Structural Function Development for Electromagnetic Interactions in the System of Multiple Resonant Magnetodielectric Spheres*

A.I. Kozar

Kharkov National University of Radio Engineering and Electronics,
14, Lenin Ave, Kharkov, 61166, Ukraine

ABSTRACT: An approach to derive structure functions of electromagnetic interactions in the system of multiple resonant magnetodielectric spheres is considered. Expressions for magntic- and electrical-type tensor functions of electromagnetic interactions in the system of spheres are obtained, the expressions for structure functions of magnetic and electric type are analyzed.

The investigations of resonance properties of electromagnetic interactions in many-body systems attract considerable attention. In the present paper, the analysis of electromagnetic interactions in many-body systems is restricted to the case of spatial systems of the small uniform resonant magnetodielectric spheres [1]. In such spatial structures, the electromagnetic interactions between spheres and the spheres themselves have resonance properties. Structural resonances of electromagnetic interactions between the spheres can influence the internal resonances of the spheres and their fine structure, which allows a theoretical and experimental approach to be developed for examination of electromagnetic interactions in multiple-sphere systems. It is convenient to describe the features of electromagnetic interactions in sphere systems by introducing the notion of structure functions of electromagnetic interactions. In the case of crystal lattices these functions are related to the so-called Ewald lattice sums, playing an important role in the dynamic theory of crystal lattices [2]. Structure functions may be useful in theoretical and experimental studies of resonance electromagnetic interactions in multiple-sphere systems, in studying physical influence of the lattice structural resonances on the internal resonances of spheres in the lattice and their fine structure [3], and in the experimental estimation of the Ewald sums.

The aim of the present study is to develop a new approach to derivation of structure functions of electromagnetic interactions in the system of small uniform

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resonant magnetodielectric spheres and to analyze their properties in several spatial arrangements.

In this problem, the wavelength of scattered wave may be comparable to the distances between spheres in the spatial structure.

PROBLEM FORMULATION AND SOLUTION

Topological structure of the sphere system. Let us consider a complex spatial system of nodes, consisting of C sublattices c ($c \in C$). The sublattices c are generated by the coordinate presentation, which in Cartesian rectangular coordinates have the following form:

$$
\begin{aligned}
x_{c,s} &= [s - 0,5\{(-1)^s - 1\}]d - (-1)^{s-1} x_{c,s=0} \left(s = 0, \pm 1, \pm 2, ...\right), \\
y_{c,t} &= [t - 0,5\{(-1)^t - 1\}]h - (-1)^{t-1} y_{c,t=0} \left(t = 0, \pm 1, \pm 2, ...\right), \\
z_{c,p} &= [p - 0,5\{(-1)^p - 1\}]l - (-1)^{p-1} z_{c,p=0} \left(p = 0, \pm 1, \pm 2, ...\right),
\end{aligned}
\quad (1)
$$

where d, h, l are defined by the conditions $y = 0$, $x = d$, $y = 0$, $y = h$, $z = 0$, $z = l$, and $x_{c,s=0}$, $y_{c,t=0}$, $z_{c,p=0}$ are the coordinates of the node that generates the sublattice c and that is located inside region defined by eq.(2), Fig. 1,

$$
\begin{aligned}
0 &\le x_{c,s=0} \le d, \\
0 &\le y_{c,t=0} \le h, \\
0 &\le z_{c,p=0} \le l.
\end{aligned}
\quad (2)
$$

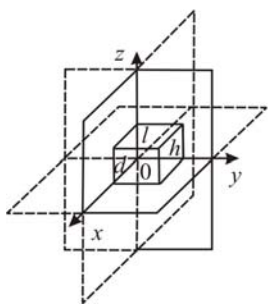


FIGURE 1. Problem geometry.

In coordinate presentation of eq.(1) the parameter p may be a function of parameters s, t [3].

Coordinates $x_{c,s}, y_{c,t}, z_{c,p}$ define the positions of nodes in sublattice c outside the region eq.(2) and depend on coordinates $x_{c,s=0}, y_{c,t=0}, z_{c,p=0}$. Time dependence can be introduced into coordinate presentation of eq.(1), if $x_{c,s=0}, y_{c,t=0}, z_{c,p=0}$ are considered to be some functions of time. An ordered triad of numbers $u = c(p,s,t)$ is associated with every node of the spatial sublattice c eq.(1). Separate node of the structure will be denoted as $u' = c'(p',s',t')$, whereas the node inside region eq.(2) will be denoted as $c(p=0, s=0, t=0)$. By specifying maximum values of (p,s,t) in eq.(1) it is possible to consider both finite and infinite spatial structures.

Required unit cell type (primitive, body-centered, face-centered, etc.) is formed from C nodes inside region eq.(2) and repeated outside region eq.(2) by coordinate presentation eq.(1) giving a spatial system of nodes of defined type.

Figure 2 shows two systems of nodes for the cases:

a) $p = 0,\pm1,\pm2$; $s = 0,\pm1,\pm2$; $t = 0,\pm1,\pm2$;

b) $p = 0,\pm1,\pm2,\ldots,\pm\left[\frac{(|s|+|t|)!}{|s|!|t|!} - 1\right]$; $s = 0,\pm1,\pm2$; $t = 0,\pm1,\pm2$, the case where the spatial arrangement of nodes is defined by the structure of the Fermat numbers [4].

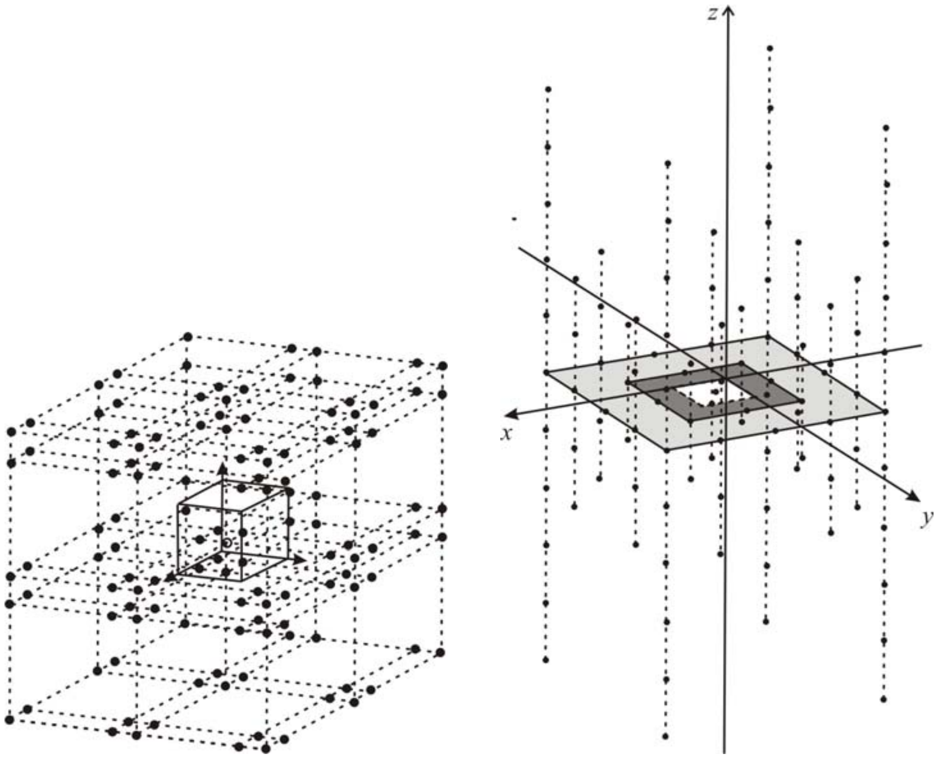
If coordinates of the nodes in region eq.(2) are changed, positions of the nodes outside region eq.(2) shift accordingly leading to the cell rearrangement and formation of spatial configuration in the structure. When the generating node lies in the center of region eq.(2) values *d*, *h*, and *l* represent lattice constants of a regular orthogonal lattice with respect to *x*, *y*, and *z* axes, respectively.

The distance between nodes is defined as

$$r_{c'(p',s',t'),c(p,s,t)} = \sqrt{\left(x_{c',s'} - x_{c,s}\right)^2 + \left(y_{c',t'} - y_{c,t}\right)^2 + \left(z_{c',p'} - z_{c,p}\right)^2}. \quad (3)$$

Located in the nodes of sublattices eq.(1) are centers of spheres with permittivities $\varepsilon_{c(p,s,t)}$, permeabilities $\mu_{c(p,s,t)}$, and radii $a_{c(p,s,t)}$ (further in the text denoted as ε_c, μ_c, and a_c, respectively). The spheres are located in free space. Fields are represented as $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{i\omega t}, \vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{i\omega t}$.

Internal field of spheres in the spatial structures. To solve the problem it is necessary to determine the internal field induced in the spheres by the external field. Using the expressions for internal field it is possible to derive required structure functions of electromagnetic interactions in a specific sphere system. We will consider the case where relationship $a_c/\lambda << 1$ is object outside the sphere, but inside the spheres the resonance condition $a_c/\lambda_g \sim 1$ is possible, λ and λ_g are the wavelengths of scattered wave outside and inside the spheres., respectively.



a) b)

FIGURE 2. Spatial systems of nodes

The internal field of the spheres can be found by solving the system of heterogeneous equations that are constructed using integral equations [5] and the results obtained in [3,6]. The heterogeneous equations for an arbitrary separate sphere $c'(p',s',t')$ in the spatial system can be written as

$$\vec{E}_{0c'(p',s',t')}(\vec{r}',t) = \left(\frac{\left(\varepsilon_{c'eff} + 2\varepsilon_0\right) + \theta_{1c'}^2 \varepsilon_{c'eff} + i\theta_{1c'}\left(\varepsilon_{c'eff} + 2\varepsilon_0\right)}{3\varepsilon_0 e^{i\theta_{1c'}}} \vec{E}^0{}_{c'(p',s',t')}(\vec{r}',t) - \right.$$

$$- \sum_{\substack{p \\ c'(p,s,t) \neq c'(p',s',t')}} \sum_s \sum_t \left\{ \left(\nabla\nabla + k^2 \varepsilon_0 \mu_0\right) \frac{1}{4\pi} \left(\frac{\varepsilon_{c'eff}}{\varepsilon_0} - 1 \right) W^E_{c'(p,s,t)}(\vec{r}) \vec{E}^0_{c'(p,s,t)}(\vec{r}',t) - \right.$$

$$-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^M_{c'(p,s,t)}(\vec{r})\vec{H}^0_{c'(p,s,t)}(\vec{r}',t)\right]\Bigg\}\Bigg)-$$

$$-\sum_{\substack{c=1\\(c\neq c')}}^{C}\left(\sum_p\sum_s\sum_t\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^M_{c(p,s,t)}(\vec{r})\vec{E}^0_{c(p,s,t)}(\vec{r}',t)-\right.\right.$$

$$\left.\left.-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^M_{c(p,s,t)}(\vec{r})\vec{H}^0_{c(p,s,t)}(\vec{r}',t)\right]\right\}\right), \tag{4}$$

$$\vec{H}_{0c'(p',s',t')}(\vec{r}',t)=\left(\frac{\left(\mu_{c'eff}+2\mu_0\right)+\theta^2_{1c'}\mu_{c'eff}+i\theta_{1c'}\left(\mu_{c'eff}+2\mu_0\right)}{3\mu_0e^{i\theta_{1c'}}}\vec{H}^0{}_{c'(p',s',t')}(\vec{r}',t)-\right.$$

$$-\sum_{\substack{p\ s\ t\\ c'(p,s,t)\neq c'(p',s',t')}}\sum\sum\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^M_{c'(p,s,t)}(\vec{r})\vec{H}^0_{c'(p,s,t)}(\vec{r}',t)+\right.$$

$$\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{c'eff}}{\varepsilon_0}-1\right)W^E_{c'(p,s,t)}(\vec{r})\vec{E}^0_{c'(p,s,t)}(\vec{r}',t)\right]\right\}\Bigg)-$$

$$-\sum_{\substack{c=1\\(c\neq c')}}^{C}\left(\sum_p\sum_s\sum_t\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^M_{c(p,s,t)}(\vec{r})\vec{H}^0_{c(p,s,t)}(\vec{r}',t)+\right.\right.$$

$$\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^E_{c(p,s,t)}(\vec{r})\vec{E}^0_{c(p,s,t)}(\vec{r}',t)\right]\right\}\right),$$

where $\vec{E}_{0c'(p',s',t')}(\vec{r}',t)$; $\vec{H}_{0c'(p',s',t')}(\vec{r}',t)$ and $\vec{E}^0_{c'(p',s',t')}(\vec{r}',t)$; $\vec{H}^0_{c'(p',s',t')}(\vec{r}',t)$ are the field of the incident wave and the internal field of sphere $c'(p',s',t')$, and $\vec{E}^0_{c(p,s,t)}(\vec{r}',t)$; $\vec{H}^0_{c(p,s,t)}(\vec{r}',t)$ are the internal fields of other spheres, $k=2\pi/\lambda$; $\theta^2_{1c}=k^2a^2_c\varepsilon_0\mu_0$; ε_0 and μ_0 are the permittivity and the permeability of space between the spheres.

Values $W^E_{c(p,s,t)}(\vec{r}')$; $W^M_{c(p,s,t)}(\vec{r}')$ have the form ($k_1=k\sqrt{\varepsilon_0\mu_0}$)

$$W^E_{c(p,s,t)}(\vec{r}')=\frac{4\pi}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\frac{e^{-ik_1r_{c'(p',s',t'),c(p,s,t)}}}{r_{c'(p',s',t'),c(p,s,t)}},$$

$$W^M_{c(p,s,t)}(\vec{r}')=-\frac{4\pi}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\frac{e^{-ik_1r_{c'(p',s',t'),c(p,s,t)}}}{r_{c'(p',s',t'),c(p,s,t)}},$$

and values ε_{ceff} , μ_{ceff} [7] can be represented as

$$\begin{aligned} \varepsilon_{ceff} &= \varepsilon_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) , \\ \mu_{ceff} &= \mu_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) , \end{aligned} \qquad (5)$$

where (Fig. 3)

$$F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = \frac{2\left(\sin ka_c\sqrt{\varepsilon_c\mu_c} - ka_c\sqrt{\varepsilon_c\mu_c}\cos ka_c\sqrt{\varepsilon_c\mu_c}\right)}{\left(k^2a_c^2\varepsilon_c\mu_c - 1\right)\sin ka_c\sqrt{\varepsilon_c\mu_c} + ka_c\sqrt{\varepsilon_c\mu_c}\cos ka_c\sqrt{\varepsilon_c\mu_c}} .$$

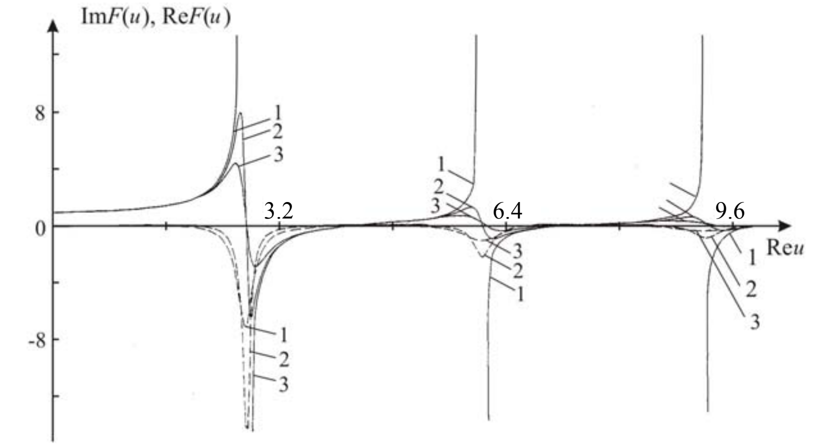


FIGURE 3. Function $F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$

First terms in the right-hand side of eqs.(4) are related to the internal field of the sphere $c'(p',s',t')$. If the influence of other spheres is neglected, the rest of the terms take account of the influence of other spheres on the scatterer $c'(p',s',t')$. In system of equations eq.(4) account is taken of the mutual electromagnetic influence of spheres in the spatial structure, and the main matrix of this algebraic system of equations contains information about the electromagnetic interaction between spheres in the structure.

Figure 3 shows peculiarities in the behavior of $\operatorname{Re} F(\theta)$ (solid line) and $\operatorname{Im} F(\theta)$ (dashed line) plotted versus $\operatorname{Re}\theta$ for different values of dielectric dissipation $\tan\delta_\varepsilon$ (1 – $\tan\delta_\varepsilon = 0$; 2 – $\tan\delta_\varepsilon = 0.05$; 3 – $\tan\delta_\varepsilon = 0.1$) and $\mu_c = 1$, here $\theta = ka_c\sqrt{\varepsilon_c\mu_c}$.

The algebraic system of equations consists of $2N = 2\sum_{c=1}^{C} N_c$ heterogeneous vector equations eq.(4), where N is the total number of spheres in the arrangement, and N_c is the number of spheres in sublattice *c*. Solution for this set of equations for a separate sphere is

$$\vec{E}^0_{c'(p',s',t')}(\vec{r}',t) = \frac{1}{\Delta^{EM}}\sum_{c=1}^{C}\left(\sum_u\left[\hat{g}^{Eu'}_u \vec{E}_{0c(p,s,t)}(\vec{r}',t) + \hat{\beta}^{Eu'}_u \vec{H}_{0c(p,s,t)}(\vec{r}',t)\right]\right),$$
$$\vec{H}^0_{c'(p',s',t')}(\vec{r}',t) = \frac{1}{\Delta^{EM}}\sum_{c=1}^{C}\left(\sum_u\left[\hat{\beta}^{Mu'}_u \vec{H}_{0c(p,s,t)}(\vec{r}',t) + \hat{g}^{Mu'}_u \vec{E}_{0c(p,s,t)}(\vec{r}',t)\right]\right), \qquad (6)$$

where

$$\hat{g}^{Eu'}_u = \begin{bmatrix} g^{Eu'}_{xxu} & g^{Eu'}_{xyu} & g^{Eu'}_{xzu} \\ g^{Eu'}_{yxu} & g^{Eu'}_{yyu} & g^{Eu'}_{yzu} \\ g^{Eu'}_{zxu} & g^{Eu'}_{zyu} & g^{Eu'}_{zzu} \end{bmatrix}; \quad \hat{\beta}^{Eu'}_u = \begin{bmatrix} \beta^{Eu'}_{xxu} & \beta^{Eu'}_{xyu} & \beta^{Eu'}_{xzu} \\ \beta^{Eu'}_{yxu} & \beta^{Eu'}_{yyu} & \beta^{Eu'}_{yzu} \\ \beta^{Eu'}_{zxu} & \beta^{Eu'}_{zyu} & \beta^{Eu'}_{zzu} \end{bmatrix};$$
$$\hat{\beta}^{Mu'}_u = \begin{bmatrix} \beta^{Mu'}_{xxu} & \beta^{Mu'}_{xyu} & \beta^{Mu'}_{xzu} \\ \beta^{Mu'}_{yxu} & \beta^{Mu'}_{yyu} & \beta^{Mu'}_{yzu} \\ \beta^{Mu'}_{zxu} & \beta^{Mu'}_{zyu} & \beta^{Mu'}_{zzu} \end{bmatrix}; \quad \hat{g}^{Mu'}_u = \begin{bmatrix} g^{Mu'}_{xxu} & g^{Mu'}_{xyu} & g^{Mu'}_{xzu} \\ g^{Mu'}_{yxu} & g^{Mu'}_{yyu} & g^{Mu'}_{yzu} \\ g^{Mu'}_{zxu} & g^{Mu'}_{zyu} & g^{Mu'}_{zzu} \end{bmatrix},$$

and Δ^{EM} is the determinant of the main matrix of set eq.(4).

The component of the sphere internal field $E^0_{xu'}(\vec{r}',t)$ in eq.(6) can be written as

$$E^0_{xu'}(\vec{r}',t) = \frac{1}{\Delta^{EM}}\sum_{c=1}^{C}\left(\sum_u\left[g^{Eu'}_{xxu}\vec{E}_{0xu}(\vec{r}',t) + g^{Eu'}_{xyu}E_{0yu}(\vec{r}',t) + g^{Eu'}_{xzu}E_{0zu}(\vec{r}',t) + \right.\right.$$
$$\left.\left. + \beta^{Eu'}_{xxu}H_{0xu}(\vec{r}',t) + \beta^{Eu'}_{xyu}H_{0yu}(\vec{r}',t) + \beta^{Eu'}_{xzu}H_{0zu}(\vec{r}',t)\right]\right).$$

Other components are obtained similarly from eq.(6).

In the case where the internal fields of all spheres with the same index *p* in each sublattice eq.(1) may be assumed equal to field $\vec{E}^0_{c(p,s=0,t=0)}(\vec{r}',t)$ of sphere $c(p,s=0,t=0)$, the system of equations for the complex spatial structure of spheres can be reduced to the system with $2(2|p_m|+1)C$ equations, where p_m is the maximum value of index *p* (eq.1) in sublattice *c*, here the value of p_m for all sublattices is the same.

When the conditions $a_c/\lambda << 1$ and $r_{cc'}/\lambda << 1$ ($r_{cc'}$ is the distance between sphere centers, eq.(3)) are fulfilled and the internal fields in all spheres of sublattice *c* may be assumed equal to field $\vec{E}^0_{c(p,s=0,t=0)}(\vec{r}',t)$ of sphere located inside region eq.(2), set of equations eq.(6) can be reduced to a system of $2C$ heterogeneous vector equations.

If the electromagnetic interaction between spheres in the arrangement may be neglected, then expressions for the internal field eq.(6) of an arbitrary sphere in the arrangement will be

$$\begin{aligned} \vec{E}^0_{c(p,s,t)}(\vec{r}',t) &= \frac{3\varepsilon_0 e^{i\theta_{1c}}}{(\varepsilon_{ceff}+2\varepsilon_0)+\theta_{1c}^2\varepsilon_{ceff}+i\theta_{1c}(\varepsilon_{ceff}+2\varepsilon_0)}\vec{E}_{0c(p,s,t)}(\vec{r}\,',t), \\ \vec{H}^0_{c(p,s,t)}(\vec{r}',t) &= \frac{3\mu_0 e^{i\theta_{1c}}}{(\mu_{ceff}+2\mu_0)+\theta_{1c}^2\mu_{ceff}+i\theta_{1c}(\mu_{ceff}+2\mu_0)}\vec{H}_{0c(p,s,t)}(\vec{r}',t). \end{aligned} \qquad (7)$$

Shown in Fig. 4 are curves for modulus $|\eta^m|$ and argument φ^m and of real Re and imaginary Im parts of the internal magnetic field of the sphere (eq.(7)) as functions of the wavelength of incident wave λ in the vicinity of the first internal resonance of magnetic type. In this example $a_c = 0.15$ cm, $\varepsilon_c = 100$, dielectric dissipation $\tan\delta_\varepsilon = 0$, $\mu_c = \mu_0 = \varepsilon_0 = 1$, λ_r is the resonance wavelength.

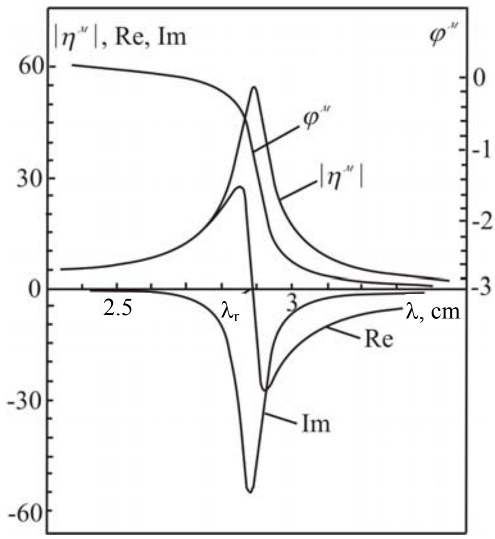


FIGURE 4. First internal resonance of magnetic type

As can be seen in Fig. 4, when the electromagnetic interaction between spheres in the spatial structure is neglected, real part of the expression (eq.(7)) for the internal magnetic field of an arbitrary sphere in the arrangement may become zero at the resonance (point λ_r in Fig. 4):

$$\operatorname{Re}\vec{H}^{0}_{c(p,s,t)}\left(\vec{r}\,',t\right)=0, \tag{8}$$

which is also the case for the internal electrical field of spheres. Condition eq.(8) is also fulfilled in the case where the electromagnetic interaction between spheres is present.

Analysis of eq.(8) shows that the following equality is valid:

$$\det \operatorname{Re}\left\|\alpha_{ij}^{EM}\right\|=0, \tag{9}$$

where $\left\|\alpha_{ij}^{EM}\right\|$ is the main matrix of algebraic system of equations eq.(4). Using eq.(9) resonance conditions for the *x*-, *y*-, and *z*-components of internal fields can be obtained.

Matrix $\operatorname{Re}\left\|\alpha_{ij}^{EM}\right\|$ contains information about the electromagnetic interactions between spheres in the spatial system.

In the case where all spheres in the arrangement are identical the order of eq.(9) is determined by the order of matrix $\operatorname{Re}\left\|\alpha_{ij}^{EM}\right\|$, which is 6*N*, where $N=\sum_{c=1}^{C}N_c$ is the total number of spheres in the spatial structure, N_c is the number of spheres in sublattice *c*.

When in the spatial structure all spheres of each sublattice are identical and spheres in one of the sublattices *c* are in resonance state, while spheres of the rest of sublattices are not in resonance state, the order of eq.(9) will be $6N_c$.

If permittivities ε_c and permeabilities μ_c of identical spheres in the resonant sublattice *c* are real, the resonance conditions for N_c spheres in sublattice *c* can be obtained from eq.(9) by solving it with respect to function $F(ka_c\sqrt{\varepsilon_c\mu_c})$ with the assumption that spheres of other sublattices is not resonant. Resonance conditions for spheres in the spatial structures consisting of identical spheres are derived in a similar way.

The order of eq.(9) can be reduced by assuming the elements of matrix $\operatorname{Re}\left\|\alpha_{ij}^{EM}\right\|$ that are related to *rot* (eq.(4)) to be zero due to their smallness at resonance. Then, eq.(9) decomposes into two independent equations:

$$\det \operatorname{Re}\left\|\alpha_{ij}^{M}\right\|=0,\quad \det \operatorname{Re}\left\|\alpha_{ij}^{E}\right\|=0, \tag{10}$$

where matrices $\left\|\alpha_{ij}^{M}\right\|$ and $\left\|\alpha_{ij}^{E}\right\|$ are related, respectively, to the magnetic and the electrical internal fields of spheres. Using eqs.(10) it is possible to obtain required resonance conditions. Simplification of eq.(9) may result in loss of some information about the fine resonance structure of the internal fields of spheres. The order of eqs.(9-10) can be reduced by choosing a corresponding spatial configuration of the sphere structure and orientation of the scattered field with respect to the structure.

If the electromagnetic interaction between spheres in the arrangement is neglected, the conditions of resonance for the internal electrical (*E*) and internal magnetic (*M*) fields for arbitrary sphere in the arrangement (from eq.(9)) are:

$$F_0^E\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=-\frac{2\varepsilon_0\left(\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right)}{\varepsilon_c\left[\left(1+\theta_{1c}^2\right)\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right]},$$
$$F_0^M\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=-\frac{2\mu_0\left(\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right)}{\mu_c\left[\left(1+\theta_{1c}^2\right)\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right]}. \qquad (11)$$

The Hertz potentials for the field scattered by spheres of the spatial structures can be represented as a superposition of the Hertz potentials for separate spheres in the arrangement (eq.(6)) [3]:

$$\vec{\Pi}^E(\vec{r},t)=\sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t\frac{1}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)\vec{E}^0_{c(p,s,t)}(\vec{r}',t)\frac{e^{-ik_1r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right],$$
$$\vec{\Pi}^M(\vec{r},t)=-\sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t\frac{1}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\left(\frac{\mu_{ceff}}{\mu_0}-1\right)\vec{H}^0_{c(p,s,t)}(\vec{r}',t)\frac{e^{-ik_1r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right]. \qquad (12)$$

Here

$$r_{c(p,s,t)}=\sqrt{\left(x-x_{c,s}\right)^2+\left(y-y_{c,t}\right)^2+\left(z-z_{c,p}\right)^2},$$

where coordinates (x,y,z) define the observation point for the scattered field outside the spheres; coordinates $(x_{c,s},y_{c,t},z_{c,p})$ define the center of scattering sphere in the spatial system (eq.(1)).

The expressions for the Hertz potentials eq.(12) can be employed for studying the spatial structures composed of magnetodielectric and metal spheres.

Structure functions of electromagnetic interactions. Let us derive the structure functions of electromagnetic interactions for spatial systems having identical spheres in each sublattice, but with spheres of one sublattice *c* being in resonance state and spheres in other sublattices being non-resonant; permittivities ε_c and permeabilities μ_c of the resonant spheres are real.

To derive the structure functions we apply the resonance conditions for internal fields of spheres in the arrangement. Resonance conditions for the *x*-, *y*-, and *z*-components of internal magnetic, *E*(*M*), and electrical, (*E*)*M*, fields of spheres in the resonant sublattice *c*, as obtained from eq.(9), are ($i,k=x,y,z$):

$$F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = f_{cik}^{E(M)}\left(\vec{r}_{cc'}\right),$$
$$F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = f_{cik}^{(E)M}\left(\vec{r}_{cc'}\right), \tag{13}$$

here $F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ and $F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ are the values of function eq.(5) (Fig. 3) at resonance of the internal magnetic and electrical fields of spheres, respectively (eq.(6)), and $f_{ci\kappa}^{E(M)}\left(\vec{r}_{cc'}\right)$, $f_{cik}^{(E)M}\left(\vec{r}_{cc'}\right)$ are the functions depending on the topological structure of the sphere arrangement, radii, permittivities, and permeabilities of the spheres, and the wavelength of scattered wave.

For a separate sphere, the resonance conditions for the *x*-, *y*-, and *z*-components of internal fields (eq.(7)) coincide and can be written ($\theta_{1c} << 1$, eq.(11)) as

$$F_0^M\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = -\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)},$$
$$F_0^E\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = -\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}. \tag{14}$$

By subtracting the right-hand and left-hand sides of eqs.(14) from the right-hand and left-hand sides of eqs.(13), respectively, we obtain for the internal resonances of magnetic type

$$F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) - F_0^M\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = \frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)} + f_{cik}^{E(M)}\left(\vec{r}_{cc'}\right), \tag{15}$$

and for the internal resonances of electrical type

$$F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) - F_0^E\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = \frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)} + f_{cik}^{(E)M}\left(\vec{r}_{cc'}\right). \tag{16}$$

The subtraction operation allows us to isolate terms that cause the shift of resonance conditions eq.(13) with respect to conditions eq.(14).

Functions in eqs.(15-16),

$$\Phi_{cik}^{E(M)}\left(\vec{r}_{cc'}\right)=\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}+f_{cik}^{E(M)}\left(\vec{r}_{cc'}\right),$$
$$\Phi_{cik}^{(E)M}\left(\vec{r}_{cc'}\right)=\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}+f_{cik}^{(E)M}\left(\vec{r}_{cc'}\right). \tag{17}$$

will be further referred to as the components of structure functions for electromagnetic interactions of magnetic and electrical type.

Now, let us introduce the structure functions for electromagnetic interactions of magnetic and electrical type for a separate sphere in the form of tensor functions

$$\hat{\Phi}_{c}^{E(M)}\left(\vec{r}_{cc'}\right)=\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}\hat{I}+\hat{f}_{c}^{E(M)}\left(\vec{r}_{cc'}\right)=\begin{pmatrix}\Phi_{cxx}^{E(M)}\left(\vec{r}_{cc'}\right) & 0 & 0\\ 0 & \Phi_{cyy}^{E(M)}\left(\vec{r}_{cc'}\right) & 0\\ 0 & 0 & \Phi_{czz}^{E(M)}\left(\vec{r}_{cc'}\right)\end{pmatrix},$$
$$\hat{\Phi}_{c}^{E(M)}\left(\vec{r}_{cc'}\right)=\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}\hat{I}+\hat{f}_{c}^{E(M)}\left(\vec{r}_{cc'}\right)=\begin{pmatrix}\Phi_{cxx}^{(E)M}\left(\vec{r}_{cc'}\right) & 0 & 0\\ 0 & \Phi_{cyy}^{(E)M}\left(\vec{r}_{cc'}\right) & 0\\ 0 & 0 & \Phi_{czz}^{(E)M}\left(\vec{r}_{cc'}\right)\end{pmatrix}. \tag{18}$$

Structure functions eq.(18) are also valid for the spatial systems of identical spheres, provided that all sublattices in the sphere arrangement are resonant.

Structure functions eq.(18) depend on the permittivities and permeabilities of all spheres in the arrangement, however, it is possible to introduce the approximate, with respect to eq.(18), structural functions that are independent of the permittivities and permeabilities of resonant spheres in the structure. Such functions can be obtained by omitting the terms related to *rot* (eq.(4)) in eq.(9), which make no significant contribution at resonance, and by using eqs.(10). To do this, let us write the conditions eqs.(15-16) in the form

$$F_{cik}^{M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)-F_0^{M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=\frac{\mu_0}{\mu_c}\left[\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{M}\left(\vec{r}_{cc'}\right)\right],$$
$$F_{cik}^{E}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)-F_0^{E}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=\frac{\varepsilon_0}{\varepsilon_c}\left[\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{E}\left(\vec{r}_{cc'}\right)\right], \tag{19}$$

then functions eq.(17) become to the following type

$$\Phi_{cik}^{M}\left(\vec{r}_{cc'}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{M}\left(\vec{r}_{cc'}\right),$$
$$\Phi_{cik}^{E}\left(\vec{r}_{cc'}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{E}\left(\vec{r}_{cc'}\right). \tag{20}$$

The superscript notation in eqs.(19-20) is changed (cf. eqs.(15-16)) according to the notation in eqs.(10).

Finally, the functions in eq.(18) can be rearranged to give

$$\hat{\Phi}_c^M\left(\vec{r}_{cc'}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}\hat{I}+\hat{f}_c^M\left(\vec{r}_{cc'}\right)=\begin{pmatrix}\Phi_{cxx}^M\left(\vec{r}_{cc'}\right) & 0 & 0\\ 0 & \Phi_{cyy}^M\left(\vec{r}_{cc'}\right) & 0\\ 0 & 0 & \Phi_{czz}^M\left(\vec{r}_{cc'}\right)\end{pmatrix},$$
$$\hat{\Phi}_c^E\left(\vec{r}_{cc'}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}\hat{I}+\hat{f}_c^E\left(\vec{r}_{cc'}\right)=\begin{pmatrix}\Phi_{cxx}^E\left(\vec{r}_{cc'}\right) & 0 & 0\\ 0 & \Phi_{cyy}^E\left(\vec{r}_{cc'}\right) & 0\\ 0 & 0 & \Phi_{czz}^E\left(\vec{r}_{cc'}\right)\end{pmatrix}. \qquad (21)$$

For the arrangements of identical resonant spheres functions eq.(21) do not depend on the permittivity and permeability of sphere material, which is important, for example, in studies of resonance properties of crystal lattices.

In the case of infinite crystal lattices the functions eq.(18,21) incorporate the triple Ewald sums with infinite limits of summation.

Structure functions for interactions in the linear lattice of spheres. Let us derive the structure functions in the form of eq.(21) for the linear lattice of identical spheres with real permittivities and permeabilities of sphere material and with spatial configurationn shown in Fig. 5.

Consider the case where the linear lattice is exposed to the plane wave propagating in the z direction with the electrical field vector of the wave $\vec{E}_{0x}$ being parallel to $0x$ axis (Fig. 5). Assume that in this case the internal fields of all spheres in the linear lattice are identical.

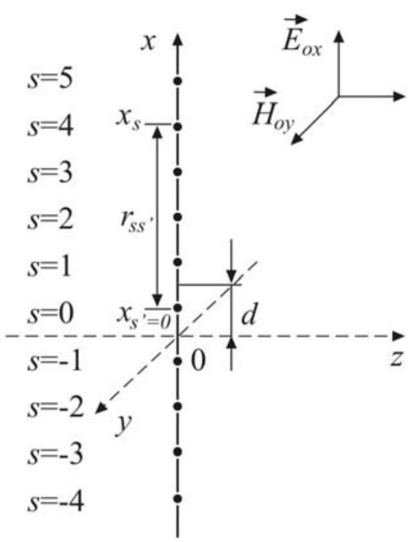


FIGURE 5. Linear lattice of spheres

Resonance conditions for the internal fields of spheres in the lattice can be obtained from eq.(10). In this case the matrix $\left\|\alpha_{ij}^{E}\right\|$ (eq.(10)) has the form

$$\mathrm{Re}\left\|\alpha_{ij}^{E}\right\| = \begin{bmatrix} \psi_{xx}^{E0'} + \psi_{xx}^{E'} & 0 & 0 \\ 0 & \psi_{yy}^{E0'} + \psi_{yy}^{E'} & 0 \\ 0 & 0 & \psi_{zz}^{E0'} + \psi_{zz}^{E'} \end{bmatrix} \tag{22}$$

The elements of matrix $\left\|\alpha_{ij}^{E}\right\|$ (eq.(22)) can be represented (Fig. 5; $x_{c,s} = x_s$, $x_{c,s=0} = x_{s=0}$; eqs.(1,5)) as

$$\left(\psi_{xx}^{E0'} + \psi_{xx}^{\text{э}'}\right) = A_{\varepsilon}^{0'} - A_{\varepsilon}\tau_{xx}^{E'}, \quad \left(\psi_{yy}^{E0'} + \psi_{yy}^{E'}\right) = \left(\psi_{zz}^{E0'} + \psi_{zz}^{E'}\right) = A_{\varepsilon}^{0'} - A_{\varepsilon}\tau_{yy}^{E'},$$

$$\tau_{xx}^{E'} = B \sum_{\substack{-s \\ s\neq s'=0}}^{s} (c_{xx}\cos k_1 r_{ss'} + a_{xx}\sin k_1 r_{ss'}), \quad k = 2\pi/\lambda, \quad k_1^2 = k^2\varepsilon_0\mu_0,$$

$$\tau_{yy}^{E'} = \tau_{zz}^{E'} = B \sum_{\substack{-s \\ s\neq s'=0}}^{s} (c_{yy}\cos k_1 r_{ss'} + a_{yy}\sin k_1 r_{ss'}), \quad \theta_1^2 = k^2 a^2 \varepsilon_0\mu_0,$$

$$A_{\varepsilon}^{0'} = \frac{(\varepsilon_{eff} + 2\varepsilon_0) + \theta_1^2\varepsilon_{eff} + \theta_1^2(\varepsilon_{eff} + 2\varepsilon_0)}{3\varepsilon_0}, \quad A_{\varepsilon} = \left(\frac{\varepsilon_{eff}}{\varepsilon_0} - 1\right),$$

$$B = \frac{1}{k_1^3}(\sin k_1 a - k_1 a\cos k_1 a),$$

$$a_{xx} = k_1\frac{2}{r_{ss'}^2}, \; a_{yy} = k_1\left|-\frac{1}{r_{ss'}^2}\right|, \; c_{xx} = k_1^2\frac{1}{r_{ss'}} + \left|\frac{2}{r_{ss'}^3} - k_1^2\frac{1}{r_{ss'}}\right|, \; c_{yy} = k_1^2\frac{1}{r_{ss'}} + \left|-\frac{1}{r_{ss'}^3}\right|,$$

$$r_{ss'} = \left|x_s - x_{s'=0}\right|.$$

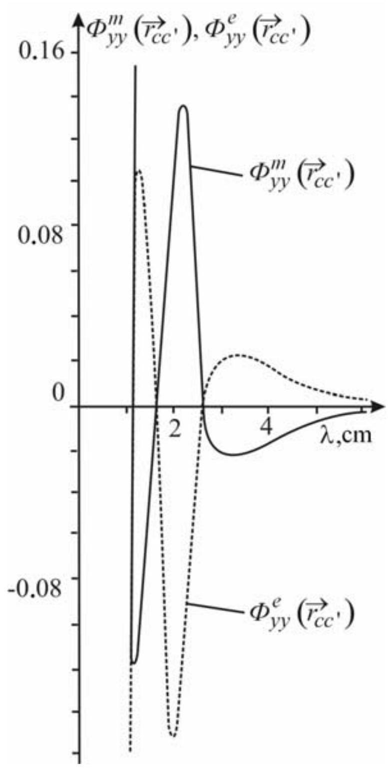
Using relationships eq.(19-20) we obtain the components of tensor function $\hat{\Phi}_c^E\left(\vec{r}_{cc'}\right)$ (eq.(21), $\theta_1 << 1$)

$$\begin{aligned} \Phi_{xx}^{E}\left(\vec{r}_{cc'}\right) &= \frac{3+4\theta_1^2}{1+2\theta_1^2} - \frac{3+4\theta_1^2}{1+2\theta_1^2-3\tau_{xx}^{E'}} \approx 3 - \frac{3}{1-3\tau_{xx}^{E'}} \approx -9\tau_{xx}^{E'}, \\ \Phi_{yy}^{E}\left(\vec{r}_{cc'}\right) &= \frac{3+4\theta_1^2}{1+2\theta_1^2} - \frac{3+4\theta_1^2}{1+2\theta_1^2-3\tau_{yy}^{E'}} \approx 3 - \frac{3}{1-3\tau_{yy}^{E'}} \approx -9\tau_{yy}^{E'}, \\ \Phi_{zz}^{E}\left(\vec{r}_{cc'}\right) &= \Phi_{yy}^{E}\left(\vec{r}_{cc'}\right), \end{aligned} \tag{23}$$

if the wavelength of scattered wave $\lambda \to \infty$, then

$$\tau_{xx}^{E'} = \frac{2}{3} a^3 \sum_{\substack{-s \\ s \neq s'=0}}^{s} \frac{1}{r_{ss'}^3}, \quad \tau_{yy}^{E'} = \tau_{zz}^{E'} = \frac{1}{3} a^3 \sum_{\substack{-s \\ s \neq s'=0}}^{s} \frac{1}{r_{ss'}^3}.$$

The components of tensor function $\hat{\Phi}_c^M\left(\vec{r}_{cc'}\right)$ (eq.(21)) can be derived from eq.(23) by reversing the signs of $\tau_{xx}^{E'}$, $\tau_{yy}^{E'}$.



FIGUERE 6. Components $\Phi_{yy}^M\left(\vec{r}_{cc}\right), \Phi_{yy}^E\left(\vec{r}_{cc}\right)$ of structure functions for interactions of magnetic and electrical type in the linear lattice of spheres

Figure 6 shows the dependences of tensor function (eq.(21)) components $\Phi_{yy}^M\left(\vec{r}_{cc}\right), \Phi_{yy}^E\left(\vec{r}_{cc}\right)$ (eq.(23)) for the linear lattice (Fig. 5) on the wavelength of scattered wave $x_{s=0}$ in the case where sphere radius $a = 0.15$ cm, linear lattice constant $d = 2.3$ cm, $x_{s'=0} = 1.15$ cm.

Analysis of eqs.(23) and Fig. 6 shows that the structure resonances of electromagnetic interactions of magnetic and electrical types occur in the lattice when the wavelength of scattered wave is comparable to the linear lattice constant $\lambda \sim d$. The structural resonances of magnetic and electrical type coexist in the lattice and have virtually identical resonance wavelengths, depending on the configuration of spatial structure and the radii of spheres in the lattice. Each type of structural resonance exerts influence only on the analogous type of internal resonance of magnetodielectric sphere in the linear lattaice. It is possible to physically match the lattice structure resonances with the internal resonances of the spheres in order to enhance their effect on the fine structure of the latter [1,3].

Relation between structure functions and resonance conditions for spheres. Resonance conditions for *x*-, *y*-, and *z*-components of the internal fields of a separate sphere (eq.(6)) are related to the components of structure functions (eqs.(15-16))

$$\begin{aligned} F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) &= -\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{1+2\theta_{1c}^2}+\Phi_{cik}^{E(M)}\left(\vec{r}_{cc'}\right), \\ F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) &= -\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{1+2\theta_{1c}^2}+\Phi_{cik}^{(E)M}\left(\vec{r}_{cc'}\right). \end{aligned} \tag{24}$$

Discussion on the possibility of experimental estimation of structure functions. Expressions for structure functions eq.(18,21) may contain triple alternating sums with infinite limits of summation, which are quite difficult to compute or estimate. This problem is especially critical in the dynamic theory of crystal lattices, where computation or estimation of Ewald lattice sums is necessary. In [7] these triple sums are substituted with integrals, but such substitution results in the error, the magnitude of which is not known. Comparison of the results obtained in [1,3] and in the present study leads us to the following conclusion. If the resonance values of functions $F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$, $F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ (eq.(13)) are found experimentally using fine structure of the internal field in spheres, then the components $\Phi_{cik}^{E(M)}\left(\vec{r}_{cc'}\right), \Phi_{cik}^{(E)M}\left(\vec{r}_{cc'}\right)$ containing the Ewald sums can be determined from eq.(24) and the experimental estimates of the sums can be obtained.

CONCLUSIONS

In the present paper, a novel approach to derivation of the structure functions describing structural electromagnetic interactions in the system of resonant magnetodielectric spheres is considered. A concept of structure functions of electromagnetic interactions of magnetic and electrical types is introduced. Using a linear lattice of magnetodielectric spheres as an example some properties of structure functions and lattice structure resonances are considered, along with the effect of the latter on the internal resonances of spheres in the lattice. The possibility for experimental estimation of structure functions is discussed.

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