Spectral Features of a Multi-Periodical Metamaterials

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***Abstract*—We numerically investigate the spectral properties of multi-periodic metamaterials of various configurations (the planar double-periodic photonic crystal and the bulk double-periodic magnetophotonic crystal) in the frequency range 15-45 GHz. In the first part of this paper we describe investigation of structure of the planar double periodic photonic crystal (PDPPC) which consists of two planar photonic crystals with large and small periods. A PDPPC unit cell consists of a series-connected narrow and wide segments of a microstrip transmission line. In the second part of this paper we will discuss results of investigation of a double-periodic magnetophotonic crystal (DPMPC) which consists of 3 unit cells. Each cell consists of periodically located 9 dielectric layers of quartz and teflon. On the border of the cell there is one layer of ferrite.**

We showed that both for a PDPPC and for a DPMPC in the transmission spectrum of these structures under study, a number of features arise: namely, narrow transmission peaks caused by the superposition of two spectra (with large and small periods) are observed in the stop bands of the spectrum. Such features may further allow the creation of a narrow-band filter for several frequencies for both planar structures and bulk ones.

Keywords—multi-periodical metamaterial, planar photonic crystal, magnetophotonic crystal, transmission spectrum, pass band, stop band

I. INTRODUCTION

It is known that planar photonic crystals (PPC) or planar metamaterial with a periodically varied signal conductor width are the subject intensive study of many authors [1-4]. A special feature of PC is the appearance of stop band in a specific frequency range (analogue of the energy band gap in the solid state). This allows the use of PPC, and structures based on the PPC as a compact broadband and narrowband frequency filters.

It is also known that the combination of photonic crystals of different periods in one structure can lead to the appearance of narrow pass bands in the stop band of such metamaterial [5].

Today the bulk metamaterials such as photonic crystals, magnetophotonic crystals attract interest of researchers [6, 7]. Interest in investigation of such media is due to the fact that a number of features arise in the spectrum of such artificial media such as Tamm states, defect modes, etc. These features in the transmission spectra allow using the above described artificial media in the production of microwave filters, attenuators, etc.

Here a new type of 1D magnetocontroled metamaterial (multi-periodic magnetophotonic crystal) in which periodically arranged dielectric multilayers (dielectric PC) are spaced by magnetic layers has been studied.

In this paper, we numerically investigate the spectral properties of multi-periodic metamaterials of various configurations (the planar double-periodic photonic crystal and the bulk double-periodic magnetophotonic crystal).

II. PLANAR DOUBLE-PERIODICAL PHOTONIC CRYSTAL

In this paper, we have numerically investigated the transmission spectrum of a planar double periodical photonic crystal (PDPPC). PDPPC consists of two photonic crystals with large and small periods (Fig. 1a). The unit cell of the PDPPC consists of sequentially connected narrow and wide segments of the microstrip transmission line (MSTL). The size of the unit cell is comparable with the wavelength.

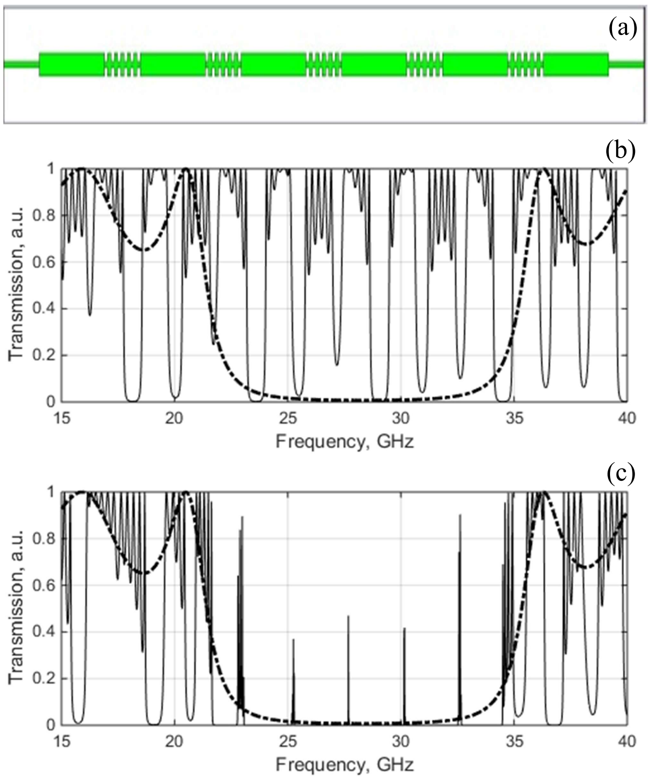

Fig. 1 (a) Investigated structure. (b) Transmission spectra of planar PC with big period (solid line) and planar PC with small period (dash-dotted line) (c) Transmission spectra of PDPPC (solid line) and planar PC with small period (dash-dotted line)

The parameters of the unit cells for the small PPC are: the length - $d_1 = 1mm$ (narrow), $d_2 = 1mm$ (wide) and the

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width - $w_1 = 0.22\,mm$ (narrow), $w_2 = 0.65\,mm$ (wide). The parameters of the unit cells for the large PPC are: $d_1 = 11\,mm$ (narrow), $d_2 = 20\,mm$ (wide). The dielectric constant of the substrate ε_r is $\varepsilon_r = 10$ (corresponding to technological brand "Rogers"), and the thickness h is $h = 0.254\,mm$. The number of unit cells is $N_1 = 5$ for small PPC and N_2=6 for large PPC. The structure are shown in Fig. 1a.

The transmission spectrum of the PDPPC (solid line) and planar photonic crystal with small period (dash-dotted line) was investigated numerically (Fig. 1c). It is seen from Fig. 1c, that in the stop band of a photonic crystal with a small period there are narrow pass bands. Moreover, the number of peaks in these pass bands is M-1, where M is the number of large periods in the structure. We have shown that narrow pass bands are caused by superposition of the stop bands for photonic crystals with a large and a small periods (Fig. 1b).

To calculate the transmission spectrum, a matrix method for microstrip transmission lines was used [1]. For this purpose, well-known expressions were used [1]:

$$\hat{T}_S = \begin{pmatrix} \exp(ik_S d_S) & 0 \\ 0 & \exp(-ik_S d_S) \end{pmatrix},$$
$$\hat{T}_{S,S+1} = \begin{pmatrix} \dfrac{r_{S,S+1}+1}{2\sqrt{r_{S,S+1}}} & \dfrac{r_{S,S+1}-1}{2\sqrt{r_{S,S+1}}} \\ \dfrac{r_{S,S+1}-1}{2\sqrt{r_{S,S+1}}} & \dfrac{r_{S,S+1}+1}{2\sqrt{r_{S,S+1}}} \end{pmatrix}, \qquad (1)$$
$$r_{S,S+1} = \rho_{S+1}/\rho_S, (s = 1,2).$$

where $k_S = 2\pi\Lambda_S^{-1}$, d_S is the length of the s-th segment.

$$\rho_S = \frac{377 h_S}{\sqrt{\varepsilon_S} W_S \left(1 + 1,735\varepsilon^{-0,0724}\left(\dfrac{W_S}{h_S}\right)^{-0,836}\right)}, \qquad (2)$$

where h_S is the substrate height, W_S is the segment width, ε_S is the substrate permittivity.

The wavelength of the electromagnetic wave, at the s-th segment of the microstrip line is defined by the known expression [1]:

$$\Lambda_S = \begin{cases} \dfrac{\lambda}{\sqrt{\varepsilon_S}} \sqrt{\dfrac{\varepsilon_S}{1 + 0.63(\varepsilon_S - 1)\left(\dfrac{W_S}{h_S}\right)^{0,1255}}} & \text{for } \dfrac{W_S}{h_S} \geq 0.6 \\ \dfrac{\lambda}{\sqrt{\varepsilon_S}} \sqrt{\dfrac{\varepsilon_S}{1 + 0.6(\varepsilon_S - 1)\left(\dfrac{W_S}{h_S}\right)^{0,0297}}} & \text{for } \dfrac{W_S}{h_S} < 0.6 \end{cases} \qquad (3)$$

The transmission-wave matrix of the unit cell has known form:

$$\hat{T} = \hat{T}_1 \hat{T}_{1,2} \hat{T}_2 \hat{T}_{2,1} \qquad (4)$$

An expression that describes the dependence of the transmittance on frequency has a known form:

$$D = |T_{11}|^{-2} \qquad (5)$$

III. SPECTRAL FEATURES OF A MULTI-PERIOD MAGNETOPHOTONIC CRYSTAL

And numerically demonstrate that in such structure presents possibility to control the internal (narrow) bandwidth of the transmission spectrum by external magnetic field.

Let call the PC with periodic magnetic defects as double-periodical magnetophotonic crystal (DPMPC). The DPMPC primary cell consists of dielectric PC and ferrite layer. The structure under study is presented on Fig. 2. The elementary dielectric PC bi-layered cell consists of teflon and quartz layers (Fig. 2).

Fig. 2 Scheme and photo of structure under study: teflon layer – grey line, quartz layer – white line, ferrite layer – black line.

The teflon layer permittivity is $\varepsilon_t = 2.05$ and thickness is $d_t = 2\,\text{mm}$. The quartz layer permittivity is $\varepsilon_q = 4.5$ and thickness is $d_q = 1\text{mm}$. The ferrite layer has the permittivity of about $\varepsilon_f = 13.2$, saturation magnetization is $M_s = 382G$, damping coefficient is $\alpha = 0.024$ and thickness is $d_f = 10\text{mm}$.

A numerical model for calculating the transmission spectrum through the structure under study was built on the basis of the well-known transfer-matrix method using the open access program Octave [8,9]. The matrix of transfer within one structure period has the form:

$$m(z)=\begin{pmatrix}\cos k_{z_1}z & -k_{z_1}^{-1}\sin k_{z_1}z\\ k_{z_1}\sin k_{z_1}z & \cos k_{z_1}z\end{pmatrix}. \qquad (6)$$

The transfer matrix $\hat{M}$ over the entire periodic structure is the product of matrices of transfer in separate periods of $m(z)$:

$$\hat{M}=\prod_{i=1}^{n} m_i(z_i). \qquad (7)$$

The amplitude transmission factor of the entire structure t is related to the elements of the transfer matrix $\hat{M}$ by the following expression:

$$t=\frac{2}{M_{11}+M_{12}+M_{21}+M_{22}}. \qquad (8)$$

The transmission factor is equal to

$$T=|t|^2. \qquad (9)$$

The permeability of a ferrodielectric layer is the second-rank tensor of the following form [5]:

$$\hat{\mu}=\begin{pmatrix}\mu & i\mu_a & 0\\ -i\mu_a & \mu & 0\\ 0 & 0 & 1\end{pmatrix}. \qquad (10)$$

For an infinite ferrodielectric magnetized to saturation the expressions for the tensor components are determined from the solution of the Landau-Lifshits magnetization equation of motion and are written as follows:

$$\mu=\frac{\omega_H(\omega_H+\omega_M)-\omega^2}{\omega_H^2-\omega^2},$$

$$\mu_a=\frac{\omega\omega_M}{\omega_H^2-\omega^2},\ \omega_H=\gamma\vec{H}-ia\omega, \qquad (11)$$

$$\omega_M=\gamma 4\pi M_s,$$

where γ is the gyromagnetic electron ratio, $\vec{H}$ is the external magnetic field, M_s is the ferromagnetic saturation magnetization, ω is the frequency of the variable electromagnetic field.

To control the spectral properties of a structure under study using the magnetic field, it would be more preferable to make use of the “extraordinary” wave. Here the vector of the alternative magnetic field is perpendicular to the permanent magnetic field vector $\vec{h}\perp\vec{H}$. In this case the effective magnetic permeability takes the well-known form [5]:

$$\mu_\perp=\frac{\mu^2-\mu_a^2}{\mu} \qquad (12)$$

A typical transmission spectrum for this type of structure is shown in Fig. 3.

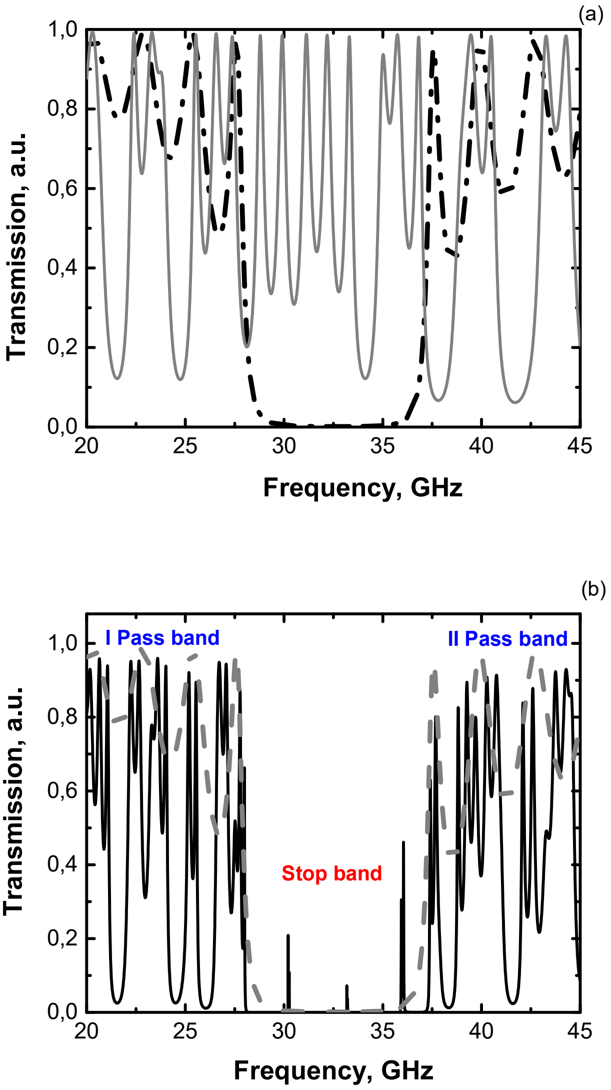

Fig. 3 (a) Transmission spectra of MPC with big period (solid line) and dielectric PC with small period (dash-dotted line) (b) Transmission spectra of DPMPC (solid line) and dielectricar PC with small period (dash-dotted line)

Numerical transmission spectra of investigation structure: dielectric multilayers PC (dashed line), DPMPC (solid line).

Thus, during this investigation we obtain that the transmission spectrum with wide stop and pass bands corresponds to the spectrum of a structure with dielectric multilayers (quartz-teflon, Fig. 3, dashed line) while the transmission spectrum with narrow stop and pass bands corresponds to the spectrum of a structure with a large period (elementary cell of DPMPC, Fig. 3, solid line).

Further, it can be seen that an increasing of ferrite permeability leads to a change in the frequency of only narrow pass bands within wide pass bands. As well it was seen that with increasing of external DC magnetic bias field we can tune resonant frequency of family narrow transmission peaks in wide stop bands (Fig. 4).

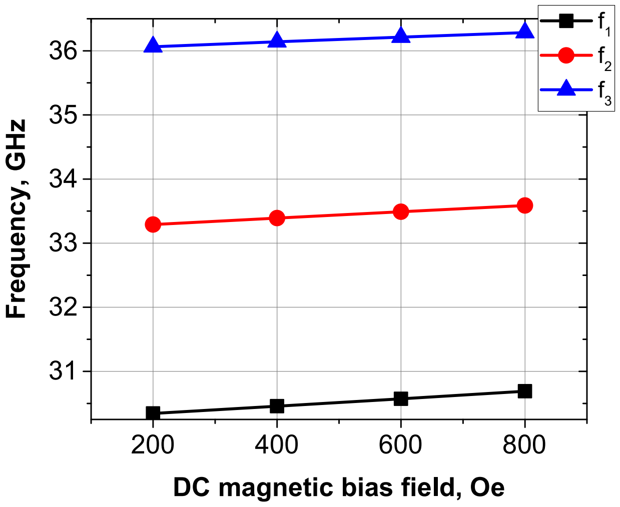

Fig. 4 Dependence of the center frequency of the narrow pass band on the DC magnetic bias field. Here f_1, f_2 and f_3 is the central frequencies of the first second and third narrow pass bands

IV. CONCLUSIONS

Thus, in summary in this paper the numerical investigation of the transmission properties of the multi-periodical metamaterials has been carried out. It was shown that the family of narrow peaks of the transmission coefficient can appear in the frequency range where the stop bands of periodic structures with small and big periods overlap.

The possibility of external DC magnetic bias field tuning of the transmission peaks position was demonstrated for DPMPC structure.

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