Sensor-Polarimeter Based on Anisotropic Photonic Crystal for Solids and Liquids

Alexandr A. Shmat'ko
School of Radiophysics, Biomedical Electronics and Computer Systems
V. N. Karazin Kharkiv National University
Kharkiv, Ukraine
alexandr.a.shmatko@univer.kharkov.ua

Eugene N. Odarenko
School of Electronics and Biomedical Engineering
Kharkiv National University of Radio Electronics
Kharkiv, Ukraine
yevhen.odarenko@nure.ua

Alexey A. Vertiy
School of Electronics and Biomedical Engineering
Kharkiv National University of Radio Electronics
Kharkiv, Ukraine
alexey.vertiy@gmail.com

***Abstract*—We solved the problem of determining the angle of polarization plane rotation for wave transmitted through anisotropic one-dimensional photonic crystal. Transfer matrix method is used to obtain analytical expressions for reflection and transmission coefficients of a plane wave of orthogonal polarizations for a limited number of photonic crystal periods. Crystalline quartz layers are used as anisotropic elements of photonic crystal. Effect of the gasoline permittivity varying that refers to octane number changing on the angle of polarization plane rotation is investigated.**

Keywords—anisotropic photonic crystal, polarization plane rotating, permittivity, sensor-polarimeter

I. Introduction

Photonic crystals are artificial periodic structures that represent unique physical properties stipulated by their spectral characteristics [1]. One-dimensional photonic crystals are in fact periodic multilayer media consisting of alternating layers with different material parameters and geometrical sizes. Theoretical analysis of such media is usually performed on the base of Floquet theory and transfer matrix method [2-6]. Dielectric and metallic photonic crystals are used for the development of waveguides, resonators, filters, and other functional devices [7-11].

Characteristics of the photonic crystals are defined by the physical properties of their elements. If at least one of the layers on the period of 1D photonic crystal characterized by tensor permittivity or permeability then periodic structures are identified as anisotropic or gyrotropic photonic crystals [6, 12-14]. Such structures have advanced capabilities for controlling electromagnetic radiation. Application of anisotropic uniaxial crystals such as crystalline quartz in photonic crystals results in the possibility of radiation polarization plane rotation [15, 16]. It is apparent that rotation angle is function not only anisotropic layers parameters but entire structure ones [16, 17]. Therefore anisotropic photonic crystals can be used as sensors or measuring cells for controlling of solids and liquids parameters. It should be noted that ellipsometric methods are well known for measurements of material parameters of various media [18, 19]. Moreover, these methods have benefits in terahertz and optical bands in comparison with convenient microwave measuring techniques [20, 21].

In this work, we consider theoretically multilayer sensor-polarimeter on the base of 1D anisotropic photonic crystal. The sensor consists of parallel quartz plates separated by air gaps in which solid or liquid isotropic substances can be placed. Varying of these substances permittivity results in rotating of transmitted radiation polarization plane. Thus measurement of polarization plane rotating angle allows to controlling the changing of physical properties of medium under investigation.

II. Statement and Solution of the Problem

It is necessary to know the transmission coefficients for two perpendicular polarizations (*p*-polarization and *s*-polarization) to determine the rotation angle of the polarization plane of the wave passing through anisotropic medium. In this regard, we will consider the problem of scattering of the electromagnetic plane wave with arbitrary polarization by anisotropic Bragg reflector, consisting of a finite number of periods of the two-layer structure. Each period contains anisotropic crystal layer and isotropic dielectric layer (Fig. 1).

Anisotropic layers of a one-dimensional photonic crystal are characterized by a diagonal permittivity tensor $\vec{\varepsilon}_1$:

$$\overset{\leftrightarrow}{\varepsilon}_1 = \begin{Vmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{Vmatrix}. \quad (1)$$

Other layers of the Bragg reflector are solid materials or liquids with isotropic dielectric constant. The period of the structure can be represented as $L = a + b$.

Note that by virtue of the principle of permutation duality [6, 13] it is sufficient to consider only one of the problems of electromagnetic plane wave scattering by a Bragg reflector: H_z -polarization (*p*-polarization) or E_z -polarization (*s*-polarization). The characteristics of the scattered fields for another polarization can be obtained through the replacements $H_z \leftrightarrow E_z$ and $\vec{\varepsilon} \leftrightarrow -\vec{\mu}$.

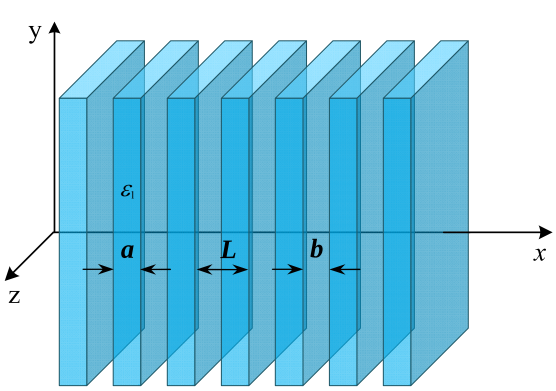

Fig. 1. Schematic of anisotropic photonic crystal.

First, we will consider the scattering of a H_z -polarized plane wave $H_z^{in} = e^{i\xi_{in}x+i\beta y}$ by a Bragg anisotropic reflector with layer parameters $\overset{\leftrightarrow}{\varepsilon}_1$ and ε_2. Here $\xi_{in} = \sqrt{k^2\varepsilon_{in}\mu_{in} - \beta^2}$ is the transverse wave number, β is the longitudinal one, ε_{in} and μ_{in} are permittivity and permeability of the medium outside the Bragg reflector.

The angle of the polarization plane of the incident wave is found through the amplitudes of the waves of two polarizations, namely: $\varphi = \arctan\left(E_y / E_z\right)$, where $E_y = \left(1/ik\varepsilon_{in}\right)\partial H_z / \partial x.$ When waves of different polarizations pass through the Bragg reflector, the transmission coefficients for a given frequency take different values due to the fact that one of the reflector layers is anisotropic. This fact leads to rotation of the polarization plane of the wave. Let us consider the solution of the problem of scattering of the initial field by the Bragg anisotropic reflector.

From the Maxwell equations follows the Helmholtz equation for the field component H_z

$$\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon(x)}\frac{\partial}{\partial x}H_z\right) + \frac{1}{\varepsilon(x)}\frac{\partial^2 H_z}{\partial y^2} + k^2\mu_j H_z = 0 . \quad (2)$$

Here $\varepsilon(x) = \begin{cases} \varepsilon_1, & 0 + NL < x < a + NL \\ \varepsilon_2, & a + NL < x < \left(N+1\right)L \end{cases}.$

The Helmholtz equation for E_z -polarization has the form

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu\left(x\right)}\frac{\partial E_z}{\partial x}\right) + \frac{1}{\mu\left(x\right)}\frac{\partial^2 E_z}{\partial y^2} + k^2\varepsilon_{\|j}\left(x\right)E_z = 0 . \quad (3)$$

In general case components of tensor (1) $\varepsilon_\perp$ and $\varepsilon_\|$ have different values. Then it is seen from equations (2) and (3) that the anisotropic dielectric affects the scattered field differently.

As can be seen from the presented equations, the problem reduces to finding solutions of Hill's equation with variable coefficients. To find the transmission and reflection coefficients of plane waves of two polarizations through the anisotropic photonic crystal, we use the transfer matrix method [5, 6].

Equations (2, 3) can be reduced using the method of variables separation $H_z\left(x,y\right) = X\left(x\right)e^{i\beta y}$ to one form of the Hill's equation with periodic coefficients, namely:

$$\frac{\partial}{\partial x}\left(p(x)\frac{\partial X}{\partial x}\right) + q(x)X_z = 0 , \quad (4)$$

where $q(x) = p(x)\left(k^2\varepsilon(x)\mu - \beta^2\right) = p(x)\xi^2(x) = p_j\xi_j^2$, $p(x) = \varepsilon(x)^{-1}$, for *p*-polarized waves.

To find the reflection and transmission coefficients R_N^s and T_N^s for plane *s*-polarized wave passing through the anisotropic Bragg reflector, it is necessary to find a solution of the Hill's equation (4) for three regions. As a rule, the main attention is paid to solving Hill's equation in the domain of the reflector itself.

Taking into account the solution of Hill's equation (4) for two layers ($j = 1,\ 2$) on the period of the anisotropic Bragg reflector, we represent the tangential components of the s-polarized fields in the form:

$$H_z^1(x,y) = \left(a_1 e^{i\xi_1 x} + b_1 e^{-i\xi_1 x}\right)e^{i\beta y},$$
$$E_y^1(x,y) = \frac{1}{k\varepsilon_1}\xi_1\beta\left(a_1 e^{i\xi_1 x} - b_1 e^{-i\xi_1 x}\right)e^{i\beta y}, \quad (5)$$

$$H_z^2(x,y) = \left(a_2 e^{i\xi_2 x} + b_2 e^{-i\xi_2 x}\right)e^{i\beta y},$$
$$E_y^2(x,y) = \frac{1}{k\varepsilon_2}\xi_2\beta\left(a_2 e^{i\xi_2 x} - b_2 e^{-i\xi_2 x}\right)e^{i\beta y}, \quad (6)$$

where a_1 , b_1 and a_2 , b_2 are unknown fields amplitudes; $\xi_j = \sqrt{k^2\varepsilon_j\mu_j - \beta^2}$ ($j = 1,\ 2$) are transversal wave numbers in structure period layers.

Using the boundary conditions for the tangential components (5, 6) at the boundaries of each layer, we obtain matrix equations for determining unknown amplitudes through the transfer matrix, namely:

$$\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix} = \ldots$$
$$\ldots = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^N \begin{pmatrix} a_N \\ b_N \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} a_N \\ b_N \end{pmatrix}. \quad (7)$$

Here

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^N$$

is unimodular matrix. For unimodular matrices the following identity should be satisfied:

$$M = \begin{pmatrix} AU_{N-1} - U_{N-2} & BU_{N-1} \\ CU_{N-1} & DU_{N-1} - U_{N-2} \end{pmatrix}, \quad (8)$$

where

$$U_N = \frac{\sin(N+1)K_{TE}L}{\sin K_{TE}L},$$

$$M_{11} = A\frac{\sin(N)K_{TE}L}{\sin K_{TE}L} - \frac{\sin(N-1)K_{TE}L}{\sin K_{TE}L},$$

$$M_{12} = BU_{N-1} = B\frac{\sin NK_{TE}L}{\sin K_{TE}L},$$

$$M_{21} = CU_{N-1} = C\frac{\sin NK_{TE}L}{\sin K_{TE}L},$$
$$M_{22} = D\frac{\sin NK_{TE}L}{\sin K_{TE}L} - \frac{\sin(N-1)K_{TE}L}{\sin K_{TE}L}.$$

Here K_{TE} is Floquet-Bloch wave number which для TE-waves is defined by the dispersion equation for infinite photonic anisotropic crystals [6, 13], namely:

$$K_{TE} = \frac{1}{L}\arccos\left\{\begin{matrix}\cos\xi_1 a\cos\xi_2 b - \\ -\frac{1}{2}\left[\frac{\xi_1}{\xi_2}\frac{\varepsilon_2}{\varepsilon_1} + \frac{\xi_2}{\xi_1}\frac{\varepsilon_1}{\varepsilon_2}\right]\sin\xi_1 a\sin\xi_2 b\end{matrix}\right\}. \quad (9)$$

Elements of the transfer matrix $ABCD$ for the periodic structure have the following form:

$$A = \left[\cos\xi_2 b - i\frac{1}{2}\left(\frac{\xi_1}{\xi_2}\frac{\varepsilon_2}{\varepsilon_1} + \frac{\xi_2}{\xi_1}\frac{\varepsilon_1}{\varepsilon_2}\right)\sin\xi_2 b\right]e^{-i\xi_1 a},$$
$$D = \left[\cos\xi_2 b + i\frac{1}{2}\left(\frac{\xi_1}{\xi_2}\frac{\varepsilon_2}{\varepsilon_1} + \frac{\xi_2}{\xi_1}\frac{\varepsilon_1}{\varepsilon_2}\right)\sin\xi_2 b\right]e^{-i\xi_1 a},$$
$$B = i\frac{1}{2}\sin\xi_2 b\left(\frac{\xi_1}{\xi_2}\frac{\varepsilon_2}{\varepsilon_1} - \frac{\xi_2}{\xi_1}\frac{\varepsilon_1}{\varepsilon_2}\right)e^{i\xi_1 a},$$
$$C = -i\frac{1}{2}\sin\xi_2 b\left(\frac{\xi_1}{\xi_2}\frac{\varepsilon_2}{\varepsilon_1} - \frac{\xi_2}{\xi_1}\frac{\varepsilon_1}{\varepsilon_2}\right)e^{-i\xi_1 a}.$$

Elements of matrix M can be calculated numerically by multiplication of matrix $ABCD$ or analytically from Chebyshev polynomial U_N. Using the property of inversed matrices one can also find:

$$\begin{pmatrix} a_N \\ b_N \end{pmatrix} = \left[\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1}\right]^N \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}^N \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} \quad (10)$$

Fields in last layer of the Bragg reflector can be written as

$$H_z^N(x,y) = \left[a_N e^{i\xi_1(x-NL)} + b_N e^{-i\xi_1(x-NL)}\right]e^{i\beta y},$$
$$E_y^N(x,y) = \frac{\xi_1}{k\varepsilon_1}\left[a_N e^{i\xi_1(x-NL)} - b_N e^{-i\xi_1(x-NL)}\right]e^{i\beta y}. \quad (11)$$

Fields in the space before the Bragg reflector and behind it have the form:

$$H_z^0(x,y) = \left(a^0 e^{i\xi_{in}x} + b^0 e^{-i\xi_{in}x}\right)e^{i\beta y},$$
$$E_y^0(x,y) = \frac{\xi_{in}}{k\varepsilon_{in}}\left(a^0 e^{i\xi_{in}x} - b^0 e^{-i\xi_{in}x}\right)e^{i\beta y}. \quad (12)$$

$$H_z^3(x,y) = \left(a_3 e^{i\xi_3(x-Nl)} + b_3 e^{-i\xi_3(x-Nl)}\right)e^{i\beta y},$$
$$E_y^3(x,y) = \frac{\xi_3}{k\varepsilon_3}\left(a_3 e^{i\xi_3(x-Nl)} - b_3 e^{-i\xi_3(x-Nl)}\right)e^{i\beta y}. \quad (13)$$

From equations (11) and (13) we obtain the matrix equation of a standard type, connecting the unknown coefficients in the last layer of the reflector ($j = N$) with the field coefficients behind the reflector:

$$\begin{pmatrix} a_N \\ b_N \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix}, \quad (14)$$

where

$$k_{11} = k_{22} = \frac{1}{2}\left[1 + \frac{\varepsilon_1}{\xi_1}\frac{\xi_3}{\varepsilon_3}\right],\ k_{12} = k_{21} = \frac{1}{2}\left(1 - \frac{\varepsilon_1}{\xi_1}\frac{\xi_3}{\varepsilon_3}\right).$$

In the same way, it is possible to determine the elements of the input matrix n_{ij}, which have the form:

$$n_{11} = \frac{1}{2}\left[1 + \frac{\xi_1}{\varepsilon_1}\frac{\varepsilon_{in}}{\xi_{in}}\right]e^{-i\xi_1 a},\ n_{12} = \frac{1}{2}\left[1 - \frac{\xi_1}{\varepsilon_1}\frac{\varepsilon_{in}}{\xi_{in}}\right]e^{i\xi_1 a},$$
$$n_{21} = \frac{1}{2}\left[1 - \frac{\xi_1}{\varepsilon_1}\frac{\varepsilon_{in}}{\xi_{in}}\right]e^{-i\xi_1 a},\ n_{22} = \frac{1}{2}\left[1 + \frac{\xi_1}{\varepsilon_1}\frac{\varepsilon_{in}}{\xi_{in}}\right]e^{i\xi_1 a}.$$

Finally one can simply find the reflection R_N^H and transmission T_N^H coefficients for a plane wave that incident on the Bragg reflector, namely:

$$R_N^H = \frac{\left[\left(n_{21}M_{11} + n_{22}M_{21}\right)k_{11} + \left(n_{21}M_{12} + n_{22}M_{22}\right)k_{21}\right]}{\left[\left(n_{11}M_{11} + n_{12}M_{21}\right)k_{11} + \left(n_{11}M_{12} + n_{12}M_{22}\right)k_{21}\right]}, \quad (15)$$

$$T_N^H = \frac{1}{\left[\left(n_{11}M_{11} + n_{12}M_{21}\right)k_{11} + \left(n_{11}M_{12} + n_{12}M_{22}\right)k_{21}\right]}. \quad (16)$$

To determine the angle of rotation of the polarization plane of the wave after passing through the Bragg reflector it is necessary to determine transmission coefficients for both the H-polarized wave T_N^H and the E-polarized one T_N^E. For a normal incidence of the wave on Bragg reflector the rotation angle of the polarization plane is

$$\varphi = \arctan\left(T_N^H / T_N^E\right). \quad (17)$$

As noted above, to determine the characteristics of the scattered field for E-polarization, it is necessary to use the principle of permutation duality with substitutions $H_z \leftrightarrow E_z$, $\ddot{\varepsilon} \leftrightarrow -\ddot{\mu}$ in expressions (9), (15), (16), where $\xi_1 = \sqrt{k^2\varepsilon_{\parallel}\mu_1 - \beta^2}$.

III. Analysis of Results

Fig. 2 shows results of transmittance calculations for bounded one-dimensional photonic crystal that contains 10 periods. Solid curve corresponds to TM polarization (E_z polarization) and dashed curve indicates transmittance for TE polarization (H_z polarization). Calculations are performed for such parameters values: $a = 0.9L$; $\varepsilon_1 = 4.643$; $\varepsilon_{\parallel} = 4.847$; $\varepsilon_2 = 2$; $\mu_1 = \mu_2 = 1$. Values of permittivity tensor diagonal elements for layer 1 correspond to α-quartz SiO_2 [22]. It is supposed that layer 2 of photonic crystal period is the gasoline with octane number (RON) near 80 [23]. Anisotropy of SiO_2 layers of photonic crystal results in difference between spectral characteristics for two perpendicular polarizations. Therefore polarization plane of radiation is rotated during passing through the photonic crystal. It's obvious that rotating angle is the function of material and structure parameters of photonic crystal. Thus such periodical structure can be used for precision measurements of samples material parameters including liquids. Measurement accuracy is determined by sensitivity of polarization plane rotating angle to changing of material parameters of photonic crystal elements. For example, let's consider polarization plane rotating for changing of gasoline octane number from 80 to 95. This deviation of octane number leads to permittivity ε_2 changing from 2 to 2.2 [23].

Fig. 3 shows transmittances of photonic crystal for different values of isotropic layer permittivity ε_2 . This permittivity varies from 2 to 2.2 by step 0.05. Fig. 3a and 3b correspond to TE and TM polarization respectively. It is clear to see that most pronounced changes in spectral characteristics occur near high-frequency edges of forbidden zones for both polarizations. Moreover forbidden zones for different polarizations are shifted along frequency axis in such manner that high-frequency edge of TM band gap lays near center of TE band gap. Because of that one can choose operation frequency providing varying transmittance only for TM polarization when permittivity ε_2 is changing. It should be noted that to ensure the accuracy of measurements it is necessary to use a sufficiently large number of structure periods providing sufficient steepness of the characteristics at forbidden zones edges.

Let's consider changing of transmittance for TM polarization by virtue of gasoline octane number varying in layer 2 (Fig. 4). Vertical dashed line indicates operation frequency value ($\omega L / 2\pi c = 1.678$). Black circles show transmittance values for different octane numbers. It is apparent from Fig. 3a that transmittance changing for chosen operation frequency for TE polarization is very small. Therefore the polarization plane rotating, in this case, is mainly due to transmittance changing for TM polarization. This transmittance varies approximately from 0.1 to 1 (Fig. 4), providing ability of precision measurements of polarization plane rotating angle.

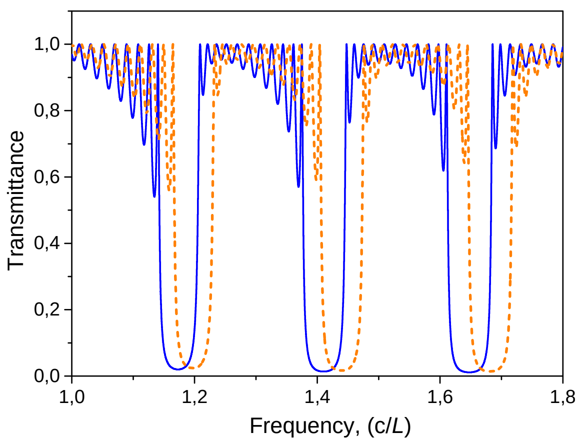

Fig. 2. Spectral characteristic of one-dimensional photonic crystal.

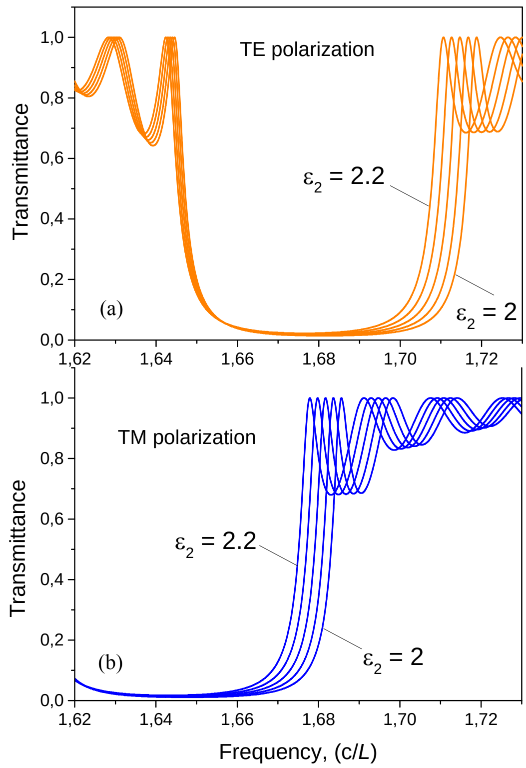

Fig.3. Transmittance of photonic crystal for different values of isotropic layer permittivity; (a) TE polarization; (b) TM polarization.

Fig. 5 shows the polarization plane rotating angle as a function of permittivity of an isotropic layer of the periodical structure under investigation. It is supposed that zero angle

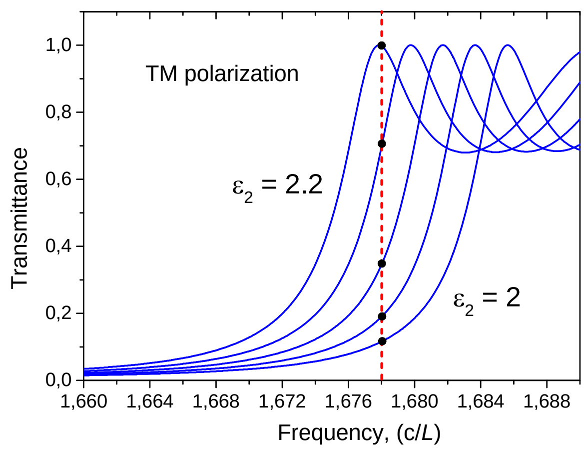

Fig. 4. Transmittance vs frequency for different values of permittivity ε_2 .

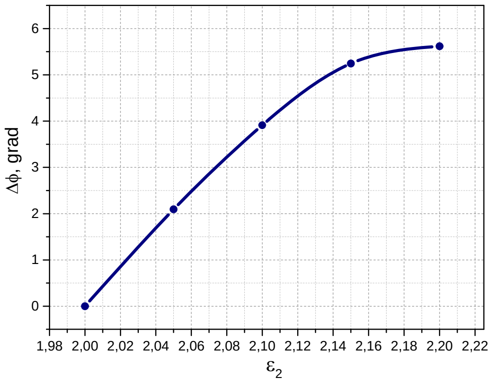

Fig. 5. Polarization plane rotating angle vs permittivity of isotropic layer.

corresponds to initial value of permittivity ($\varepsilon_2 = 2$). Permittivity changing by 0.05 results in polarization plane rotating on some degrees. Such rotating angle values can be easy measured with modern equipment that provides ability of precise defining of different media material parameters.

IV. Conclusion

Sensor-polarimeter on the base of one-dimensional anisotropic photonic crystal has been considered. Theoretical analysis is performed for a bounded multilayer structure that contains anisotropic parallel plates separated by isotropic layers. Floquet theory and transfer matrix method are used for deriving analytical expressions for sensor transmittance and reflectance for two orthogonal linear polarizations of incident radiation. Sensor response for isotropic layers permittivity changing is investigated. The effect of the gasoline octane number changing on the rotation angle of transmitted radiation polarization plane is shown.

References

[1] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, Photonic Crystals: molding the flow of light. Princeton University Press, 2008.

[2] G. Floquet, "Sur les equations differentielles lineaires a coefficients periodiques," Annales Scientifiques de l'Ecole Normale Superieure, vol.12, pp. 47–88, 1883.

[3] R. L. Mooney, "Theory of an efficient interference filter," J. of the Opt. Soc. of America, vol. 36, no. 5, pp. 256–260, May 1946.

[4] G. V. Morozov, and D. W. L. Sprung, "Floquet-Bloch waves in one-dimensional photonic crystals," EPL, vol. 96, 54005, 2011.

[5] P. Yen, A. Yariv, and Chi-Shain Hong, "Electromagnetic propagation in periodic stratified media. I. General theory," J. Opt. Soc. Am, vol. 67, no. 4, pp. 423–438, 1977.

[6] Theoretical Foundations and Applications of Photonic Crystals, (Ed. A. Vakhrushev) (Ch, 3, A. A. Shmat'ko, V. N. Mizernik, E. N. Odarenko, V. T, Lysytsya. Dispersion Properties of TM and TE Modes of Gyrotropic Magnetophotonic Crystals). InTech, 2018.

[7] Q. Gong, X. Hu, Photonic Crystals. Principles and Applications. Taylor & Francis Group, 2013.

[8] E. N. Odarenko, Y. V. Sashkova, and A. A. Shmat'ko, "Localized field enhancement in slow-wave modes of modified Bragg waveguide," Proc. IEEE 5th Microwaves, Radar and Remote Sensing Symposium (MRRS), Kiev, Ukraine, 29-31 Aug., 2017, pp. 147–150.

[9] L. I. Ivzhenko, E. N. Odarenko, and S. I. Tarapov, "Mechanically tunable wire medium metamaterial in the millimeter wave band," Progress in Electromagnetics Research Letters, vol. 64, pp. 93–98, 2016.

[10] T. F. Krauss, "Planar photonic crystal waveguide devices for integrated optics," Phys. Stat. Sol., vol. 197, no. 3, pp. 688–702, 2003.

[11] E. N. Odarenko, Y. V. Sashkova, A. A. Shmat'ko, and N. G. Shevchenko, "Analysis of slow wave modes in modified photonic crystal waveguides using the MPB package," Proc. of Int. Conference on Mathematical Methods in Electromagnetic Theory (MMET), Kyiv, Ukraine, 2-5 July, pp. 164–167, 2018.

[12] M. Inoue, K. Arai, T. Fujii, and M. Abe, "One-dimensional magnetophotonic crystals," J. of Applied Physics, 85, pp. 5768–5770, 1999.

[13] A. A. Shmat'ko, V. N. Mizernik, E. N. Odarenko, V. A. Yampol'skii, T. N. Rokhmanova, and A. Yu. Galenko, "Dispersion properties of a one-dimensional anisotropic magnetophotonic crystal with a gyrotropic layer," Proc. IEEE 7th Int. Conf. On Advanced Optoelectronics and Lasers (CAOL), Odessa, Ukraine, 12-15 Sept., pp. 126–128, 2016.

[14] A. A. Shmat'ko, V. N. Mizemik, E. N. Odarenko, and T. N. Rokhmanova, "Bragg reflection and transmission of light by one-dimensional gyrotropic magnetophotonic crystal," Proc. IEEE Int. Conf. on Advanced Information and Communication Technologies (AICT), Lviv, Ukraine, 4-7 July, pp. 232–236, 2017.

[15] W. Peddie, "On the rotation of the plane of polarization by quartz, and its relation to wave-length," Proc. of the Royal Society of Edinburgh, vol. 11, pp. 815–818, 1882.

[16] A. A. Vertiy, S. P. Gavrilov, V. N. Derkach, and V. P. Shestopalov, "Application of multilayer structures at millimeter wavelength," Int. J. of Infrared and Millimeter Waves, vol. 10, no. 10, pp. 1193–1206, Oct. 1989.

[17] A. A. Vertiy, S. P. Gavrilov, S. Helhel, B. Colak, and R. Samedov, "Quasioptical multilayered interferometer for gasoline testing," Int. J. of Infrared and Millimeter Waves, vol. 17, no. 3, pp. 543–557, 1996.

[18] T. Tumkur, Y. Barnakov, S. T. Kee, M. A. Noginov, and V. Liberman, "Permittivity evaluation of multilayered hyperbolic metamaterials: Ellipsometry vs. reflectometry," J. of Applied Physics, vol. 117, 103104, 2015.

[19] J. H. Jung, S. Yoo, and Q. H. Park, "Measuring the optical permittivity of two-dimensional materials without *a priori* knowledge of electronic transitions," Nanophotonics, vol. 8, no. 2, pp. 263–270, 2019.

[20] M. T. Jilani, M. Z. Rehman, A. M. Khan, M. T. Khan, and S. M. Ali, "A brief review of measuring techniques for characterization of dielectric materials," Int. J. of Information Techn. and Electrical Engineering, vol. 1, no. 1, pp. 1–5, Dec. 2012.

[21] J. Krupka, "Frequency domain complex permittivity measurements at microwave frequencies," Meas. Sci. Technol., vol. 17, R55–R70, 2006.

[22] C. Sevik, and C. Bulutay, "Theoretical study of the insulating oxides and nitrides: SiO_2, GeO_2, Al_2O_3, Si_3N_4, and Ge_3N_4," J. Mater. Sci., vol. 42, pp. 6555–6565, 2007.

[23] B. D. Zaitsev, A. A. Teplykh, I. A. Borodina, I. E. Kuznetsova, and E. Verona, "Gasoline sensor based on piezoelectric lateral electric field excited resonator," Ultrasonics,vol.80, pp. 96–100, 2017.