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PARAMETRIC SETTING OF THE MODIFIED WADE FORECAST MODEL

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The problem of parametric setting of the modified Wade forecast model is considered. The similarity of the predictive models of Brown and Wade is analytically evaluated. Critical regions are determined on the extended admissible set of the smoothing parameter, within which the quality of the Brown and Wade models is significantly different. Recommendations are given regarding the parametric tuning of the Wade predictive model based on retrospective analysis by criterion. The necessity of independent parametric tuning of Brown and Wade models is shown. An example of the practical use of the proposed recommendations is given. The results allow the user to intelligently select and tune the specified predictive models.

Introduction

Modern technologies for analysis and forecasting of time series include dozens of classical methods [1], as well as an impressive number of their modifications [2]. Their diversity can be estimated by the theoretical and methodological base of classification of forecasting methods, one of the variants of which [3] is shown in fig. 1.

Despite the variety of complex models and methods tested in practice, simple and reliable one-parameter models do not lose popularity, including the model of simple exponential smoothing or Brown's model [4, 5].

The predictive estimate in Brown's model is determined by the exponential weighted value of the time series:

$$\hat{y}_t = \alpha y_{t-1} + \alpha(1-\alpha) y_{t-2} + \ldots + \alpha(1-\alpha)^{n-1} y_{t-n} = \sum_{i=1}^{n} \alpha(1-\alpha)^{i-1} y_{t-i}, \quad (1)$$

where $\hat{y}_t$ is an estimation (forecast) of the time series value for a time point t; $y_{t-1}, y_{t-2}, \ldots, y_{t-n}$ – the values of the series at the corresponding times; n – sample length of the time series; α – smoothing parameter.

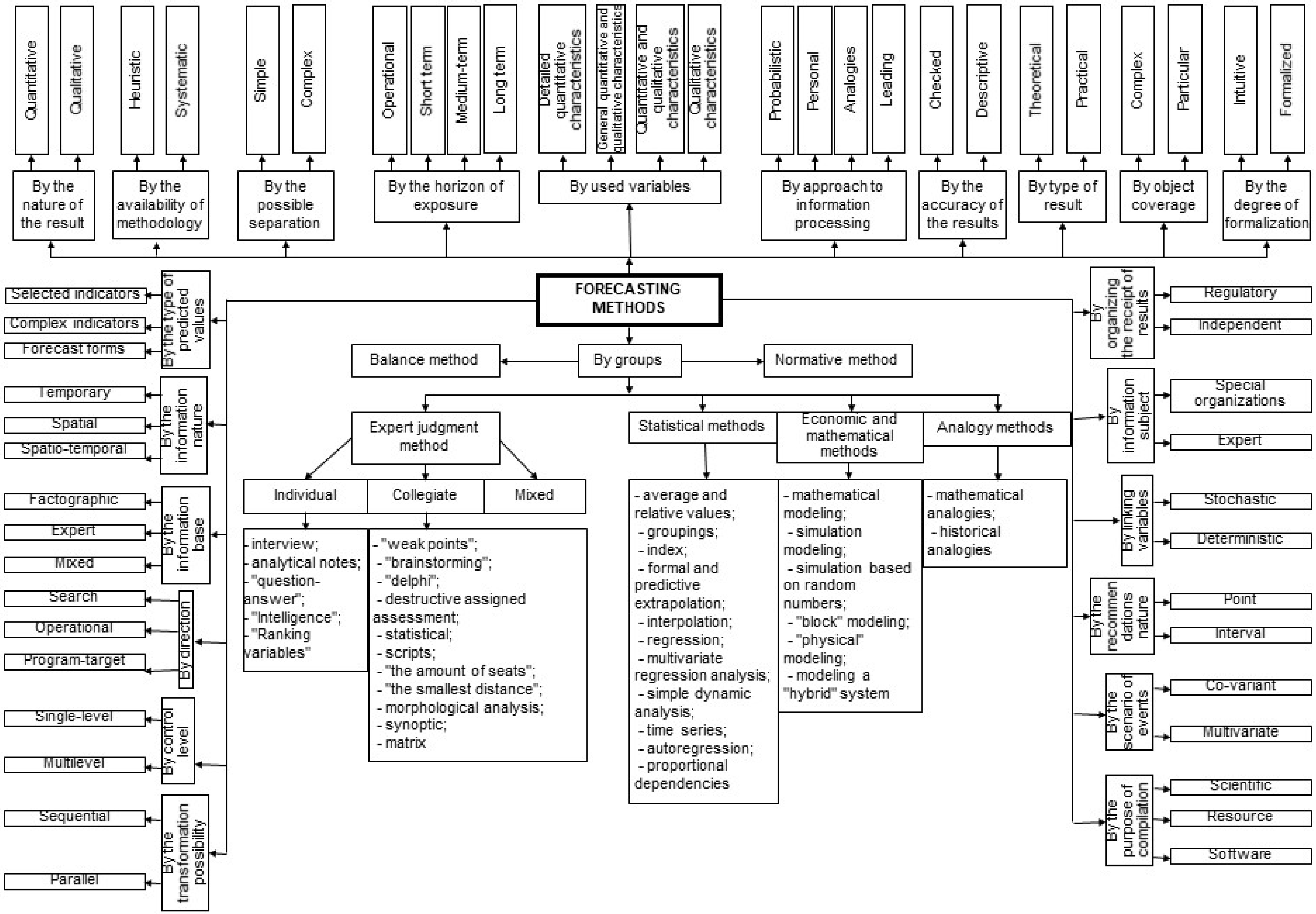

Fig. 1. Theoretical and methodological basis for the classification of forecasting methods [3]

Brown's recurrent formula for smoothing the time series [5] can be obtained by grouping from the formula (1):

$$\hat{y}_t = \alpha y_{t-1} + (1-\alpha)\left(\alpha y_{t-2} + \alpha(1-\alpha)y_{t-3} + \ldots + \alpha(1-\alpha)^{n-1} y_{t-n}\right) = \alpha y_{t-1} + (1-\alpha)\hat{y}_{t-1}, \quad (2)$$

where $\hat{y}_{t-1}$ – estimate for the previous point in time $(t-1)$.

The smoothing parameter is usually selected from the classical range of admissible values $K_c = \{\alpha :\ 0 \le \alpha \le 1\}$, which ensures the convergence of the sum of the sequence of weight coefficients in the formula (1)

$$\{a_k\}_{k=1}^{n} = \alpha,\ \alpha(1-\alpha),\ \ldots,\ \alpha(1-\alpha)^{n-1} \quad (3)$$

to one.

The classical range can be extended to $K_{ext} = \{\alpha :\ 0 \le \alpha \le 2\}$ without violating the convergence conditions [8, 9]. In this case, sequence (3) changes from being constant on the interval $\alpha \in [0,\ 1]$ to alternating on the interval $\alpha \in (1,\ 2]$

Let's consider the function of the sum of the sequence (3) on the number of its elements n on the extended set K_{ext} of the smoothing parameter α (fig. 2):

$$S_n = \sum_{i=1}^{n} \alpha(1-\alpha)^i = 1-(1-\alpha)^n. \quad (4)$$

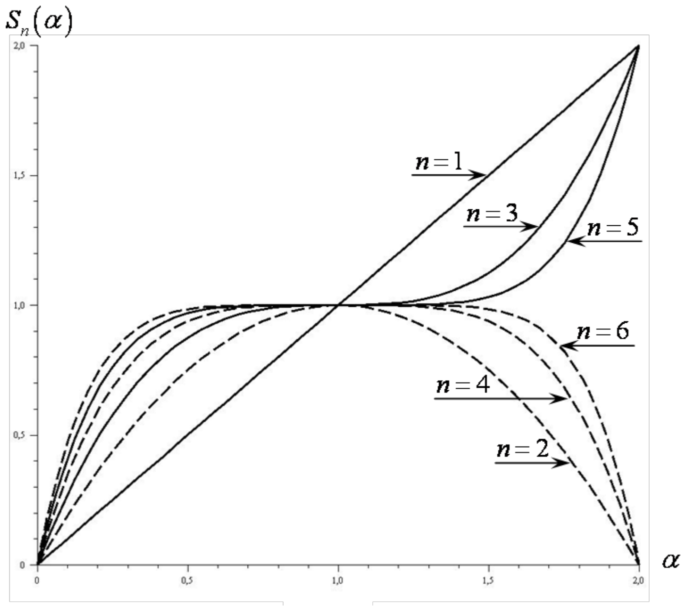

Fig. 2. Dependence of the sum of the coefficients of the Brown model on the smoothing parameter α and the number of elements of the series n on the extended set K_{ext}

Fig. 2 shows that the sum of the coefficients in formula (1) is close to unity not in the entire range K_{ext}. In order to correct this feature, a modification was proposed by R. Wade [5–7]. It consists in multiplying Brown's predictive estimate by the coefficient

$$k_W = \frac{1}{\sum_{i=1}^{n} \alpha (1-\alpha)^{i-1}} \tag{5}$$

to ensure that the sum of the weighting factors is equal to one.

The modified Wade model has the form

$$\hat{y}_t = k_W = \frac{\sum_{i=1}^{n} \alpha (1-\alpha)^{i-1} y_{t-i}}{\sum_{i=1}^{n} \alpha (1-\alpha)^{i-1}} = \frac{\sum_{i=1}^{n} \alpha (1-\alpha)^{i-1} y_{t-i}}{1-(1-\alpha)^n}. \tag{6}$$

When using a model of the form (6), the user faces two main tasks: the estimation of the similarity of the Brown and Wade models, as well as the choice of the smoothing parameter. Moreover, both problems should be solved on the extended set K_{ext}.

Estimating the similarity of the Brown and Wade models

Since the predictive estimates in models (2) and (5) differ by a coefficient $k_W = \frac{1}{S_n}$, it is its closeness to unity that determines the degree of similarity of the two models. Let us estimate it analytically, transforming dependence (4) as follows

$$S_n' = 1 - |1-\alpha|^n, \tag{7}$$

i.e. by mirroring a group of growing branches relative to a unit level (Fig. 3).

In fig. 3, in addition to dependence (7), a plane is shown at the level of 0.995, which the author of the modification R. Wade himself recommended as the boundary of the transition from the modified model to the classical one [5]. It cuts off the region of parameters in the plane (α, n), within which the predictive estimates of the two models differ by less than 0.5%.

The boundaries of this area can be found analytically from the relation

$$1-|1-\alpha|^n \ge 0.995\,. \tag{8}$$

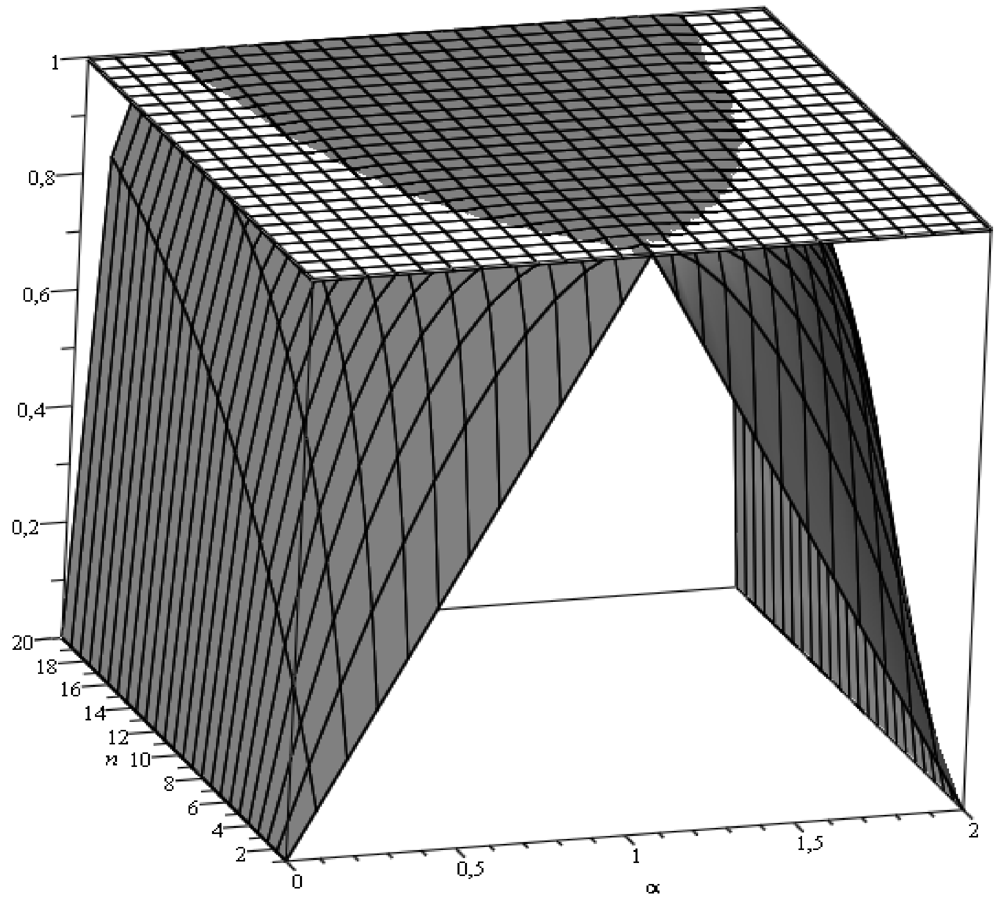

Fig. 3. Dependence of the similarity coefficient $S_n'(\alpha,n)$ of the Brown and Wade model on the smoothing parameter and the number of elements of the series on the extended set K_{ext}

Thus

$$|1-\alpha|^n \le 0.005 \tag{9}$$

or finally

$$1-(0.005)^{1/n} \le \alpha \le 1+(0.005)^{1/n}\quad . \tag{10}$$

The area satisfying (10) is shown in fig. 4.

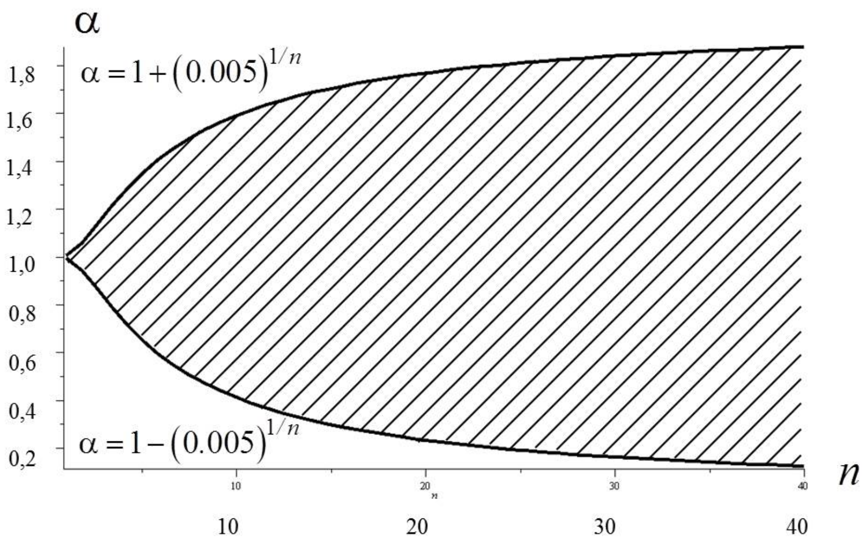

Рис. 4. The area on the parameter v plane, within which the predictive estimates in the Brown and Wade models coincide by more than 99.5%

Thus, the similarity of the two predictive models (2) and (5) can be estimated graphically analytically. For example, from fig. 3 and 4, it can be seen that short time series ($n \leq 15$) when choosing the smoothing parameter $0 < \alpha \leq 0{,}3$ and $1{,}7 \leq \alpha < 2$ the difference between the models is significant and Wade's modification makes practical sense.

Parametric synthesis of the Wade model

The choice of the smoothing parameter is proposed to be carried out on the basis of the hypothesis of retrospective analysis [10]. It consists in the assumption that the quality of the retrospective forecast estimates obtained for the values of the time series in the past points in time tends to be preserved at the next point in time.

In this case, the quality of the retro-forecast can be evaluated analytically using statistical criteria for the quality of predictive models [12], for example, by the sum of squares of errors m of the latest ($m < n$) retro-forecasts $SSE = \sum_{i=1}^{m} e_i^2$. To do this, you need to solve an optimization problem of the following form [11]

$$SSE(\alpha) = \sum_{i=1}^{m} e_{t-i}^2(\alpha) \to min, \ \alpha \in K_{ext} . \qquad (11)$$

Expressions for errors $e_{t-i}(\alpha)$ for the modified Wade model can be written in general form using the expression (6):

$$e_{t-1}(\alpha)=\hat{y}_{t-1}(\alpha)-y_{t-1}=\frac{\sum_{i=1}^{n-1}\alpha(1-\alpha)^{i-1}y_{t-i-1}}{1-(1-\alpha)^{n-1}}-y_{t-1},$$

$$e_{t-2}(\alpha)=\hat{y}_{t-2}(\alpha)-y_{t-2}=\frac{\sum_{i=1}^{n-2}\alpha(1-\alpha)^{i-1}y_{t-i-2}}{1-(1-\alpha)^{n-2}}-y_{t-2}, \quad (12)$$

$$\ldots$$

$$e_{t-m}(\alpha)=\hat{y}_{t-m}(\alpha)-y_{t-m}=\frac{\sum_{i=1}^{n-m}\alpha(1-\alpha)^{i-1}y_{t-i-m}}{1-(1-\alpha)^{n-m}}-y_{t-m}.$$

Since $SSE(\alpha)$ is a rational function, the solution to problem (11) always exists on an extended set K_{ext} without including the extreme points 0 and 2. The values of the smoothing parameter found from (11) can be used for the current forecasting.

Thus, for the parametric tuning of the Wade model, the methods used for the classical Brown model can be used, in particular, the retrospective analysis technology.

Example

Let’s consider the time series [11] (table 1).

Table 1

Initial time series

t	1	2	3	4	5	6	7	8	9	10	11	12
y_t	15,08	16,31	16,44	16,28	15,89	15,49	15,08	15,78	15,48	15,22	14,91	14,71

To solve problem (11), we will depict dependencies $SSE(\alpha)$ for Brown's model - and for Wade's model - $SSE_W(\alpha)$ (fig. 5).

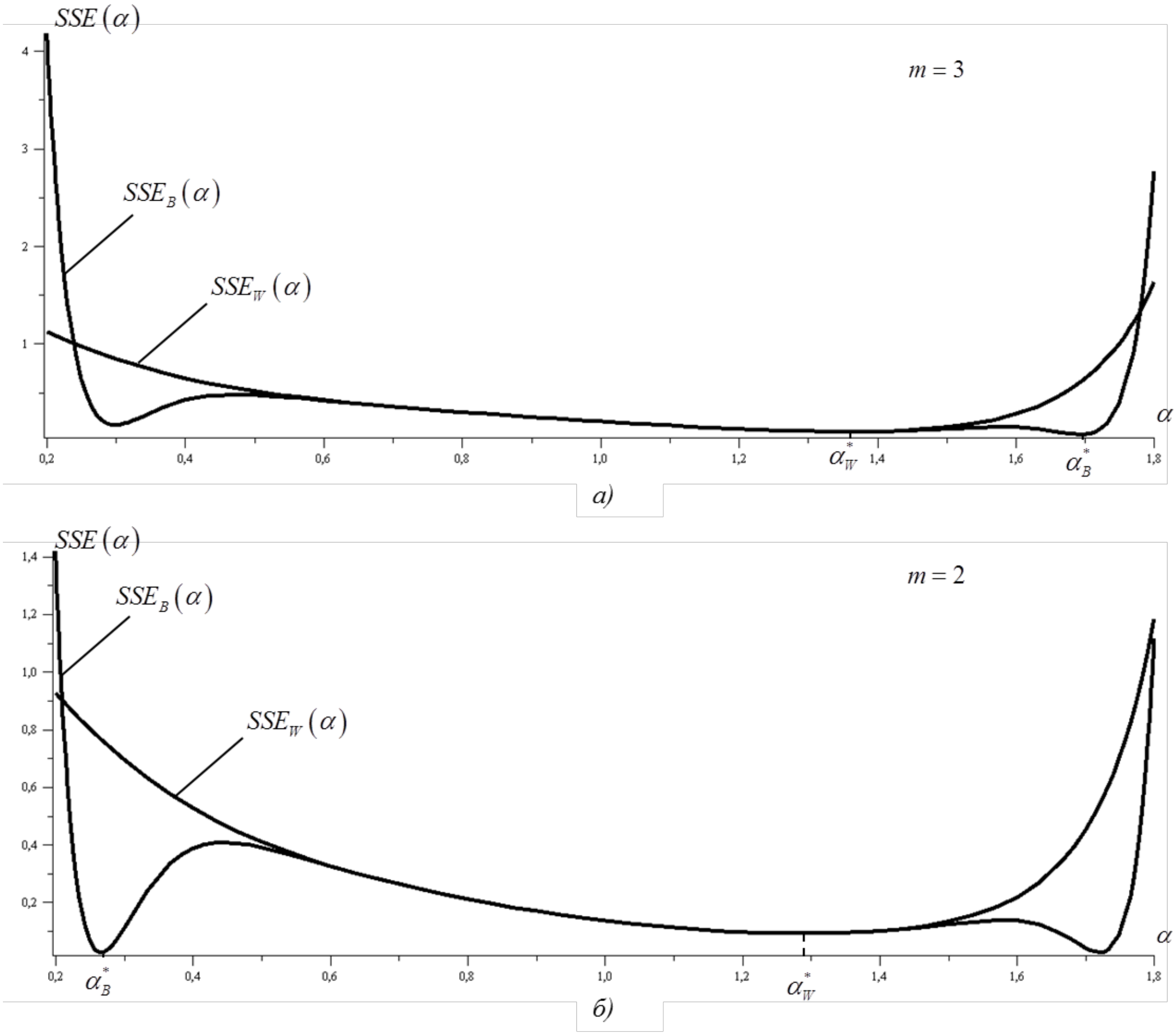

Fig. 5. Graphs of retrospective dependencies $SSE(\alpha)$ for the brown and Wade models: a) for $m = 3$, b) for $m = 2$

Analysis of graphs allows us to make the following practical conclusions.

1) Optimal values of the criterion smoothing parameter for the two models do not match:

$$\alpha_B^* = 1{,}694\,,\ \alpha_W^* = 1{,}360\,,\ m = 3\,, \tag{13}$$

$$\alpha_B^* = 0{,}267\,,\ \alpha_W^* = 1{,}297\,,\ m = 2\,. \tag{14}$$

This means that the parametric tuning of the considered models should be performed independently.

2) On the interval $0{,}5 < \alpha < 1{,}5$, the quality of the two models by the criterion coincides, which is due to the high degree of similarity of the models in this interval.

3) The Wade model is superior in quality to Brown's model only in the critical areas $\alpha < 0{,}25$ and $1{,}75 < \alpha$, while the optimal values of the smoothing parameter are in the vicinity of the value $\alpha = 1{,}3$.

Conclusions

The problem of parametric tuning of the modified Wade forecast model is considered. The similarity of the predictive models of Brown and Wade is analytically evaluated. Critical regions are determined on the extended admissible set of the smoothing parameter, within which the quality of the Brown and Wade models is significantly different. Recommendations are given regarding the parametric tuning of the Wade predictive model based on retrospective analysis by criterion SSE. The necessity of independent parametric tuning of Brown and Wade models is shown. An example of the practical use of the proposed recommendations is given. The results allow the user to intelligently select and tune the specified predictive models.

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