dependencies discrete form including the nodal values of a function corresponded to the primarily chosen nodes of the argument variable, and the linear interpolation is proposed to compute the required values of the represented function. These principal ideas allow us to have the generalized uniform approaches to representation of different functional dependencies used in mathematical maintenance of automated systems. The main difficulties in using of the functional dependencies discrete form are in not existing required data required to have this discrete form, and it is proposed to use the cubic spline interpolation for correct transformation of the primarily existing not full data to the data set providing the reliability of the linear interpolation. The shown example of using of the proposed approaches deals with estimating the temperature fields in the pellets of the ceramic nuclear fuel used taking into account the temperature dependences of its heat conductivity.

***Keywords*:** automated system, mathematical support, data, functional dependence, linear interpolation, cubic splines, temperature, heat conductivity, nuclear fuel.

## I. Introduction

Developing of industrial automation systems is principally required to improve the quality of controlled technological processes, and it is widely discussed in different modern researches in general point of view like in the research [1] or for different separate applications like in research [2] for example. The traditional approaches to automation are based on using the automation loops with the suitable controllers (governors) reach the limits of perfectness and it is actually difficult to have significant improvements of the controlled processes quality by using such traditional approaches at present. Due to this circumstance, modern automation systems are built using the computer simulations [3], artificial intelligent [4], machine learning [5, 6] and other computerised technologies. Mathematical support is the principally required component of the of the computerised automation systems implementing of which is the main general trend of modern industrial automation at present. Thus, the theme of this research dealing with one particular task required to improve the industrial automation systems mathematical support is in current interests due to the existed agreement with the modern general trends of implementing of the computerised technologies to the industrial automation systems.

The different data are one of the principal components of the mathematical support of the industrial automation systems. These data can be different regarding with the nature of processes in the automated systems. The simplest of these data can be the number defining some parameters values like in the researches [7, 8], but the functional dependencies like in the researchers [9, 10], the graphs-schemes and the flow-charts representing for example the different kinds of spatial configurations [11], as well as the knowledges bases and others are the more complicated data for representing using the computer technologies. It is naturally that developing the more suitable approaches for complicated data representation is required to improve the mathematical support of the computerised automation systems. The purpose of this research is in develop of the approaches to functional dependencies generalised uniform representation, which will be suitable for using in industrial automation systems mathematical support. To realize the purpose of this research, the following tasks will be accomplished:

- the general uniform methodology for functional dependencies representing suitable to use in mathematical support of different purposed industrial computerised automated systems will be proposed;
- the universal approaches for building the data required to realize the proposed general uniform methodology for functional dependencies representing in the mathematical software will be proposed;
- estimation of the temperature fields in the ceramic nuclear fuel pellets taking into account the temperature dependence of the heat conductivity will be considered as example of using the proposed methodology and approaches for functional dependencies representation.

Realization of the formulated tasks will allow us to have the clear imaginations about the proposed methodology and approaches.

## II. General Methodology

Let we have the functional dependence between two values, so that the $y$ value is dependent on the $x$ independent value, and we will represent it as follows:

$$y = f(x), \ a \le x \le b, \qquad (1)$$

where $y$ and $x$ are the dependent and independent values, $f$ is the kind of the functional dependence; $a$ and $b$ are the boundaries of the interval of the possible $x$ values.

Functional dependence (1) can represent the different situations which must be took into account in the mathematical support of the industrial computerised automation systems. The view of the functional dependencies (1) can be different regarding the sense of each particular functional dependence (1), but it is required to represent each of the functional dependence (1) in the form suitable for developing the correspondent software. It is naturally for us the wish to have generalized uniform methodology of representing the functional dependences

(1) independently to the view of the function $f$. We can realize programmatically the different ways to define the functional dependencies including analytical, but the discrete representations (fig. 1) are the more universal. The discrete representation of some functional dependence (1) can be obtained by introducing the grid of the enumerated $x$ independent values covering the interval $[a,b]$ of the possible values (fig. 1):

$$x_1 = a,\ x_2 = x_1 + \Delta x_1, \dots, x_{k+1} = x_k + \Delta x_k, \dots, x_n = b\ , \qquad (2)$$

where $x_k$ and $\Delta x_k$ are the grid node and the grid step with the $k = 1,2,\dots,n$ number; $n$ is the count of the grid nodes covering the possible values interval.

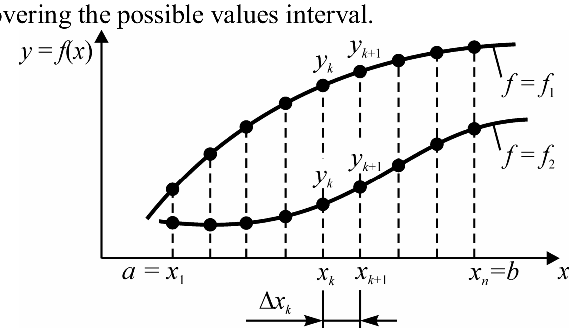


Fig.1. The discrete representation (markers) of the functional dependencies (curves)

Due to the introduced grid (2) of the independent $x$ value, we can define the following nodal values of the dependent $y$ value:

$$y_k = f\left(x_k\right),\ k = 1,2,\dots,n\ . \qquad (3)$$

It is obviously (fig. 1) that the line segments connecting the close nodal values (3) can represent the dependent values with the enough small errors, so that these small errors will have no principal influence on the further results. These line segments connecting the close nodal values (3) are represent the linear interpolation:

$$f\left(x\right) = y_k \frac{x_{k+1} - x}{x_{k+1} - x_k} + y_{k+1} \frac{x - x_k}{x_{k+1} - x_k},\ x_k \le x \le x_{k+1}\ . \quad (4)$$

It is clearly understood (fig. 1) that such linear interpolation (4) can be used for the different functional dependences (1). The errors of the linear interpolation (4) are defined by choosing the count of nodes and the grid steps.

We can see that the discrete form including the grid (2) and the nodal values (3) with the linear interpolation (4) allows us to represent the different kind of the functional dependencies (1) by means the correspondent choosing of the grid steps. Thus, the general uniform methodology for functional dependencies (1) representing suitable to use in mathematical support of different purposed industrial computerised automated systems is reduced to defining of two data sets:

$$X_n = \left\{x_1, x_2, \dots, x_k, \dots, x_n\right\}, \qquad (5)$$

$$Y_n = \left\{y_1, y_2, \dots, y_k, \dots, y_n\right\}, \qquad (6)$$

where $X_n$ and $Y_n$ are the data sets about the grid nodes and the corresponded nodal values.

Representing the functional dependencies (1) by means the data sets (5), (6) with the linear interpolation (4) is suitable for programming on different languages, because of these data sets can be easily realized by using the arrays or the lists, which are the typical programable data.

## III. APPROACHES TO REALISE THE METHODOLOGY

To represent any kinds of the functional dependencies (1) by using the proposed generalized methodology we must develop the linear interpolation (4) programme code and we must build the data sets (5) and (6). Development of the program code realizing the linear interpolation (4) has no any principal difficulties, and can be easily made on different program languages; one the same code will be applicable for any kinds of the functional dependencies (1). Building the data sets (5) and (6) which will represent the functional dependence (1) is the more complicated problem, because of such building is significantly dependent on the existed knowledge about the dependence (1), so we will consider further exactly building of the data sets (5), (6).

Building of the data set (5) representing the $x$ variable grid is actually relatively independent on building of the data set (6) representing the $y$ nodal values of the dependent variable. It is suitable and typical to use the uniform grids with the constant step:

$$x_k = a + \left(k-1\right)\Delta x,\ \Delta x = \frac{b-a}{n-1},\ k = 1,2,\dots,n\ , \qquad (7)$$

where $\Delta x$ is the step of the uniform grid.

Due to the relations (7) we have the set of different grids of the x independent value, and each element of this data set is defined by the count n of the grid nodes. Thus, the relations (7) define the data set (5) required for representing the functional dependencies (1). It is necessary to note, that building the data set (5) is not fully independent because of the built grid must provide the wished small errors of representing the functional dependency (1), so the count of the grid (7) nodes must be substantiated for each particular functional dependency (1), but such substantiations will not be discussed in this research. It is necessary to note also that it is possible to use not only the uniform grids (7) to build the data set (5), and it is possible to use the different not uniform grids, but we will consider in this research only the uniform grids (7) because it will allow without the not necessary complications to represent the proposed universal approaches for building the data required to realize the proposed general uniform methodology for the functional dependencies (1) representing in the mathematical software.

It is naturally that building the data set (6) representing the nodal values of the $y$ dependent value will be significantly depended of the view of the given function $f$ defining the functional dependence (1). The function defining the functional dependence (1) can be defined by different ways including by means the analytical relation, by means the tabled form, by means the differential equation as well as by means others ways. The analytical relations and the tabled data are widely used for giving the functions $f$ defining the functional dependencies (1), but the differential equations and other ways for giving such functions are the specific and are not widely used. So, we will consider further building the data set (6) required to represent the functional dependencies (1) only for the functions $f$ given by means the analytical relation or by means the tabled data.

The case of the exactly defined and given in the analytical view function $f$ representing the dependence (1) is the simplest for building the data sets (5), (6). Really, having the analytical exactly given function we can build the data set (6) by directly using of the relations (3) for the data set (5) representing any previously defined grid (2) and the uniform grid (7) in particular. Thus, we can build the dada sets (5), (6) corresponded to any arbitrary count $n$ nodes of the uniform grid (7).

The case of the function $f$ defined by means the tabled data is more complicated for building the data set (5) required for representing the functional dependence (1). The tabled data representing the function $f$ have the following view:

$$(\xi_1,\eta_1),(\xi_2,\eta_2),\ldots,(\xi_k,\eta_k),\ldots,(\xi_N,\eta_N), \quad (8)$$

where $N$ is the count of the existed data about the function $f$ representing the functional dependence (1); $\xi_k$ and $\eta_k$ are the numbered by the value $k=1,2,\ldots,N$ given values of the $x$ and $y$ values corresponded to the function $f$.

The tabled data (8) are ordinary represent the results of experimental measuring, and exactly such tabled data (8) are most widely used to represent the different kinds of functional dependencies (1) required for mathematical support of the industrial automation systems. Let introduce the follows values:

$$\xi_{\min}=\xi_1,\ \xi_{\max}=\xi_N, \quad (9)$$

where $\xi_{\min}$ and $\xi_{\max}$ are the minimum and maximum values between the independent variable $x$ given values $\xi_k$, $k=1,2,\ldots,N$ used in the function $f$ tabled data (8)

We will consider further the case of the given data (8) covering the interval $[a,b]$ of the functional dependence (1)

$$\xi_{\min}\le a_1,\ \xi_{\max}\ge b, \quad (10)$$

The sense of the conditions (10) is that are the necessary requirements to have the full information about the functional dependence (1) by the given tabled data (8) representing the $f$ function. The existing tabled data (8), (9) and the relations (10) allow us to define the data sets (5), (6) required to represent the functional dependence (1) in the corresponded to the $n=N$ following view (fig. 2):

$$X_N=\{\xi_1,\xi_2,\ldots,\xi_k,\ldots,\xi_N\}, \quad (11)$$

$$Y_N=\{\eta_1,\eta_2,\ldots,\eta_k,\ldots,\eta_n\}. \quad (12)$$

At the same time, the typical is the case of the tabled data (8)
corresponded to the small counts N of the existed data, because of limited possibilities of the experimental measuring or other reasons. The linear interpolation (4) can have the noticed errors of representing the functional dependence (1) in the cases of small counts N of the existed data (8) as schematically shown on fig. 2, so using the data sets (11), (12) based on the existed data (8) can lead to the noticeable errors, and it is necessary have the approaches of extracting the more accurate data from the small existing tabled data (8) by using the hidden regularities of these data to improve the functional dependence (8) representation.

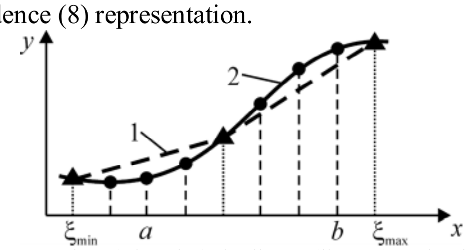


Fig.2. Known data (triangles), its linear (lines 1) and cubic (curve 2)
interpolations as well as the built improved data (circles)

It is proposed to use the continuity of first and second derivatives of the functional dependence (1) as the hidden regularities for extracting the more accurate data from the small existing tabled data (8). Taking into account such regularities will lead us to the well-known [13] cubic spline interpolation of the existed tabled data (8), and we can represent the function $f$ defining the functional dependence (1) in the following view [13]:

$$f(x)=M_{k-1}\frac{(\xi_k-x)^3}{6h_k}+M_k\frac{(x-\xi_{k-1})^3}{6h_k}+$$
$$+\left(\eta_{k-1}-\frac{M_{k-1}h_k^2}{6}\right)\frac{\xi_k-x}{h_k}+\left(\eta_k-\frac{M_kh_k^2}{6}\right)\frac{x-\xi_{k-1}}{h_k},$$
$$\xi_{k-1}\le x\le\xi_k, \quad (13)$$

where $h_k=\xi_k-\xi_{k-1}$; $M_k, k=1,2,\ldots,N$ are the nodal values
of the functional dependence (1) second derivatives.

It is well-known [13] that the nodal values of the second derivative $M_k, k=1,2,\ldots,N$ defining the cubic spline (13) can be find form the linear algebraic equations system:

$$M_1=0,$$
$$\frac{h_k}{6}M_{k-1}+\frac{h_k+h_{k+1}}{3}M_k+\frac{h_{k+1}}{6}M_{k-1}=$$
$$=\frac{\eta_{k+1}-\eta_k}{h_{k+1}}-\frac{\eta_k-\eta_{k-1}}{h_k},k=2,3,\ldots,N-1,$$
$$M_N=0. \quad (14)$$

It seems naturally (fig. 2) that the cubic spline (13), (14) will $R_g$ lead to more accurate results then the linear interpolation (4), because the cubic spline is in agreement with continuity of the values and two derivatives, but the linear interpolation is in agreement with continuity only of the nodal values for the functional dependence (1). Thus, using the cubic spline (13), (14) based on the small existed tabled data (8) we can compute the additional nodal values for having the reliable linear interpolation (4), as it shown on fig. 2. So, to build the data set (6) based on the any wished suitable grid (5) we can use the relations (3) with the function $f$ represented by means the cubic spline (13), (14), even in the case of the small existed tabled data (8). We have the substantiated hope that such data sets (5), (6) defined by means the cubic spline (13), (14) will be best with the linear interpolation (4) even for the small existed data (8) about the function $f$ (fig. 2).

## IV. USING THE PROPOSED METHODOLOGY

We will consider here the example of using the proposed methodology as well as the approaches of its realisation in the particular task about estimation of the temperature of the nuclear fuel inside the cladding of the fuel rods, which can be the component of the mathematical software of the nuclear power plants computerised automation systems as well as the different simulators used for teaching and training of the nuclear power plants operational staff.

The fuel rod (fig. 3a) is the principal structural element of the nuclear fuel for the most modern nuclear power reactors [7, 9, 10]. The nuclear reactions providing the heat power are in the volumes of the nuclear fuel pellets (fig. 3b) inside the sealed cladding of the fuel rods (fig. 3a), and these reactions lead to the temperature field in the fuel pellets (fig. 3b) which can be approximately estimated by the follows relations [7]:

$$T(r) = T_{\mathrm{HC}} - \frac{Q}{4\lambda_{\mathrm{f}}} r^2 + \frac{QR_{\mathrm{h}}^2}{2\lambda_{\mathrm{f}}} \ln\frac{r}{R_{\mathrm{f}}} + Q\cdot C,\; R_{\mathrm{h}} \le r \le R_{\mathrm{f}},$$

$$C = \frac{R_{\mathrm{f}}^2}{4\lambda_{\mathrm{f}}} + \frac{R_{\mathrm{f}}}{2k} - \frac{R_{\mathrm{h}}^2}{2kR_{\mathrm{f}}},\; Q = \frac{W}{n\pi\left(R_{\mathrm{f}}^2 - R_{\mathrm{h}}^2\right)L},$$

$$k = \left(\frac{R_{\mathrm{f}}}{\lambda_{\mathrm{g}}} \ln\frac{R_{\mathrm{g}}}{R_{\mathrm{f}}} + \frac{R_{\mathrm{f}}}{\lambda_{\mathrm{c}}} \ln\frac{R_{\mathrm{c}}}{R_{\mathrm{g}}} + \frac{R_{\mathrm{f}}}{\alpha R_{\mathrm{c}}}\right)^{-1}, \qquad (15)$$

where $T$ is the temperature in a point of the fuel pellet; $r$ is the radial coordinate; $T_{\mathrm{HC}}$ is the temperature of the heat carrier; $Q$ is the intensity of volume heat sources due to the nuclear fission reactions; $\lambda_{\mathrm{f}}$ is the average heat conductivity of the ceramic nuclear fuel; $R_{\mathrm{h}}$ and $R_{\mathrm{f}}$ are the internal and external radii of the pellet; $k$ is the heat transfer coefficient from the fuel pellet to the heat carrier; $R_{\mathrm{g}}$ and $R_{\mathrm{c}}$ are the internal and external radii of the cladding; $\lambda_{\mathrm{g}}$ and $\lambda_{\mathrm{c}}$ are the average heat conductivities in the gases inside the cladding and the structural material of the cladding; $\alpha$ is the heat transfer coefficient between the cladding and the heat carrier; $W$ is the heap power of the reactor; $n$ is the number of the fuel elements in the core of the nuclear reactor.

The relations (15) show us that the temperature of the nuclear pellet is significantly depended on the heat conductivity average value

$$\lambda_{\mathrm{f}} = \frac{1}{T_{\max} - T_{\min}} \int_{T_{\min}}^{T_{\max}} \lambda_{\mathrm{f}}(T)\, dT, \qquad (16)$$

where $T_{\min}$ and $T_{\max}$ are the averaging boundaries; $\lambda_{\mathrm{f}}(T)$ is the temperature dependence of the nuclear fuel pellet heat conductivity.

It is well-known [14] that the ceramic nuclear fuel has the significant temperature dependence $\lambda_{\mathrm{f}}(T)$, and we can see (fig. 4) that the linear and the spline interpolations of the existed data about the temperature dependency $\lambda_{\mathrm{f}}(T)$ has the noticeable differences between some data points. We can see also (fig. 4) that correction of the existed data by using the cubic spline interpolation (13), (14) will allow us to have the data providing the more substantiated linear interpolation (4) which can be used for the further computations.

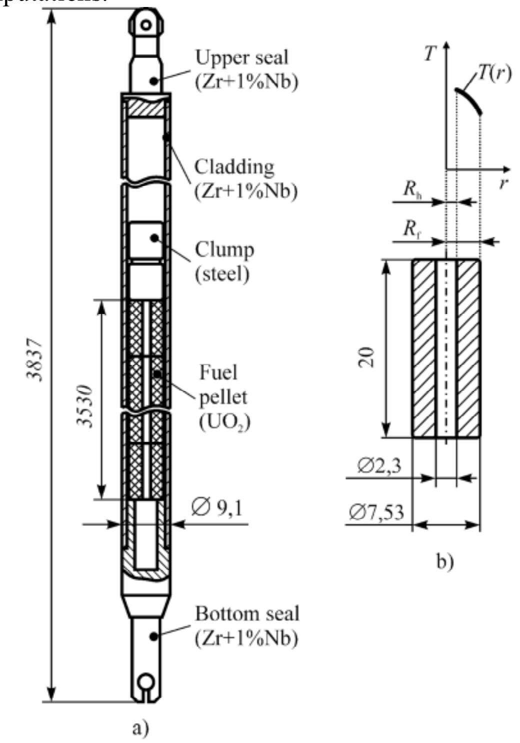


Fig.3. The typical design of the fuel rod (a) as well as the nuclear fuel pellet with their temperature field (b)

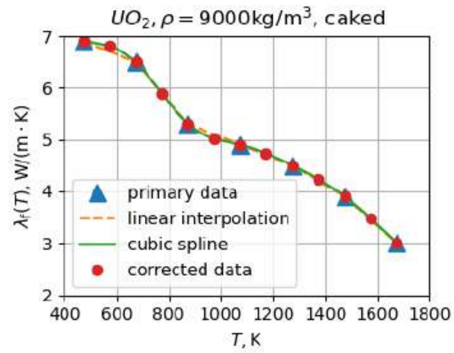


Fig.4. Temperature dependence of the ceramic nuclear fuel

It is naturally that the average value (16) must be defined by the following averaging boundaries agreed with the temperature field (14):

$$T_{\min} = \min_{R_{\mathrm{h}} \le r \le R_{\mathrm{f}}} T(r),\; T_{\max} = \max_{R_{\mathrm{h}} \le r \le R_{\mathrm{f}}} T(r). \qquad (17)$$

At the same time, the temperature field (14) can be defined only for the given value (16), so we have actually the nonlinear problem and to resolve this problem it is possible to use the iteration process started from some arbitrary given values $T_{\min}$, $T_{\max}$ and necessary times continuing with help of the temperature field (14) and the relations. (17). To show such approach we will use the following data [7] to define the temperature field (14):

$$Q = 3000\mathrm{MW}, n = 50856, L = 3530\mathrm{mm}, T_{\mathrm{HC}} = 583\mathrm{K},$$

$$R_{\mathrm{f}} = 3{,}765\mathrm{mm}, R_{\mathrm{g}} = 3{,}86\mathrm{mm}, R_{\mathrm{c}} = 4{,}55\mathrm{mm},$$

$$\alpha = 35\tfrac{\mathrm{kW}}{\mathrm{m^2 \cdot K}}, \lambda_{\mathrm{g}} = 0{,}3\tfrac{\mathrm{W}}{\mathrm{m \cdot K}}, \lambda_{\mathrm{c}} = 20{,}5\tfrac{\mathrm{W}}{\mathrm{m \cdot K}}. \qquad (18)$$

The obtained results (fig. 5) show us that the temperature field (14) in the nuclear fuel pellet is significantly depended on the $\lambda_f$ average value, and the most correct results are corresponded to the following value

$$\lambda_f \cong 5{,}1359 \tfrac{\text{W}}{\text{m}\cdot\text{K}} . \qquad (19)$$

The value (19) is the result of ten iterations with help of the temperature field (14) and the relations (16) and (17).

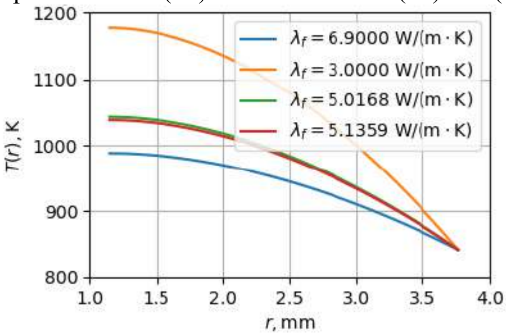


Fig.5. Temperature fields in the ceramic nuclear fuel

The result (19) and fig. 5 are obtained by using the manually built tabled data to represent the nuclear fuel heat conductivity temperature dependence, but it will be more suitable to have automated building of these data further.

## V. CONCLUSIONS

Results were obtained in this research allow formulating the follows conclusions.

The general uniform methodology for functional dependencies representing suitable to use in mathematical support of different purposed industrial computerised automated systems is proposed. This methodology is reduced to linear interpolation of the especially previously prepared tabled data.

The universal approaches for building the data required to realize the proposed general uniform methodology for functional dependencies representing in the mathematical software are proposed. These approaches are reduced to using the cubic splines interpolation of the existed data to have the required for linear interpolation of the tabled data.

Estimation of the temperature fields in the ceramic nuclear fuel pellets taking into account the temperature dependence of the heat conductivity is considered as example of using the proposed methodology and approaches for functional dependencies representation. It is shown that using the cubic interpolation will help us to have the more reliable representations of the functional dependencies even in the case of small volumes of the primarily existed data.

The further researches are planned in the field of intelligent automation of building of the required for linear interpolation tabled data, including by using the different kinds of extrapolations and the artificial intelligence technologies.

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