Mathematical Modelling of Impulse Excitation of a Superwideband PEC Cone Antenna

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***Abstract* — The method for solving initial-boundary electromagnetic problems is proposed. It uses the Green's function method and the Laplace and Meler-Fock integral transforms in a time domain. The model problem of special slotted cone antenna excitation is studied. Integral and series representation of electromagnetic field component are given.**

Keywords* — *initial boundary, cone, antenna, integral transform, approximations

I. Introduction

The superwideband (SWB) system creation and embedding is an important step in the formation of the modern radioengineering [1, 2]. The peculiarities and mechanisms of the physical electromagnetic waves scattering, radiation and receiving processes for the antennas are studied during the non-stationary electromagnetic problems solving for the probing SWB signals. Creating mathematical model of the physical process and solving corresponding mathematical problems are the first stages to produce modern radioengineering and electronics equipments. Harmonic wave diffraction boundary problems for cone and plane angular structures are considered in [3-5]. Green's function for an isotropic PEC cone in time domain is given in [6]. Properties of transient electromagnetic fields in presence of PEG isotropic biconical transmission line [7] are investigated. In this paper a mathematical model of impulse excitation of a cone antenna is proposed. The choice of such surface is caused by undirected and SWB properties conical structures. Investigating the mathematical problem is based on using the rigorous mathematical methods.

II. Formulation of the problem

Let's consider a semi-infinite perfectly conducting conical structure Σ . The point activation source (an electric, $\chi=1$, or magnetic, $\chi=2$, radial dipole) with the moment

$$\vec{P}^{(\chi)}\left(\vec{r},t\right)=\vec{M}^{(\chi)}\delta\left(\vec{r}-\vec{r}_0\right)f\left(t-t_0\right)$$

and current density

$$\vec{j}^{(\chi)}(\vec{r},t)=\frac{\partial}{\partial t}\vec{P}^{(\chi)}(\vec{r},t)$$

is settled in the $B\left(\vec{r}_0\right)$ point, where $\delta\left(\vec{r}-\vec{r}_0\right)$ is a delta-function, and $f(t-t_0)$ function defines the source field changing in time, $f(t-t_0)\equiv 0$, $t<t_0$ (the source is turned on when $t=t_0$). The medium, the conical surface and the source are placed into, is homogeneous, isotropic and stationary with permittivity ε and permeability μ . The task is to find the electromagnetic field $\vec{E}(\vec{r},t)$, $\vec{H}(\vec{r},t)$ in the conical structure and the source presence. The electromagnetic initial-boundary problem is reduced to the mathematical physics initial- boundary problem for Debye's potential $\upsilon^{(\chi)}(\vec{r},t)$, $\chi=1,2$, corresponding to the total field $\vec{E}$, $\vec{H}$ that satisfies the following conditions in each moment of time:

1) the three-dimensional wave equation

$$(\Delta-\frac{1}{a^2}\frac{\partial^2}{\partial t^2})\upsilon^{(\chi)}(\vec{r},t)=-\hat{F}^{(\chi)}\left(\vec{r},t\right),\ \vec{r}\notin\Sigma, \tag{1}$$

$$\hat{F}^{(\chi)}\left(\vec{r},t\right)=\frac{1}{\varepsilon^{2-\chi}\mu^{\chi-1}r}M_r^{(\chi)}\delta(\vec{r}-\vec{r}_0)f(t-t_0),$$

$$\varepsilon\mu=1/a^2;$$

2) initial conditions

$$\upsilon^{(\chi)}\equiv 0\equiv\frac{\partial\upsilon^{(\chi)}}{\partial t},\ t\le t_0; \tag{2}$$

3) boundary conditions

$$\frac{\partial^{\chi-1}}{\partial n^{\chi-1}}\left(\frac{\partial\upsilon^{(\chi)}}{\partial t}\right)\Bigg|_{\Sigma}=0; \tag{3}$$

4) the finite stored energy condition

$$\iiint\limits_V(\left|\frac{\partial\upsilon^{(\chi)}}{\partial t}\right|^2+\left|\nabla\upsilon^{(\chi)}\right|^2)dV<\infty. \tag{4}$$

The boundary problem of the mathematical physics in such statement (1)-(4) has the unique solution.

The Debye's potential $\upsilon^{(\chi)}(\vec{r},t)$ can be presented in the following form

$$\upsilon^{(\chi)}(\vec{r},t)=\upsilon_0^{(\chi)}(\vec{r},t)+\upsilon_1^{(\chi)}(\vec{r},t),$$

where

$$\upsilon_0^{(\chi)}=-\frac{M_r^{(\chi)}}{4\pi r_0\varepsilon^{2-\chi}\mu^{\chi-1}}\frac{1}{R}f(t-t_0-\frac{1}{a}R)\eta(t-t_0-\frac{1}{a}R)$$

is the potential corresponding to the source field, $\upsilon_1^{(\chi)}(\vec{r},t)$ - unknown Debye's potential that corresponds to the scattered field, $\eta(\xi)$ - the Heaviside function, $R=|\vec{r}-\vec{r}_0|$. Let's use Green's function method and define the Debye's potential $\upsilon^{(\chi)}(\vec{r},t)$ according to the following formula per this method:

$$\upsilon^{(\chi)}(\vec{r},t)=\frac{M_r^{(\chi)}}{r_0\varepsilon^{2-\chi}\mu^{\chi-1}}\int\limits_0^{t-t_0}G^{(\chi)}\left(\vec{r}-\vec{r}_0,z\right)f\left(t-t_0-z\right)dz,$$

where $G^{(\chi)}\left(\vec{r},t\right)$ - is the Green's function for the boundary problem (1)-(4) that satisfies the wave equation

$$(\Delta-\frac{1}{a^2}\frac{\partial^2}{\partial t^2})G^{(\chi)}(\vec{r},t)=-\delta\left(\vec{r}-\vec{r}_0\right)\delta\left(t-t_0\right),\ \vec{r}\notin\Sigma;$$

the initial condition, the boundary condition like (3), the finite stored energy condition. The Green's function $G^{(\chi)}(\vec{r},t)$ can be rewritten in the following way:

$$G^{(\chi)}(\vec{r},t)=G_0(\vec{r},t)+G_1^{(\chi)}(\vec{r},t), \tag{5}$$

where $G_0(\vec r,t)$ is the Green's function for the free space,

$$G_0(\vec r,t)=\frac{\delta\left[t-t_0-R/a\right]}{4\pi R},$$

and the Debye's potential $\upsilon_1^{(\chi)}(\vec r,t)$ for the scattered field can be written as

$$\upsilon_1^{(\chi)}(\vec r,t)=\frac{M_r^{(\chi)}}{r_0\varepsilon^{2-\chi}\mu^{\chi-1}}\int_0^{t-t_0}G_1^{(\chi)}\left(\vec r-\vec r_0,z\right)f\left(t-t_0-z\right)dz\ .(6)$$

Thus, the given electromagnetic problem solution results in finding the Green's function $G_1^{(\chi)}(\vec r,t)$ for the complex conical structure Σ.

III. The solution method

Let's construct the Green's function for the boundary problem for a single cone $\Sigma:\theta=\gamma$ with N periodic longitudinal slots with the angle slot width d and the period $l=2\pi/N$. The task is to find the Green's function $G^{(\chi)}(\vec r,t)$ for the cone Σ. The Laplace transform can be applied to the Green's function $G^{(\chi)}(\vec r,t)$ for the time parameter

$$G^{s,(\chi)}=G^{s,(\chi)}(\vec r)=\int_0^{+\infty}G^{(\chi)}(\vec r,t)e^{-st}dt\ ,\ \operatorname{Re}s>0\,.$$

We can state the boundary problem for the $G^{s,(\chi)}$ representation. One should find the $G^{s,(\chi)}$ function that satisfies:

1) the homogeneous Helmholtz equation

$$(\Delta-q^2)G^{s,(\chi)}(\vec r)=-e^{-st_0}\delta\left(\vec r-\vec r_0\right),\ \vec r\notin\Sigma_0\,,\ q=\frac{s}{a};$$

2) the boundary condition $\dfrac{\partial^{\chi-1}}{\partial n^{\chi-1}}G^{s,(\chi)}\Big|_{\Sigma}=0$;
3) the absorption limit condition;
4) the finite stored energy condition.

Let's set $q>0$ for determinacy (we will add analytical extension for q into the complex semi-plane $\operatorname{Re}q>0(\operatorname{Re}s>0)$ afterwards). Taking into account the solution uniqueness for the stated boundary problem and the Laplace transform peculiarities, the boundary problem for the $G^{s,(\chi)}$ function has the unique solution as well. The unknown function $G^{s,(\chi)}$ satisfies the Helmholtz equation beyond the cone Σ and the source. The spherical coordinate system r,θ,φ, is considered. The center of this system is the conical structure tip $(r=0)$. For finding the Green's function $G^{s,(\chi)}$ one can use the Kontorovich-Lebedev transforms:

$$\hat g(\tau)=\int_0^{+\infty}g(r)\frac{K_{i\tau}(qr)}{\sqrt r}dr\ ,\qquad(7)$$

$$g(r)=\frac{2}{\pi^2}\int_0^{+\infty}\tau sh\pi\tau\hat g(\tau)\frac{K_{i\tau}(qr)}{\sqrt r}d\tau\ ,\qquad(8)$$

here $K_{i\tau}(qr)$ is the McDonald's function of the imaginary order.

According to (5) the function $G^{s,(\chi)}$ has the following form:

$$G^{s,(\chi)}(\vec r)=G_0^s(\vec r)+G_1^{s,(\chi)}(\vec r)\,,\ G_0^s(\vec r)=e^{-st_0}\frac{e^{-qR}}{4\pi R}.$$

The unknown function $G^{s,(\chi)}$ (similar to $G_0^s(\vec r)$) can be found in the form of Kontorovich-Lebedev integral transforms (7), (8)

$$G_1^{s,(\chi)}(\vec r)=\frac{2}{\pi^2}\int_0^{+\infty}\tau sh\pi\tau\hat G_1^{s,(\chi)}\frac{K_{i\tau}(qr)}{\sqrt r}d\tau\ ,\qquad(9)$$

$$\hat G_1^{s,(\chi)}=\sum_{m=-\infty}^{+\infty}b_{m\tau}^sU_{m,i\tau}^{(\chi)}(\theta,\varphi)\,,$$

$b_{m\tau}^s$ are known coefficients, where $U_{m,i\tau}^{(\chi)}$ is an unknown function. The function dependence on the q parameter is only included into the integral transform kernel (9) and into the $b_{m\tau}^s$ coefficients.

Statement 1: The function $U_{m,i\tau}^{(\chi)}$) does not depend on the parameter q.

Statement 2: The Green's function $G_1^{(\chi)}(\vec r,t)$ of boundary problem of the wave equation for a single cone with longitudinal slots can be presented in the only possible way according to the following integral

$$G_1^{(\chi)}(\vec r,t)=\int_0^{+\infty}\tau th\pi\tau\tilde G_1^{(\chi)}P_{-1/2+i\tau}(chb)d\tau\ ,\qquad(10)$$

where

$\tilde G_1^{(\chi)}=-\dfrac{1}{r}\eta\left(t-t_0-\dfrac{r+r_0}{a}\right)\displaystyle\sum_{m=-\infty}^{+\infty}d_{m\tau}U_{m,i\tau}^{(\chi)}$, $d_{m\tau}$ - known coefficients, $chb=\dfrac{a^2\left(t-t_0\right)^2-r^2-r_0^2}{2rr_0}$, $b\in[0,+\infty)$.

The Integral representation (10) can be reduced to the Meler-Fock integral transforms

$$\hat F(\tau)=\int_0^{+\infty}shbF(b)P_{-1/2+i\tau}(chb)db\ ,\ \ b\in[0,+\infty),\qquad(11)$$

$$F(b)=\int_0^{+\infty}\tau th\pi\tau\hat F(\tau)P_{-1/2+i\tau}(chb)d\tau\qquad(12)$$

Thus, the Meler-Fock integral transform (11), (12) is generalization for the Kontorovich-Lebedev integral transform (7), (8) for the boundary problems solving for wedges, angular sectors, cones and bicones in the time domain. The potential $\upsilon_1^{(\chi)}(\vec r,t)$ corresponding to the scattered field has the following form:

$$\upsilon_1^{(\chi)}(\vec r,t)=\frac{1}{r{r_0}^2}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\times$$

$$\times\sum_{m=-\infty}^{+\infty}\int_0^{+\infty}\tau th\pi\tau\frac{\Gamma(1/2-m+i\tau)}{\Gamma(1/2+m+i\tau)}\tilde b_{m\tau}^{(\chi),p}U_{m,i\tau}^{(\chi)}(\theta,\varphi)\Phi_{i\tau}(t,r)d\tau,$$

$$\Phi_{i\tau}(t,r)=\frac{i}{\pi th\pi\tau}\left[\widehat\Psi_{i\tau}(t,r)-\widehat\Psi_{-i\tau}(t,r)\right],$$

$$\widehat\Psi_{i\tau}(t,r)=\int_{\frac{r+r_0}{a}}^{t-t_0}f(t-t_0-z)Q_{-1/2+i\tau}(chb(z))dz\,,$$

$Q_{-i/2+i\tau}(u)$ is the Legendre function of the second type. It should be noted that the function $U_{m,i\tau}^{(\chi)}(\theta,\varphi)$ can be found by using methods proposed in [8]. Thus $\upsilon_1^{(\chi)}(\vec r,t)$ can be represented in the form

$$\upsilon_1^{(\chi)}(\vec{r},t)=\frac{1}{rr_0^{\;2}}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\times$$
$$\times\sum_{m=-\infty}^{+\infty}\int_0^{+\infty}\tilde{d}_{m\tau}^{(\chi)}\hat{U}_{m,i\tau}^{(\chi)}\left[\widehat{\Psi}_{i\tau}-\widehat{\Psi}_{-i\tau}\right]d\tau. \qquad (13)$$

Let's consider the case of the open cone excitation when the source is located at its axis ($\theta_0=\pi$). Then the representation for $\upsilon_1^{(\chi)}(\vec{r},t)$ (13) is simplified

$$\upsilon_1^{(\chi)}(\vec{r},t)=\frac{\eta\left(t-t_0-\dfrac{r+r_0}{a}\right)}{rr_0^{\;2}}\int_0^{+\infty}\hat{U}_{0,i\tau}^{(\chi)}\left[\widehat{\Psi}_{i\tau}-\widehat{\Psi}_{-i\tau}\right]d\tau. \quad (14)$$

The integral representations (13), (14) are convenient, for instance, for the field analysis in the transition regions. In order to study the spatial field distribution in case of the close source location to the tip of the conical structure ($r_0<<1$) and near the conical structure tip ($r<<1$), it is convenient to use the representation for the potential $\upsilon_1^{(\chi)}(\vec{r},t)$ as a series:

$$\upsilon_1^{(\chi)}(\vec{r},t)=-\frac{2\pi}{rr_0^{\;2}}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\times$$
$$\times\sum_{s=o}^{+\infty}\frac{\overset{\frown}{g}_{\hat{\mu}}^{(\chi)}}{\dfrac{d}{d\hat{\mu}}\overset{\frown}{G}_{\hat{\mu}}^{(\chi)}}\Bigg|_{\hat{\mu}=\hat{\hat{\mu}}_s^{(\chi)}}\widehat{\Psi}_{\hat{\hat{\mu}}_s^{(\chi)}}(t,r)\,,$$

$U_{0,\hat{\mu}}^{(\chi)}(\theta,\varphi)=\dfrac{\overset{\frown}{g}_{\hat{\mu}}^{(\chi)}(\theta,\varphi)}{\overset{\frown}{G}_{\hat{\mu}}^{(\chi)}}$, $\hat{\hat{\mu}}_s^{(\chi)}$ are simple roots of the function $\overset{\frown}{G}_{\hat{\mu}}^{(\chi)}$.

IV. Field Approximations

The electric field components in case of the excitation by the electric dipole ($\chi=1$) have the following form:

$$E_{\theta,1}=\frac{a\hat{p}_1}{4\pi rr_0^{\;2}}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\times$$
$$\times\sum_{m=-\infty}^{+\infty}(-1)^m e^{-im\varphi_0}\int_0^{+\infty}\tau th\pi\tau\frac{\Gamma(1/2-m+i\tau)}{\Gamma(1/2+m+i\tau)}\hat{b}_{m\tau}^{(1),p}\frac{\partial}{\partial\theta}\hat{U}_{m,i\tau}^{(1)}(\theta,\varphi)\frac{\partial}{\partial r}\{r\Phi_{i\tau}(t,r)\}\,d\tau,$$

$$E_{\varphi,1}=\frac{a\hat{p}_1}{4\pi rr_0^{\;2}\sin\theta}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\times$$
$$\times\sum_{m=-\infty}^{+\infty}(-1)^m e^{-im\varphi_0}\int_0^{\infty}\tau th\pi\tau\frac{\Gamma(1/2-m+i\tau)}{\Gamma(1/2+m+i\tau)}\hat{b}_{m\tau}^{(1),p}\frac{\partial}{\partial\varphi}\hat{U}_{m,i\tau}^{(1)}(\theta,\varphi)\frac{\partial}{\partial r}\{r\Phi_{i\tau}(t,r)\}\,d\tau.$$

Assuming that the source is placed onto the considered surface axis ($\theta_0=\pi, m=0$), components $E_{\theta,1}$, $E_{\varphi,1}$ of the scattered electric field can be represented in the form

$$E_{\theta,1}=-\frac{a\hat{p}_1}{2\pi r^2 r_0^{\;2}}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\times$$
$$\times\sum_{s=o}^{+\infty}\hat{\hat{\mu}}_s^{(1)}\frac{\dfrac{\partial}{\partial\theta}\overset{\frown}{g}_{\hat{\mu}}^{(1)}}{\dfrac{d}{d\hat{\mu}}\overset{\frown}{G}_{\hat{\mu}}^{(1)}}\Bigg|_{\hat{\mu}=\hat{\hat{\mu}}_s^{(1)}}P_{-1/2+\hat{\hat{\mu}}_s^{(1)}}(\cos\gamma)\frac{\partial}{\partial r}\left[\widehat{\Psi}_{\hat{\hat{\mu}}_s^{(1)}}(t,r)\right], \quad (15)$$

$$E_{\varphi,1}=-\frac{a\hat{p}_1}{2\pi r^2 r_0^{\;2}\sin\theta}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\times$$
$$\times\sum_{s=o}^{+\infty}\hat{\hat{\mu}}_s^{(1)}\frac{\dfrac{\partial}{\partial\varphi}\overset{\frown}{g}_{\hat{\mu}}^{(1)}}{\dfrac{d}{d\hat{\mu}}\overset{\frown}{G}_{\hat{\mu}}^{(1)}}\Bigg|_{\hat{\mu}=\hat{\hat{\mu}}_s^{(1)}}P_{-1/2+\hat{\hat{\mu}}_s^{(1)}}(\cos\gamma)\frac{\partial}{\partial r}\left[\widehat{\Psi}_{\hat{\hat{\mu}}_s^{(1)}}(t,r)\right].$$

If the source is located near the cone tip ($r_0<<1$, $r_0<r$), the dominant mode defining the field in each of the before mentioned cases where $chb>>1$ can be selected from the series to study the field behavior near this tip ($r<<1,\ r<r_0$) . Thus the representation (15) can be transformed as follows:

$$E_{\theta,1}=-\frac{a\hat{p}_1}{2\sqrt{\pi}}\sum_{s=o}^{+\infty}(rr_0)^{-3/2+\hat{\hat{\mu}}_s^{(1)}}\frac{\Gamma\left(3/2+\hat{\hat{\mu}}_s^{(1)}\right)}{\Gamma\left(\hat{\hat{\mu}}_s^{(1)}\right)}\frac{\dfrac{\partial}{\partial\theta}\overset{\frown}{g}_{\hat{\mu}}^{(1)}(\theta,\varphi)}{\dfrac{d}{d\hat{\mu}}\overset{\frown}{G}_{\hat{\mu}}^{(1)}}\Bigg|_{\hat{\mu}=\hat{\hat{\mu}}_s^{(1)}}P_{-1/2+\hat{\hat{\mu}}_s^{(1)}}(\cos\gamma)\times$$
$$\times\int_{\frac{r+r_0}{a}}^{t-t_0}\frac{f(t-t_0-z)\times\overset{\frown}{h}(z,r,r_0)}{\left(a^2z^2-r^2-r_0^2\right)^{\hat{\hat{\mu}}_s^{(1)}+1/2}}\left[1+O\left(\left(\frac{rr_0}{a^2z^2-r^2-r_0^2}\right)^2\right)\right]dz\,. \quad (16)$$

It is supposed in the representation (16) that $t-t_0>\left(r+r_0\right)/a$. The summation in (16) is made by the numbers of the spectral values, the least of which ($\hat{\hat{\mu}}_0^{(1)}$) is more than 0.5 for each of the conical structure parameters. One of the spatial spectrum features is the fact that its neighbor values differ from each other almost by unit. This means that in case of $rr_0<<1$, the most significant deposit into the series sum corresponds to the first series term (the least spectrum value $\hat{\hat{\mu}}_0^{(1)}$). The mode amplitude factor will be big if $rr_0<<1$, while the absolute value of the second series term corresponding to $\hat{\hat{\mu}}_1^{(1)}>3/2$ is much smaller than the first one. The third series term is much smaller than the second provided that $rr_0<<1$ etc. Thus, if the condition $rr_0<<1$ is valid, it is enough to consider only the first term of the series (16) to obtain the single-mode approach in the considered cases. In this particular case ($\chi=1$) the following approximation for $E_{\theta,1}$ is obtained from (16):

$$E_{\theta,1}^{(0)}=-\frac{a\hat{p}_1}{2\sqrt{\pi}}\eta\left(t-t_0-\frac{r+r_0}{a}\right)r_0^{-3/2+\hat{\hat{\mu}}_0^{(1)}}r^{-3/2+\hat{\hat{\mu}}_0^{(1)}}\frac{\Gamma\left(3/2+\hat{\hat{\mu}}_0^{(1)}\right)}{\Gamma\left(\hat{\hat{\mu}}_0^{(1)}\right)}P_{-1/2+\hat{\hat{\mu}}_0^{(1)}}(\cos\gamma)\times$$
$$\times\frac{\dfrac{\partial}{\partial\theta}\overset{\frown}{g}_{\hat{\mu}}^{(1)}(\theta,\varphi)}{\dfrac{d}{d\hat{\mu}}\overset{\frown}{G}_{\hat{\mu}}^{(1)}}\Bigg|_{\hat{\mu}=\hat{\hat{\mu}}_0^{(1)}}\hat{\hat{F}}_{\hat{\hat{\mu}}_0^{(1)}}^{(1)}\left(t;r,r_0\right), \quad (17)$$

$$\hat{\hat{F}}_{\hat{\hat{\mu}}_0^{(1)}}^{(1)}\left(t;r,r_0\right)=\int_{\frac{r+r_0}{a}}^{t-t_0}\frac{f(t-t_0-z)}{\left(a^2z^2-r^2-r_0^2\right)^{\hat{\hat{\mu}}_0^{(1)}+1/2}}\overset{\frown}{h}\left(z,r,r_0\right)dz\,. \quad (18)$$

In the considered excitation case the electromagnetic field near the center of the conical surface behaves as follows:

$$\left|\vec{E}\right|\sim r^{-1+\alpha}\,,\ \left|\vec{H}\right|\sim r^{\alpha}\,,\ r<<1\,,$$
$$\alpha=-1/2+\hat{\hat{\mu}}_0^{(1)}\,,\ \hat{\hat{\mu}}_0^{(1)}=\min_s\hat{\hat{\mu}}_s^{(1)}\,.$$

As it follows from (17), the field components dependence on the time parameter t in case of the close source location to the apex ($r_0 << 1$), is described by the function $\hat{\hat{F}}^{(1)}_{\hat{\mu}_0^{(1)}}(t;r,r_0)$ (18) that includes the $f(t-t_0)$ function, defining the considered physical process changing in time. The form of the function $\hat{\hat{F}}^{(1)}_{\hat{\mu}_0^{(1)}}(t;r,r_0)$ for some specific cases is given below:

1) δ -impulse

$$f(t-t_0)=\delta(t-t_0)\,, \qquad (19)$$

$$\hat{F}^{(1)}_{\hat{\mu}_0^{(1)}}(t;r,r_0)=\frac{\overset{\frown}{h}(t-t_0,r,r_0)}{\left(a^2(t-t_0)^2-r^2-r_0^2\right)^{\hat{\mu}_0^{(1)}+1/2}}\,, \qquad (20)$$

$$\overset{\frown}{h}(z,r,r_0)=\frac{a^2z^2+r^2-r_0^2}{\sqrt{a^2z^2-(r+r_0)^2}\sqrt{a^2z^2-(r-r_0)^2}}\,;$$

2) a rectangular impulse

$$f(t-t_0)=\hat{A}\left[\eta(t-t_0)-\eta(t-t_0-\Delta\hat{\tau})\right],$$

where $\hat{A}$ is the impulse amplitude, $\Delta\hat{\tau}$ is the impulse length,

$$\hat{F}^{(1)}_{\hat{\mu}_0^{(1)}}(t;r,r_0)=\hat{A}\int\limits_{t-t_0-\Delta\hat{\tau}}^{t-t_0}\frac{\overset{\frown}{h}(z,r,r_0)}{\left(a^2z^2-r^2-r_0^2\right)^{\hat{\mu}_0^{(1)}+1/2}}dz\,;$$

3) $f(t-t_0)=\hat{A}e^{-\hat{\beta}^2(t-t_0-\Delta\hat{\tau}/2)^2}\left[\eta(t-t_0)-\eta(t-t_0-\Delta\hat{\tau})\right]$,

$$\hat{F}^{(1)}_{\hat{\mu}_0^{(1)}}(t;r,r_0)=\hat{A}\int\limits_{t-t_0-\Delta\hat{\tau}}^{t-t_0}\frac{e^{-\hat{\beta}^2(t-t_0-\Delta\hat{\tau}/2-z)^2}}{\left(a^2z^2-r^2-r_0^2\right)^{\hat{\mu}_0^{(1)}+1/2}}\overset{\frown}{h}(z,r,r_0)dz\,.$$

In case of the δ - impulse source (19), (20) $E^{(0)}_{\theta,1}$ is represented as follows:

$$E^{(0)}_{\theta,1}=-\frac{a\hat{p}_1}{2\sqrt{\pi}}\eta\left(t-t_0-\frac{r+r_0}{a}\right)r_0^{-3/2+\hat{\mu}_0^{(1)}}r^{-3/2+\hat{\mu}_0^{(1)}}\times$$

$$\times\frac{\Gamma\left(3/2+\hat{\mu}_0^{(1)}\right)}{\Gamma\left(\hat{\mu}_0^{(1)}\right)}P_{-1/2+\hat{\mu}_0^{(1)}}(\cos\gamma)\frac{\frac{\partial}{\partial\theta}\overset{\frown}{g}^{(1)}_{\hat{\mu}}(\theta,\varphi)}{\frac{d}{d\hat{\mu}}\overset{\frown}{G}^{(1)}_{\hat{\mu}}}\Bigg|_{\hat{\mu}=\hat{\mu}_0^{(1)}}\times$$

$$\times\frac{\overset{\frown}{h}(t-t_0,r,r_0)}{\left(a^2(t-t_0)^2-r^2-r_0^2\right)^{\hat{\mu}_0^{(1)}+1/2}} \qquad (21)$$

The expression for $E^{(0)}_{\theta,1}$ (21), provided that $a(t-t_0)>>1$, can be obtained from (21):

$$E^{(0)}_{\theta,1}=-\frac{a\hat{p}_1}{2\sqrt{\pi}}r_0^{-3/2+\hat{\mu}_0^{(1)}}r^{-3/2+\hat{\mu}_0^{(1)}}\frac{\left(\hat{\mu}_0^{(1)}+1/2\right)}{\frac{d}{d\hat{\mu}}\overset{\frown}{G}^{(1)}_{\hat{\mu}}\Big|_{\hat{\mu}=\hat{\mu}_0^{(1)}}}\frac{\Gamma\left(\hat{\mu}_0^{(1)}+1/2\right)}{\Gamma\left(\hat{\mu}_0^{(1)}\right)}\times$$

$$\times P_{-1/2+\hat{\mu}_0^{(1)}}(\cos\gamma)\frac{\frac{\partial}{\partial\theta}\overset{\frown}{g}^{(1)}_{\hat{\mu}_0^{(1)}}(\theta,\varphi)}{\left(a(t-t_0)\right)^{2\hat{\mu}_0^{(1)}+1}}\left[1+O\left(a^{-2}(t-t_0)^{-2}\right)\right].$$

Such approximation is the late-time one. As a result, the asymptotic decomposition of $E_{\theta,1}$ by little parameter $\overset{\frown}{\xi}$ is derived

$$E_{\theta,1}=\frac{a\hat{p}_1}{4\pi r r_0^2}\eta\left(t-t_0-\frac{r+r_0}{a}\right)\sum_{m=-\infty}^{+\infty}(-1)^m e^{-im\varphi_0}\int\limits_0^{+\infty}\tau th\pi\tau\times$$

$$\times\left[1+\frac{r+r_0}{2r_0}\left(\tau^2+1/4\right)\right]\frac{\Gamma(1/2-m+i\tau)}{\Gamma(1/2+m+i\tau)}P_{-1/2+i\tau}(\cos\gamma)\times$$

$$\times\frac{\partial}{\partial\theta}\hat{U}^{(1)}_{m,i\tau}(\theta,\varphi)d\tau+O\left(\overset{\frown}{\xi}\right);$$

$$\overset{\frown}{\xi}=\frac{a^2(t-t_0)^2-(r+r_0)^2}{2rr_0}<<1\,.$$

Another field approximations can be obtained from the representations (13)-(15).

V. Conclusions

The strict method for solving a model problem of impulse superwideband antenna excitation is proposed. This method is based on using the Green's function apparatus and integrals transforms. The Meler-Fock integral transforms are applied to solving the wave equation in case of cone excitation and arbitrary field dependence on a time parameter. Field approximations are given for the source placement onto the considered surface axis. Results of this article can be used for the non-antenna theory and producing wideband and superwide antennas.

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