Telecommunications and Radio Engineering, 67(9):833-841 (2008)

Multiple Plane Waves Expansion Method for Dispersive Media

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ABSTRACT: This paper presents the multiple plane waves expansion method. The method is based on plane waves expansion technique but it oriented primarily to the computation of band structures of photonic crystals containing dispersive materials. The verification of the method is also given in the work by comparison of results obtained for the same photonic crystal by multiple plane waves expansion method and by the finite difference time domain method with recursive convolution. Results obtained in the work show high accuracy of the method as well as its high performance and universality.

INTRODUCTION

Photonic crystals (PhCs) today play one of the most crucial roles in optoelectronics and telecommunications due to their unique possibility to prohibit light propagation inside some frequency range owing to existence of so-called photonic band gap. First investigations describing unique possibilities provided by PhCs are described in works of Yablonovitch [1] and John [2] in 1987. Due to optical properties as well as structure peculiarities such as strict periodicity, the effect of strong light localization in the defect region of the structure appears. Multiple devices utilizing PhCs are already available, such as photonic crystal fibers [3], laser applications [4], highly-efficient waveguides [5], waveguide bends [6] passive optical elements such add-drop filters [7] WDM multiplexers [8], and couplers [9]. The theoretical and experimental investigations demonstrate that the range of possible applications of PhCs covers almost whole telecommunications and optical information processing area. Many of devices provided by the PhCs in these areas such as splitters, combiners, DWDM multiplexers and add-drop filters are already investigated numerically and even created in laboratories.

One of the most important characteristics used for the design and investigation of PhCs-based devices is the band structure. Conventional

ISSN 0040-2508

where $\vec{G}$ is the reciprocal lattice vector, $h_{k,n}(\vec{G})$ is the field distribution function in representation of wave vectors, $\chi(\vec{G})$ is the Fourier expansion coefficient of the reversed permittivity function. Substituting (3) to (1), we obtain the system of linear equations [10]:

$$\sum_{\vec{G}} \chi\left(\vec{G}-\vec{G'}\right)\left(\vec{k}+\vec{G}\right)\left(\vec{k}+\vec{G'}\right)h_{k,n}\left(\vec{G'}\right)=\frac{\omega_{k,n}^2}{c^2}h_{k,n}\left(\vec{G}\right). \quad (4)$$

After that, we solve the eigen-problem for the operator represented by matrix, which elements are taken in following form:

$$k_{\vec{G},\vec{G'}}=\chi\left(\vec{G}-\vec{G'}\right)\left(\vec{k}+\vec{G}\right)\left(\vec{k}+\vec{G'}\right). \quad (5)$$

As a result, we obtain the eigen-values $\frac{\omega_{k,n}^2}{c^2}$ of the equation (4) which give us the eigen-frequencies of the PhC at given value of the wave vector. Such computations are carried out for each of wave vectors on so-called k-path inside the Brilluoin zone.

As a result of the computation we have the band structure – the number of eigen-frequencies corresponding to different values of wave vector.

FDTD -RC METHOD

As was mentioned above, the FDTD method is the one that allows treating for chromatic dispersion due to the possibility to find the dynamic input signal response of the investigated system. The dispersion is introduced via the polarization relaxation functions which are different for each material.

The method consists in digitization of Maxwell's equations.

$$\begin{aligned}\nabla\times\vec{H}(\vec{r},t)&=\frac{\partial\vec{D}(\vec{r},t)}{\partial t}+\sigma(\vec{r})\vec{E}(\vec{r},t)\,,\\ \nabla\times\vec{E}(\vec{r},t)&=-\frac{\partial\vec{B}(\vec{r},t)}{\partial t}+\sigma^*(\vec{r})\vec{H}(\vec{r},t)\,.\end{aligned} \quad (6)$$

where $\sigma(\vec{r})$ is the electric conductivity, $\sigma^*(\vec{r})$ is magnetic conductivity. Considering only dispersive permittivity, material equations can be taken in following form:

$$
\begin{aligned}
H_x(x+a, y+b) &= H_x(x,y)\exp\left\{i\left(k_x a + k_y b\right)\right\},\\
H_y(x+a, y+b) &= H_y(x,y)\exp\left\{i\left(k_x a + k_y b\right)\right\},\\
E_x(x+a, y+b) &= H_z(x,y)\exp\left\{i\left(k_x a + k_y b\right)\right\},
\end{aligned}
\tag{9}
$$

where a, b determine the size of the 2D computation domain.

During the computation process the field is measured in some point. After the computation measured, the field is being Fourier-transformed. The analysis of the response spectrum gives us the eigen-frequencies at the certain value of the wave vector.

The method was implemented on Celeron 1.7 GHz and the computation with 5% accuracy has taken about 3 minutes.

PWE METHOD FOR THE DISPERSIVE MEDIA

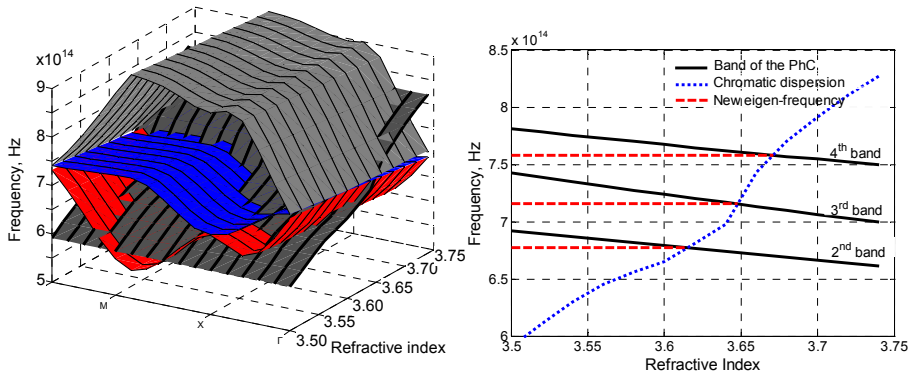
It is well-known fact that one of the disadvantages of the PWE method is the impossibility to treat the dispersive materials. However, the frequency range covered by the band structure is extremely large so the chromatic dispersion cannot be neglected.

The dispersion cannot be taken into account for one reason: the method searches for eigen-frequencies of the structure at given wave vector $\vec{k}$. In order to be able to find eigen-values of the matrix, the information about the permittivity distribution must be uniquely determined and this cannot be done if the material is dispersive.

In our work we propose the method that consists in multiple solution for eigen-frequencies of the PhCs. At each solution, the refractive indices of its materials are changed in accordance with dispersion low so at different computation steps different frequencies correspond to actual refractive index with a glance of chromatic dispersion.

After that, we consider the chromatic dispersion curve which is a referenced experimental data presented in form of tabulated function. Then we search for cross-points of such curve and bands of the PhCs with certain wave vector computed at different refractive indices. The cross-point of the band and the dispersion curve gives a new eigen-frequency of the PhCs with a glance of chromatic dispersion (see Fig. 1).

In our work we computed the band structure for the PhCs represented by the infinite square lattice of rods made of GaAs placed in air. The PhCs has following parameters: the distance between elements centers $a = 1.4\mu m$, radii of rods $r = 0.28\ \mu m$.



a) b)

FIGURE 1. Intersection of the band structure and the material dispersion curve (bands from 2nd to 4th) (a) and formation of new eigen-frequencies at the cross-section between band structures and chromatic dispersion curve (b).

To prove the validity of proposed method, we compared results with ones obtained by FDTD-RC method. Unfortunately, the FDTD-RC does not allow us to consider dispersion in form of tabulated function so the experimental data should be approximated by some analytic function. In our work we investigate the PhCs made of GaAs which dispersion is taken from [13]. The dispersion was approximated by the single-resonance Lorentz curve given in [14] and have taken the following form:

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\Delta\omega_0^2}{\omega_0^2 + 2j\omega\delta - \omega^2}, \tag{10}$$

here $\varepsilon_\infty = 3.3$, $\Delta = 7.3$, $\omega_0 = 12 \cdot 10^{14}$, $\delta = 2.5 \cdot 10^{14}$. Having this values of parameters, the Lorentz function approximates the experimental data with good accuracy. The data and approximation function are depicted in Fig. 2

Once the approximation of the experimental data is complete, the FDTD-RC method can be implemented. In Fig. 3 it is shown the comparison between the band structures obtained by PWE method with band structure obtained by FDTD-RC method (Fig. 3(a)) and multiple PWE method (Fig. 3(b)). As is seen there is insignificant difference between results obtained by these methods. However, it is 10 times lower than the shift of the band structure provided by consideration of the chromatic dispersion.

The method was implemented on Celeron 1.7 GHz and the computation with 5% accuracy has taken about 2 sec.

CONCLUSION

In the work there was developed the method of the computation of the band structure of the PhCs made of dispersive materials. The method was implemented for 2D PhCs and results was compared with traditionally used FDTD-RC method. Comparison have shown that the method gives the same accuracy as FDTD-RC, however it requires essentially lower computation time.

Both FDTD-RC and multiple PWE methods work on macroscopic (phenomenological) level. However, in contrast to FDTD-RC, the multiple PWE method can be implemented for "pure" experimental data without any approximation, so it does not depend on the type of polarization relaxation [14].

One more advantage of the method is the universality. Once implemented, it can be used for any dimensionality of the PhCs and for any representation of the band structure.

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