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PLENARY SESSION

Section I GENERAL ASPECTS OF METROLOGY. MEASUREMENT METHODS. UNITY AND ACCURACY OF MEASUREMENTS

Section II SENSORS, TRANSDUCERS AND DEVICES FOR MEASUREMENT OF PHYSICAL QUANTITIES

Section III **MEASUREMENT AND INFORMATION SYSTEMS AND TECHNOLOGIES**

Section IV **MEASUREMENTS IN THE INDUSTRY**

Section V **MEASUREMENTS IN THE ELECTRICAL POWER ENGINEERING**

Section VII **MEASUREMENTS IN THE ECOLOGY, BIOTECHNOLOGY, MEDICINE, AND SPORT**

Section VIII **METROLOGY PRACTICE**

Measuring instruments calibration: advanced realisation of key elements

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***Abstract*—The main elements of measuring instruments calibration are described. The basic models of calibration of indicating measuring instruments and material measures, and the corresponding procedures for measurement uncertainty evaluating based on the kurtosis method are presented. Methods for validating calibration procedures for various types of measuring instruments are proposed. A technique for assessing the compliance of a calibrated measuring instruments with the given metrological characteristics is considered.**

Keywords—calibration, methods and models, measurement uncertainty, kurtosis method, validation, conformity assessment.

I. INTRODUCTION

An essential condition for ensuring the uniformity of measurements is metrological traceability – a property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty [1]. Calibrations are carried out by laboratories that have received accreditation from bodies that have concluded MRAs with similar bodies in other countries [2]. At the same time, in accordance with the requirements of the ISO/IEC 17025:2017 standard, calibration laboratories must have a procedure for measurement uncertainty evaluation (MU) [3].

The key elements of measuring instrument (MI) calibration are:

1) justification of mathematical models of measurements for the basic methods of calibration;
2) MU evaluation at the implementation of these models;
3) making a decision on the compliance of the metrological characteristics of the calibrated measuring instrument with the established requirements;
4) validation (verification) of the calibration procedure.

The effectiveness of the implementation of those listed in points 3,4 calibration elements depends on the reliability of the expanded uncertainty evaluation of the calibrated MI (point 2) performed for the corresponding calibration model (point 1).

A reliable estimate of the expanded uncertainty cannot be obtained without taking into account the laws of distribution (PDF) of input quantities, which is usually done by the Monte Carlo method (MCM) [4]. For calibration problems, a reliable estimate of the expanded uncertainty can be obtained based on the kurtosis method (KM) developed by the authors [5]. Its use makes it possible to obtain estimates of the expanded uncertainty close to the estimates obtained by the MCM method. The KM is applicable if the PDF of input quantities are symmetrical, the measurement model is linear or linearizable, the number of multiple measurements of input quantities is at least 6. All these restrictions correspond to real calibration conditions.

The report considers the application of the kurtosis method for MU evaluation for the main calibration schemes used in metrological practice [6], as well as when deciding on the compliance of the metrological characteristics of a calibrated MI with the established requirements, validation (verification) of the calibration procedures.

II. EVALUATION OF THE EXPANDED MEASUREMENT UNCERTAINTY BY THE KURTOSIS METHOD

In the kurtosis method [5], the expanded uncertainty is found by the formula:

$$U = k(\eta) \cdot u_c(y)\,, \tag{1}$$

where $u_c(y)$ is combined standard uncertainty; $k(\eta)$ is coverage factor for confides level 0,9545, calculated by the formula:

$$k = \begin{cases} 0{,}12\eta^3 + 0{,}1\eta + 2, \text{ при } \eta < 0; \\ 2, \text{ при } \eta \geq 0. \end{cases} \tag{2}$$

Here η is kurtosis of measurand, which is calculated as:

$$\eta = \left(\sum_{i=1}^{N} \eta_i u_i^4(y) \right) \Big/ u_c^4(y)\,, \tag{3}$$

where $u_i(y)$ and η_i the uncertainty contribution of the *i*-th input quantity to the uncertainty of the measurand and its kurtosis, respectively. The values of kurtosis η_i for different distribution laws (PDFs) are given in Table. 1.

TABLE I. KURTOSIS VALUES FOR DIFFEREND PDFS OF INPUT QUANTITIES

PDF	Arsine	Uniform	Triangular	Normal	Student, ν=*n*-1
Kurtosis	-1,5	-1,2	-0,6	0	6/(ν-4)

II. CALIBRATING OF INDICATING MEASURING INSTRUMENTS (IMIS)

A. Direct measurement by an IMI to be calibrated of a quantity reproduced by a reference material measure (MM)

The calibration scheme for this case is shown in Fig. 1 [6].

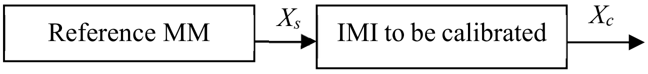


Fig. 1. – Scheme of direct measurement by the IMI to be calibrated of the value reproduced by the reference MM

When implementing this scheme, the systematic error of the IMI to be calibrated at the calibration point is determined by the formula:

$$\Delta = (X_c + \Delta_c) - (X_s + \Delta_s), \quad (4)$$

where X_c is; Δ_c is correction for the resolution of the IMI to be calibrated; X_s is the quantity reproduced by the reference MM; Δ_s is correction for the combined additional error of the reference MM associated with the drift of the value reproduced by it since the last calibration, deviations in operating conditions, the effect on the value reproduced by the MM of the IMI to be calibrated, etc.

With repeated measurements, the value X_c is determined as the arithmetic mean of the readings x_{ci}:

$$\overline{x}_c = \frac{1}{n}\sum_{i=1}^{n} x_{ci} . \quad (5)$$

An estimate of the reproducible value of a reference MM x_s is taken from its calibration certificate.

The corrections Δ_c and Δ_s are treated as centered random variables (with an estimated value of zero).

Therefore, the estimate of the systematic error (bias) of the IMI to be calibrated will be equal to:

$$\hat{\Delta} = \overline{x}_c - x_s . \quad (6)$$

The input quantities of equation (4) correspond to the following standard uncertainties:

- standard uncertainty associated with the observed variability of the readings of the IMI to be calibrated, estimated by type *A* when performing repeated measurements:

$$u_A(\overline{x}_c) = \sqrt{\frac{n-1}{n-3}} \cdot \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_{ci} - \overline{x}_c)^2} ; \quad (7)$$

- standard uncertainty of the correction associated with the resolution of the IMI to be calibrated, assuming a uniform PDF, equal to:

$$u_B(\Delta_c) = \frac{d}{2\sqrt{3}} , \quad (8)$$

where *d* is resolution of IMI to be calibrated;

- standard instrumental uncertainty of the reference MM, obtained from the values of the expanded uncertainty $U(x_s)$ and the coverage factor k_s, given in the calibration certificate:

$$u_B(x_s) = \frac{U(x_s)}{k_s} ; \quad (9)$$

- standard uncertainty of the correction Δ_s under the assumption of a uniform PDF of its distribution within the boundaries $\pm\theta_s$:

$$u_B(\Delta_s) = \frac{\theta_s}{\sqrt{3}} . \quad (10)$$

The combined standard MU of the systematic error of the IMI to be calibrated will be in this case equal to:

$$u_c(\Delta) = \sqrt{u_A^2(\overline{x}_c) + u_B^2(\Delta_c) + u_B^2(x_s) + u_B^2(\Delta_s)} . \quad (11)$$

The kurtosis of the measurand is calculated as:

$$\eta = \frac{\frac{6}{n-5}u_A^4(\overline{x}_c) + 0 \cdot u_B^4(x_s) + (-1,2)[u_B^4(\Delta_c) + u_B^4(\Delta_s)]}{u_c^4(\Delta)} . \quad (12)$$

The uncertainty budget for the case under consideration is given in Table II.

TABLE II. UNCERTAINTY BUDGET FOR DIRECT MEASUREMENTS BY AN IMI TO BE CALIBRATED OF A QANTITY REPRODUCED BY A REFERENCE MM

X_i	x_i	$u(x_i)$	η_i	c_i	$u_i(y)$
X_c	(5)	(7)	6/(*n*-5)	1	$u_A(\overline{x}_c)$
Δ_c	0	(8)	-1,2	1	$u_B(\Delta_c)$
X_s	x_s	(9)	0	-1	$-u_B(x_s)$
Δ_s	0	(10)	-1,2	-1	$-u_B(\Delta_s)$
Y	***y***	$u_c(y)$	η	***k***	***U***
Δ	(6)	(11)	(12)	(2)	(1)

B. Comparison of the reference IMI and IMI to be calibrated using a comparison device

The calibration scheme for this case is shown in Fig. 2 [6].

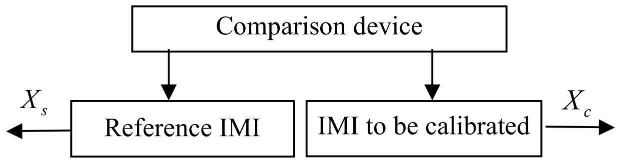


Fig. 2 – Scheme of direct comparison of the reference IMI and IMI to be calibrated

The comparison device (CD) acts as a signal source of measuring information for both IMIs.

The model equation for this case of calibration has the form:

$$\Delta = (X_c + \Delta_c) - (X_s + \Delta_s) + \Delta_{CD} , \quad (13)$$

where X_c, X_s are quantities measured, respectively, by reference IMI and IMI to be calibrated; Δ_c is correction for the resolution of the IMI to be calibrated; Δ_s is correction for the combined additional error of the reference IMI associated with the drift since the last calibration, deviations in operating conditions, etc.; Δ_{CD} is non-equivalence of the of input values for the reference IMI and IMI to be calibrated.

With repeated measurements, the estimates X_c and X_s are determined as arithmetic means for the number of measurements, respectively, n_c and n_s:

$$\overline{x}_c = \frac{1}{n_c}\sum_{i=1}^{n_c} x_{ci} \ , \quad (14)$$

$$\overline{x}_s = \frac{1}{n_s}\sum_{i=1}^{n_s} x_{si} \ . \quad (15)$$

The corrections Δ_c, Δ_s and Δ_{CD} are treated as centered random variables (with an estimated value of zero).

Therefore, the estimate of the systematic error (bias) of the IMI to be calibrated will be equal to:

$$\hat{\Delta} = \overline{x}_c - \overline{x}_s \ . \quad (16)$$

The input quantities of equation (13) correspond to the following standard uncertainties:

- standard uncertainty associated with the observed variability of the readings of the IMI to be calibrated, estimated by type A when performing repeated measurements:

$$u_A(\overline{x}_c) = \sqrt{\frac{n_c - 1}{n_c - 3}} \cdot \sqrt{\frac{1}{n_c(n_c-1)}\sum_{i=1}^{n_c}(x_{ci} - \overline{x}_c)^2} \ ; \quad (17)$$

- the standard uncertainty of the resolution of IMI to be calibrated, assuming a uniform PDF, equal to:

$$u_B(\Delta_c) = \frac{d}{2\sqrt{3}} \ , \quad (18)$$

where d is resolution of IMI to be calibrated;

- standard uncertainty associated with the observed variability of the readings of the reference IMI, estimated by type A when performing repeated measurements:

$$u_A(\overline{x}_s) = \sqrt{\frac{n_s - 1}{n_s - 3}} \cdot \sqrt{\frac{1}{n_s(n_s-1)}\sum_{i=1}^{n_s}(x_{si} - \overline{x}_s)^2} \ ; \quad (19)$$

- standard instrumental uncertainty of the reference IMI, obtained from the values of the expanded uncertainty $U(x_s)$ and the coverage factor k_s given in the calibration certificate:

$$u_B(x_s) = \frac{U(x_s)}{k_s} \ ; \quad (20)$$

- standard uncertainty of the correction Δ_s under the assumption of a uniform PDF within the boundaries $\pm\theta_s$:

$$u_B(\Delta_s) = \frac{\theta_s}{\sqrt{3}} \ . \quad (21)$$

- standard uncertainty Δ_{CD} of the correction under the assumption of a uniform PDF within the boundaries $\pm\theta_{CD}$:

$$u_B(\Delta_{CD}) = \frac{\theta_{CD}}{\sqrt{3}} \ . \quad (22)$$

The combined standard uncertainty of measuring the systematic error of the IMI to be calibrated will be equal to:

$$u_c(\Delta) = \sqrt{u_A^2(\overline{x}_c) + u_B^2(\Delta_c) + u_A^2(\overline{x}_s) + u_B^2(x_s) + u_B^2(\Delta_s) + u_B^2(\Delta_{yc})} \ . \quad (23)$$

The kurtosis of the measurand is calculated as:

$$\eta = \frac{1}{u_c^4(\Delta)}\left[\frac{6}{n_c - 5}u_A^4(\overline{x}_c) - 1{,}2 \cdot u_B^4(\Delta_c) + \frac{6}{n_s - 5}u_A^4(\overline{x}_s) + \right.$$
$$\left. + 0 \cdot u_B^4(x_s) - 1{,}2 \cdot [u_B^4(\Delta_s) + u_B^4(\Delta_{yc})]\right]. \quad (24)$$

The uncertainty budget for the considered case is given in Table III.

TABLE III. MU BUGET FOR NONSIMULTANEOUS COMPARISING OF THE READINGS OF REFERENCE IMI AND IMI TO BE CALIBRATED

X_i	x_i	$u(x_i)$	η_i	c_i	$u_i(y)$
X_c	(14)	(17)	6/(n-5)	1	$u_A(\overline{x}_c)$
Δ_c	0	(18)	-1,2	1	$u_B(\Delta_c)$
X_s	(15)	(19)	6/(n-5)	-1	$-u_A(\overline{x}_s)$
		(20)	0	-1	$-u_B(x_s)$
Δ_s	0	(21)	-1,2	-1	$-u_B(\Delta_s)$
Δ_{yc}	0	(22)	-1,2	1	$u_B(\Delta_{yc})$
Y	y	$u_c(y)$	η	k	U
Δ	(16)	(23)	(24)	(2)	(1)

With the simultaneous measurement $(n_c = n_s = n)$ by the reference IMI and IMI to be calibrated of the value reproduced by the comparison device, an observed correlation between their readings may occur. In this case, the estimate of the systematic error (bias) of the IMI to be calibrated has the form:

$$\hat{\Delta} = \overline{x_c - x_s} = \frac{1}{n}\sum_{i=1}^{n}(x_{ci} - x_{si}) \ , \quad (25)$$

and its combined standard uncertainty will be equal to:

$$u_c(\Delta) = \sqrt{u_A^2(\overline{x_c - x_s}) + u_B^2(x_s) + u_B^2(\Delta_c) + u_B^2(\Delta_s) + u_B^2(\Delta_{yc})} \ , \quad (26)$$

where $u_A(\overline{x_c - x_s})$ is standard uncertainty of the difference between the simultaneously observed readings of the reference IMI and IMI to be calibrated, determined by the formula:

$$u_A(\overline{x_c - x_s}) = \sqrt{\frac{n-1}{n-3}}\sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}[(x_{ci} - x_{si}) - (\overline{x_c - x_s})]^2} \quad (27)$$

The kurtosis of the measurand in this case must be calculated by the formula [7]:

$$\eta = \frac{1}{u_c^4(\Delta)}\left[\frac{6}{n_c - 5}u_A^4(\overline{x_c - x_s}) - 1{,}2 \cdot u_B^4(\Delta_c) + \right.$$
$$\left. + 0 \cdot u_B^4(x_s) - 1{,}2 \cdot [u_B^4(\Delta_s) + u_B^4(\Delta_{yc})]\right]. \quad (28)$$

The uncertainty budget for the considered case is given in Table IV.

TABLE IV. MU BUDGET FOR SIMULTANEOUS COMPARISING OF THE READINGS OF REFERANCE IMI AND IMI TO BE CALIBRATED

X_i	x_i	$u(x_i)$	η_i	c_i	$u_i(y)$
$X_c - X_s$	(25)	(27)	6/(*n*-5)	1	$u_A(\overline{x_c - x_s})$
Δ_c	0	(18)	-1,2	1	$u_B(\Delta_c)$
X_s	–	(20)	0	-1	$-u_B(x_s)$
Δ_s	0	(21)	-1,2	-1	$-u_B(\Delta_s)$
Δ_{yc}	0	(22)	-1,2	1	$u_B(\Delta_{yc})$
Y	y	$u_c(y)$	η	k	U
Δ	(25)	(26)	(28)	(2)	(1)

C. Validation (verification) of IMI calibration procedures

When validating (verifying) the IMI calibration procedures, the reference IMI with the values of bias $\hat{\Delta}_{cert}$ and expanded uncertainty U_{cert} specified in its calibration certificate is recalibrated in the laboratory [7]. To determine the suitability of the procedure, the following inequality should be used:

$$E_n = \frac{\left|\hat{\Delta}_{lab} - \hat{\Delta}_{cert}\right|}{\sqrt{U_{lab}^2 + U_{cert}^2}} \le 1\,, \tag{29}$$

where $\hat{\Delta}_{lab}$ and U_{lab} are, respectively, the bias of the reference IMI and its expanded uncertainty, obtained as a result of applying the calibration procedure to the reference IMI in the laboratory.

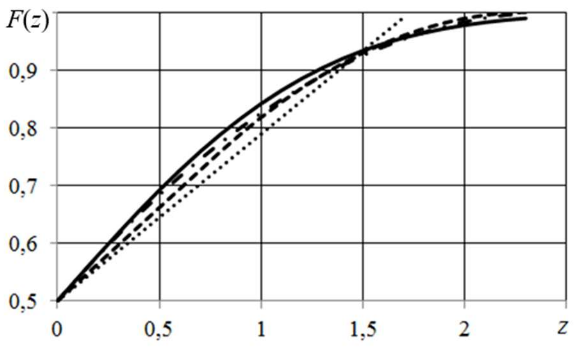
D. Conformity assessment of the calibrated IMI

Based on the results of the IMI calibration, the customer should assess the probability of compliance p_c of its metrological characteristics with the requirements of the documentation [8,9]. To do this, use the expression:

$$p_c = F\left(\frac{MPE - \left|\hat{\Delta}\right|}{u}\right), \tag{30}$$

where $F(z)$ is distribution function with variable $z = (MPE - |\hat{\Delta}|)/u$; $\hat{\Delta}$ and u are bias and standard instrumental uncertainty calibrated IMI, accordingly; MPE is maximum permissible error of IMI.

On Fig. 3 [9] shows the dependencies $F(z)$ for different distribution function.



Dependencies $F(z)$ for uniform (···), trapezoidal (---) with γ=0.5, triangular (–·–), and normal (—) distribution functions

III. CALIBRATION OF THE MATERIAL MEASURE

A. Direct measurement by reference IMI of the quantity reproduced by the MM to be calibrated

The calibration scheme for this case is shown in Fig. 4 [6].

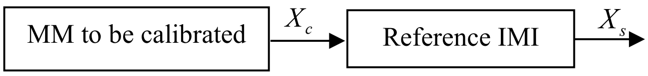


Fig. 4. Scheme of direct measurement by the reference IMI of the quantity reproduced by the MM to be calibrated

The model equation for this case has the form:

$$X_c = (X_s + \Delta_s)\,, \tag{31}$$

where X_c is the quantity reproduced by the MM to be calibrated; X_s is the quantity measured by the reference IMI; Δ_s is combined additional error of the reference IMI associated with the drift since the last calibration, deviations in operating conditions, etc.

With repeated measurements, the estimate of X_s defined as the arithmetic mean of the n_s readings of x_{si} :

$$\overline{x}_s = \frac{1}{n_s}\sum_{i=1}^{n_s} x_{si} \tag{32}$$

Usually Δ_s considered as a centered random variable (with an estimated value of zero).

Therefore, the estimate of the quantity reproduced by the MM x_c to be calibrated will be equal to:

$$x_c = \overline{x}_s\,. \tag{33}$$

The input quantities of equation (31) correspond to the following standard uncertainties:

- standard uncertainty associated with the observed variability of the readings of the reference IMI, estimated according to type A when performing repeated measurements:

$$u_A(\overline{x}_s) = \sqrt{\frac{n-1}{n-3}} \cdot \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_{si} - \overline{x}_s)^2}\ ; \tag{34}$$

- standard instrumental uncertainty of the reference IMI, obtained from the values of the expanded uncertainty $U(x_s)$ and the coverage factor k_s given in the calibration certificate:

$$u_B(x_s) = \frac{U(x_s)}{k_s}\,; \tag{35}$$

- standard uncertainty of the correction Δ_s under the assumption of a uniform law of its PDF within the boundaries $\pm\theta_s$:

$$u_B(\Delta_s) = \frac{\theta_s}{\sqrt{3}}\,. \tag{36}$$

The combined standard MU of the value of x_c , reproduced by the MM to be calibrated, will be equal to:

$$u_c(x_c) = \sqrt{u_A^2(\overline{x}_s) + u_B^2(x_s) + u_B^2(\Delta_s)}\,. \tag{37}$$

The kurtosis of the measurand is calculated as:

$$\eta = \frac{\frac{6}{n-5} u_A^4(\bar{x}_s) + 0 \cdot u_B^4(x_s) - 1{,}2 \cdot u_B^4(\Delta_s)}{u_c^4(\Delta)}. \qquad (38)$$

The uncertainty budget for the considered case is given in Table V.

TABLE V. UNCERTAINTY BUDGET OF DIRECT MEASUREMENT BY REFERANCE IMI OF THE QUNTITY REPRODUCED BY THE MM TO BE CALIBBRATED

X_i	x_i	$u(x_i)$	η_i	c_i	$u_i(y)$
X_s	(32)	(34)	6/(*n*-5)	1	$u_A(\bar{x}_s)$
		(35)	0	1	$u_B(x_s)$
Δ_s	0	(36)	-1,2	1	$u_B(\Delta_s)$
Y	**y**	**$u_c(y)$**	**η**	**k**	**U**
X_c	(33)	(37)	(38)	(2)	(1)

B. Comparison of the values reproduced by the reference MM and MM to be calibrated using a comparator

When comparing two MMs, an auxiliary MI is used – a comparator (a comparison device designed to compare measures of homogeneous quantities). The scheme for transmitting the measurement unit using the comparator is shown in Fig. 5 [6].

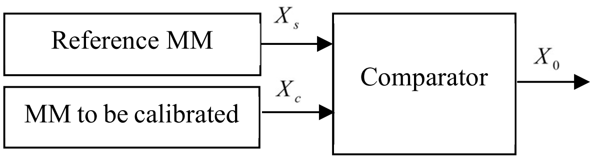


Fig. 5. Scheme for comparing the reference MM and MM to be calibrated using a comparator

The model equation for this case has the form:

$$X_c = (X_s + \Delta_s) + (X_0 + \Delta_0), \qquad (39)$$

where X_c is the quantity reproduced by the MM to be calibrated; X_s is the quantity reproduced by the reference MM; Δ_s is correction for the combined error of the reference MM associated with the drift of the value since the last calibration, deviations in operating conditions, etc.; X_0 is the quantity measured by the comparator; Δ_0 is the correction for the combined comparator error associated with the drift since the last calibration, deviations in operating conditions, etc.

With repeated measurements, the estimate of X_0 defined as the arithmetic mean of the n_s readings of x_{0i}:

$$x_0 = \bar{x}_0 = \frac{1}{n}\sum_{i=1}^{n} x_{0i}. \qquad (40)$$

An estimate of the reproducible value of a reference MM x_s is taken from its calibration certificate.

The corrections Δ_s and Δ_0 are treated as centered random variables (with an estimated value of zero).

Therefore, the estimate of the quantity reproduced by the MM to be calibrated x_c will be equal to:

$$x_c = \bar{x}_s + x_0. \qquad (41)$$

The input quantities of equation (39) correspond to the following standard uncertainties:

- standard instrumental uncertainty of the reference MM, obtained from the values of the expanded uncertainty $U(x_s)$ and the coverage factor k_s, given in the calibration certificate:

$$u_B(x_s) = \frac{U(x_s)}{k_s}; \qquad (42)$$

- standard uncertainty of the correction Δ_s under the assumption of a uniform PDF of its distribution within the boundaries $\pm\theta_s$:

$$u_B(\Delta_s) = \frac{\theta_s}{\sqrt{3}}. \qquad (43)$$

- standard uncertainty associated with the variability of the comparator readings, estimated as Type *A* when performing repeated measurements:

$$u_A(\bar{x}_0) = \sqrt{\frac{n-1}{n-3}} \cdot \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_{0i} - \bar{x}_0)^2}; \qquad (44)$$

- standard instrumental uncertainty of the comparator, obtained from the values of the expanded uncertainty $U(x_0)$ and the coverage factor k_0, given in the calibration certificate:

$$u_B(x_0) = \frac{U(x_0)}{k_0}; \qquad (45)$$

- standard uncertainty of the correction Δ_0 under the assumption of a uniform PDF of its distribution within the boundaries $\pm\theta_0$:

$$u_B(\Delta_0) = \frac{\theta_0}{\sqrt{3}}. \qquad (46)$$

The combined standard MU of the value of x_c, reproduced by the MM to be calibrated, will be equal to:

$$u_c(x_c) = \sqrt{u_B^2(x_s) + u_B^2(\Delta_s) + u_A^2(\bar{x}_0) + u_B^2(x_0) + u_B^2(\Delta_0)}. \qquad (47)$$

The kurtosis of the measurand is calculated as:

$$\eta = \frac{1}{u_c^4(x_c)}\Big[0 \cdot u_B^4(x_s) - 1{,}2 \cdot u_B^4(\Delta_s) + \frac{6}{n-5}u_A^4(\bar{x}_0) + $$
$$+ 0 \cdot u_B^4(x_0) - 1{,}2 \cdot u_B^4(\Delta_0)\Big]. \qquad (48)$$

The uncertainty budget for the considered case is given in Table VI.

TABLE VI. MU BUDGET WHEN COMPARISING REFERENCE MM AND MM TO BE CALIBRATED USING A COMPARATOR

X_i	x_i	$u(x_i)$	η_i	c_i	$u_i(y)$
X_s	x_s	(37)	0	1	$u_B(x_s)$
Δ_s	0	(38)	-1,2	1	$u_B(\Delta_s)$
X_0	(35)	(39)	6/(*n*-5)	1	$u_A(\bar{x}_0)$
		(40)	0	1	$u_B(x_0)$
Δ_0	0	(41)	-1,2	1	$u_B(\Delta_0)$
Y	**y**	**$u_c(y)$**	**η**	**k**	**U**
X_c	(36)	(42)	(43)	(2)	(1)

C. Validation (verification) of MM calibration procedures

When validating (verifying) the MM calibration procedures, the reference MM with the with value $\hat{X}_{cert}$ and expanded uncertainty U_{cert} specified in it calibration certificate is recalibrated in the laboratory [7]. To determine the suitability of the procedure, the following inequality should be used:

$$E_n = \frac{\left|\hat{X}_{lab} - \hat{X}_{cert}\right|}{\sqrt{U_{lab}^2 + U_{cert}^2}} \le 1\,, \qquad (43)$$

where $\hat{X}_{lab}$ and U_{lab} are, respectively, the value of the reference MM and its expanded uncertainty, obtained as a result of applying the calibration procedure to the reference MM in the laboratory.

D. Conformity assessment of the calibrated MM

Based on the results of the MM calibration, the customer should assess the probability of compliance p_c of its metrological characteristics with the requirements of the documentation [8,9]. To do this, use the expression:

$$p_c = F\left(\frac{\hat{X}_{н} - \hat{X}_c}{u}\right), \qquad (44)$$

where $F(z)$ distribution function with variable $z = (\hat{X}_{н} - \hat{X}_c)/u$; $\hat{X}_{н}$ is nominal value of calibrated MM; $\hat{X}_c$ and u is bias and instrumental standard uncertainty of calibrated MM, taken from calibration certificate.

On Fig. 3 [9] shows the dependencies $F(z)$ for different distribution functions.

CONCLUSIONS

1. The application of the kurtosis method for the uncertainty evaluation at calibration of indicating measuring instruments and material measures by the method of direct measurements and the method of comparison is considered. Uncertainty budgets are constructed to automate the process of measurement uncertainty evaluation.

2. When considering the comparison reference indicating measuring instruments and indicating measuring instruments of calibrated using a comparison device, the issue of taking into account the correlation between their readings, which makes it possible to reduce the combined standard uncertainty, is considered.

3. Examples of the implementation of the considered methods for measurement uncertainty evaluation are given in the article [10].

4. Expressions are given that allow validation (verification) of calibration procedures for both indicating measuring instruments and material measures.

5. Formulas are proposed for estimating the probability of compliance of indicating measuring instruments and material measures, which is a reliable indicator of the suitability of a calibrated measuring instrument. Graphs are given for normal, uniform, triangular and trapezoidal laws of distribution of the measurand.

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