Muticriteria Balance Layout Problem of 3d-Objects

Igor Grebennik[1], Tatiana Romanova[2], Anna Kovalenko[1], Inna Urniaieva[1], Sergey Shekhovtsov[1]

[1] Kharkiv National University of Radioelectronics, Kharkiv, Ukraine
igorgrebennik@gmail.com

[2] Institute for Mechanical Engineering Problems of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine
sherom@kharkov.ua

Abstract. The paper studies the optimal layout problem of 3D-objects in a container with circular racks. The problem takes into account placement constraints (non-overlapping, containment, distance constraints), as well as, behaviour characteristics of the mechanical system (equilibrium, moments of inertia and stability characteristics). We construct a mathematical model of the problem in the form of multicriteria optimisation problem and call the problem the Multicriteria Balance Layout Problem (MBLP). We also consider several realisations of MBLP problem that depend on forms of objective functions and behaviour constraints.

Keywords. Layout problem, Behaviour Constraints, Placement Constraints, Mathematical Model, Multicriteria Optimisation

1. Introduction

3D layout optimization problems have a wide spectrum of practical applications. In particular, these problems arise in space engineering for rocketry design. Their distinctive feature consists of taking into account behaviour constraints of a satellite system. Behaviour constraints specify the requirements for system's mechanical properties such as equilibrium, inertia, and stability. Many publications analyze problems of the equipment layout in modules of spacecraft or satellites (see, e.g. [1, 2]). These problems are NP-hard.

In the research we consider the balance layout problem in the following statement: arrange 3D objects in a container taking into account special constraints so that the objective function attains its extreme value (see, e.g. [3]).

As extension of the balance layout of 3D-objects considered in [3] as NLP-problem the purpose of the study is to create a mathematical model of the problem in the form of multicriteria optimisation problem. We call the problem the Multicriteria Balance Layout Problem (MBLP). Such model can be used to obtain various variants of the MBLP problem, which are determined by the variety of forms of the objective functions and the presence of the special constraints (behaviour constraints).

To describe placement constraints (non-overlapping of objects, containment of objects in a container with regard for the minimal and maximal allowable distances) analytically we employ phi-function technique [4]. We also formalise behaviour constraints (equilibrium, moments of inertia, and stability constraints) based on [3].

2. Problem Formulation

MBLP: Pack 3D objects $A_i \in A$, $i \in I_n = \{1, 2, \ldots, n\}$, inside container Ω, so that the vector function attains its extreme value with regard for placement and behaviour constraints.

The placement constraints in the MBLP problem are generated by non-overlapping of objects A_i, A_j, $i > j \in I_n$, which have to be placed inside container Ω, and containment of object A_i in container Ω, $i \in I_n$. In addition, the minimal ρ_{ij}^- and maximal $\rho_{ij}^+ \geq \rho_{ij}^-$ allowable distances between objects A_i, A_j, $i > j \in I_n$, may be specified. Also, the minimal allowable distance ρ_i^- between object $A_i \in A$, $i \in I_n$, and the lateral surface of container Ω may be given. Without loss of generality we set $\rho_{ij}^- = 0$ (or $\rho_{ij}^+ = \varpi$) if a minimal (or a maximal) allowable distance between objects A_i and A_j is not given, $i > j \in I_n$. Here ϖ is a given sufficiently great number. In particular, the condition $\rho_{ij}^+ = \rho_{ij}^-$ provides the arrangment of objects A_i and A_j on the exact distance. We also set $\rho_i^- = 0$ if a minimal allowable distance between object A_i and the lateral surface of container Ω is not given.

Placement constraints in the MBLP problem may be presented as the following:

$$\rho_{ij}^- \leq \mathrm{dist}(A_i, A_j) \leq \rho_{ij}^+, \; i > j \in I_n,$$

and

$$\mathrm{dist}(A_i, \Omega^*) \geq \rho_i^-, \; i = 1, \ldots, n,$$

where $\Omega^* = R^3 \setminus \mathrm{int}\,\Omega$.

To describe the placement constraints analytically we employ the phi-function technique (see, e.g., [4,5]).

Let us consider the constraints of mechanical characteristics of system Ω_A.

The equilibrium constaints are defined by the following system of inequalities:

$$\mu_{11}(u) = \min\{-(x_s(u) - x_e) + \Delta x_e, (x_s(u) - x_e) + \Delta x_e\} \geq 0,$$

$$\mu_{12}(u) = \min\{-(y_s(u) - y_e) + \Delta y_e, (y_s(u) - y_e) + \Delta y_e\} \geq 0,$$

$$\mu_{13}(u) = \min\{-(z_s(u) - z_e) + \Delta z_e, (z_s(u) - z_e) + \Delta z_e\} \geq 0,$$

where (x_e, y_e, z_e) is the expected position of O_s, $(\Delta x_e, \Delta y_e, \Delta z_e)$ are admissible deviations from the point (x_e, y_e, z_e).

The constraints of moments of inertia are defined as the following:

$$\mu_{21}(u) = -J_X(u) + \Delta J_X \geq 0,$$

$$\mu_{22}(u) = -J_Y(u) + \Delta J_Y \geq 0,$$

$$\mu_{23}(u) = -J_Z(u) + \Delta J_Z \geq 0,$$

where $J_X(u), J_Y(u), J_Z(u)$ are the moments of inertia of the system Ω_A with respect to the axes of coordinate system O_sXYZ, $\Delta J_X, \Delta J_Y, \Delta J_Z$ are admissible values for

$J_X(u), J_Y(u), J_Z(u)$, where

$$J_X(u) = J_{x_0} + \sum_{i=1}^{n}(J_{x_i}\cos^2\theta_i + J_{y_i}\sin^2\theta_i) + \sum_{i=1}^{n}(y_i^2 + z_i^2)m_i - M(y_s^2 + z_s^2),$$

$$J_Y(u) = J_{y_0} + \sum_{i=1}^{n}(J_{x_i}\sin^2\theta_i + J_{y_i}\cos^2\theta_i) + \sum_{i=1}^{n}(x_i^2 + z_i^2)m_i - M(x_s^2 + z_s^2),$$

$$J_Z(u) = \sum_{i=0}^{n} J_{z_i} + \sum_{i=1}^{n}(y_i^2 + z_i^2)m_i - M(x_s^2 + y_s^2),$$

$J_{x_0}, J_{y_0}, J_{z_0}$ are the moments of inertia of container Ω with respect to the axes of the coordinate system $Oxyz$, $J_{x_i}, J_{y_i}, J_{z_i}$, $i \in I_n$, are the moments of inertia of object A_i with respect to the axes of coordinate system $O_i x_i y_i z_i$ (see Appendix B).

The stability constraints are defined by the following system of inequalities:

$$\mu_{31}(u) = \min\{-J_{XY}(u) + \Delta J_{XY}, J_{XY}(u) + \Delta J_{XY}\} \ge 0,$$

$$\mu_{32}(u) = \min\{-J_{YZ}(u) + \Delta J_{YZ}, J_{YZ}(u) + \Delta J_{YZ}\} \ge 0,$$

$$\mu_{33}(u) = \min\{-J_{XZ}(u) + \Delta J_{XZ}, J_{XZ}(u) + \Delta J_{XZ}\} \ge 0,$$

where $J_{XY}(u), J_{YZ}(u), J_{XZ}(u)$ are the products of inertia of system Ω_A with respect to the axes of the coordinate system $O_s XYZ$, $\Delta J_{XY}, \Delta J_{YZ}, \Delta J_{XZ}$ are admissible values for $J_{XY}(u), J_{YZ}(u), J_{XZ}(u)$, respectively,

$$J_{XY}(u) = \frac{1}{2}\sum_{i=1}^{n}(J_{x_i} - J_{y_i})\sin 2\theta_i + \sum_{i=1}^{n} x_i y_i m_i - M x_s y_s,$$

$$J_{YZ}(u) = \sum_{i=1}^{n} y_i z_i m_i - M y_s z_s, \quad J_{XZ}(u) = \sum_{i=1}^{n} x_i z_i m_i - M x_s z_s.$$

Behaviour constraints of the BLP problem we define as the system of inequalities

$$\mu_1(u) \ge 0, \mu_2(u) \ge 0, \mu_3(u) \ge 0,$$

where

$$\mu_1(u) = \min\{\mu_{11}(u), \mu_{12}(u), \mu_{13}(u)\}, \tag{1}$$

$$\mu_2(u) = \min\{\mu_{21}(u), \mu_{22}(u), \mu_{23}(u)\}, \tag{2}$$

$$\mu_3(u) = \min\{\mu_{31}(u), \mu_{32}(u), \mu_{33}(u)\}. \tag{3}$$

Here $O_s = (x_s, y_s, z_s)$ is the center of mass of system Ω_A, where

$$x_s(u) = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_s(u) = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad z_s(u) = \frac{1}{M}\sum_{i=1}^{n} m_i z_i,$$

$\mathrm{M} = \sum_{i=0}^{n} \mathrm{m_i}$ is the mass of system Ω_A.

3. A Mathematical Model

A mathematical model of the MBLP problem can be presented in the form

$$extr F(p,u) \text{ s.t. } (u,p) \in W \tag{4}$$

$$W = \{(u,\ p) \in R^{\xi} : \Upsilon(u,\ p) \geq 0, \mu(u,\ p) \geq 0, \zeta \geq 0\}, \tag{5}$$

where $F(p,u) = (F_1(p,u), F_2(p,u), \ldots, F_k(p,u))$,
$\Upsilon(u,\ p)$ describes placement constraints, $\Upsilon(\mathrm{u,p}) = \min\{\Upsilon_1(\mathrm{u}), \Upsilon_2(\mathrm{u,p})\}$,
$\Upsilon_1(\mathrm{u})$ is responsible for non-overlapping constraints,
$\Upsilon_2(\mathrm{u,p})$ is responsible for containment constraints,
$\mu(u) = \min\{\mu_s(u), s \in U_t\}$ is responsible for behavior constraints, $U_t \in P(U)$, $P(U)$ is the power set of $U = \{1,2,3\}$, functions $\mu_1(u), \mu_2(u), \mu_3(u)$ are given by (1)-(3), $\zeta \geq 0$ is the system of additional constraints of metric characteristics of container Ω and placement parameters of objects. If $s = \varnothing$, i.e. behaviour constraints are not involved in (5), then our objective function $F(u)$ meets mechanical characteristics of system Ω_A.

Depending on the different combinations of objective functions $F_1(p,u), F_2(p,u), \ldots, F_k(p,u)$ different variants of mathematical model (4)-(5) can be generated. The most frequently occurring objective functions found in related publications are the following: 1) size of container Ω; 2) deviation of the center of mass of system Ω_A from a given point; 3) moments of inertia of system Ω_A (see, e.g., [3,6-11]).

Let us consider some of realisations of model (4) - (5):

- $F(p,u) = p$ s.t. $(p,u) \in W \subset R^{\xi}$,

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu(p,u) \geq 0, \zeta \geq 0\};$$

- $F(u) = d$, $(p,u) \in W \subset R^{\xi}$,

$$d = (x_s(u) - x_e)^2 + (y_s(u) - y_e)^2 + (z_s(u) - z_e)^2,$$

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu_2(p,u) \geq 0, \mu_3(p,u) \geq 0, \zeta \geq 0\};$$

- $F(p,u) = (F_1(p,u) = p, F_2(p,u) = d)$, $(p,u) \in W \subset R^{\xi}$,

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu_2(p,u) \geq 0, \mu_3(p,u) \geq 0, \zeta \geq 0\};$$

- $F(p,u) = (F_1(p,u) = J_X(p,u), F_2(p,u) = J_Y(p,u), F_3(p,u) = J_Z(p,u))$
$(p,u) \in W \subset R^{\xi}$,

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu_1(p,u) \geq 0, \mu_3(p,u) \geq 0, \zeta \geq 0\}.$$

4. Conclusion

In this paper we formulate the optimization layout problem of 3D objects into a container taking into account placement (non-overlapping, containment, distance) and behaviour (equilibrium, inertia and stability) constraints. We call the problem as Multicriteria Balance Layout Problem (MBLP). In order to describe placement constraints analytically we employ phi-function technique. A mathematical model of the problem in the form of multicriteria optimisation problem is proposed. We also consider some variants of the MBLP problem depending on the forms of the objective functions, shapes of objects and containers, types of distance and behaviour constraints.

References

1. Fasano, G., Pinte'r, J. (Eds.): Modeling and Optimization in Space Engineering. Series: Springer Opt. and Its Appl. 73, XII. 404 (2013).
2. Fasano, G., Pinte'r, J. (Eds.): Optimized Packings and Their Applications. Springer Opt. and its Appl. 105, 326 (2015).
3. Che, C., Wang Y., Teng, H.: Test problems for quasi-satellite packing: Cylinders packing with behaviour constraints and all the optimal solutions known. Opt. Online http://www.optimization-online.org/DB_HTML/2008/09/2093.html (2008).
4. Chernov, N., Stoyan, Y., Romanova T.: Mathematical model and efficient algorithms for object packing problem. Comput. Geom.: Theory and Appl. 43(5), 533-553 (2010).
5. Stoyan, Y., Pankratov, A., Romanova, T.: Quasi-phi-functions and optimal packing of ellipses. J. Glob. Optim. (2015). DOI: 10.1007/s10898-015-0331-2.
6. Lei, K.: Constrained Layout Optimization Based on Adaptive Particle Swarm Optimizer. Advances in Computation and Intelligence. Series: Springer-Verlag Berlin Heidelberg. Zhihua C., Zhenhua L., Zhuo K., Yong L. (Eds.). 1, 434-442. (2009).
7. Sun, Z., Teng, H.: Optimal layout design of a satellite module. Eng. Opt. 35(5), 513-530 (2003).
8. 6. Jingfa, L., Gang, L.: Basin filling algorithm for the circular packing problem with equilibrium behavioural constraints, SCIENCE CHINA Inf. Sci., 53(5), 885–895 (2010).
9. Oliveira, W.A., Moretti, A.C., Salles-Neto, L.L.: A heuristic for the nonidentical circle packing problem, Anais do CNMAC, 3, 626–632 (2010).
10. Xu, Y.-C., Xiao R.-B., Amos, M.: A novel algorithm for the layout optimization problem, Proc. 2007 IEEE Congr. Evolut. Comput. (CEC07), IEEE Press, 3938–3942 (2007).
11. Kovalenko, A., Romanova, T., Stetsyuk, P.: Balance layout problem for 3D-objects: mathematical model and solution methods. Cybern. Syst. Anal. 51(4), 556-565 (2015). DOI 10.1007/s10559-015-9746-5.