NUMERICAL SIMULATION OF UWB ELECTROMAGNETIC PULSES PROPAGATION IN DISPERSIVE ELECTRODYNAMIC LINES

A.V. Gritsunov, N.V. Ostapyuk
Kharkiv National University of Radio Electronics
Address: 14, Lenin Ave., Kharkiv, 61166, Ukraine
Tel. +38 057 7021362, E-mail gritsunov@gmail.com

The technique of wideband and ultrawideband (UWB) electromagnetic pulses is relatively new and very fast developed branch of the electrodynamics and the microwave electronics. These pulses may be useful for new radars, digital telecommunications with high data transfer rates, *etc*. The principal direction of the radio and electronics engineering development is replacement of frequency-localized (quasi-harmonic) signals by time-localized UWB pulses. As a result, the general principles of an electronic radio equipment construction that have been formulated by Hertz, Popov, Marconi *et al.* are obsolescent to a certain extent and are renewed. Naturally, new engineering solutions can be based only on advanced mathematical methods. These are enough topical today for the electrodynamics as well as for the radio engineering.

However, the modern computer simulations and design of the UWB circuits are founded mainly on the FDTD (FETD) or the FDFD (FEFD) methods. The reason is that the most of known analytical approaches to solving of the field equations was developed in the middle of the previous century exclusively for narrowband (quasi-harmonic) electromagnetic fields. Taking into account excessive „computer laboriousness" of the field equations direct solving, new theoretical approaches to the UWB pulse fields evaluation are wanted.

There is a universal approach to active UWB pulse devices theoretical investigations, which may be called as spectral [1]. This idea consists in simultaneous account of all temporal harmonics of the electromagnetic fields and exiting currents over a frequency continuum taking into consideration their non-linear interaction, and obtaining frequency dependencies of the device output parameters. The spectral method is realized in the time domain using a non-stationary (transient) simulation technique. The main stage of one is evaluation of arbitrary non-harmonic fields in a dispersive electrodynamic line with an electron beam (this might be a delay structure for TWTs and CFDs or a smoothbore waveguide for fast wave devices).

A method for computer simulations of such fields excited in a regular (i.e., longitudinally uniform or longitudinally periodic with a period *D*) electrodynamic line by a short external current or by a bunched electron beam is considered in the paper. Taking into account that the electron beam occupies a small part of the total simulated volume, the variable separation method may be effective for this procedure. The offered algorithm founds on solving so-called generalized wave equation for a regular dispersive and dissipative electrodynamic line with arbitrary geometry and dispersion characteristic [2]. This equation can take into account the delayed potentials in the transverse directions of the line as well as in the longitudinal one, if several transverse eigenmodes of the line are considered simultaneously.

So-called generic potential $\mathfrak{A}(t,x,y,z)=\Phi/c$ or $\vec{A}$, where $\Phi(t,x,y,z)$ and $\vec{A}(t,x,y,z)$ are the scalar and the vector potentials in the Lorentz gauge; c is the light velocity, is found below as a superposition of so-called regular modes of the line $\mathfrak{A}_{gq}(x,y,z,\beta)$, one per each wave type (transverse eigenmode, passband) q = 0, 1, 2, …, where $-\pi/D\le\beta\le\pi/D$ is the longitudinal wavenumber (propagation factor). The regular mode is defined as a complex envelope of the line eigenmode $\mathfrak{A}_{eq}(x,y,z,\beta)$ in the longitudinal direction *z*, i.e.:

$$\mathfrak{A}_{eq}(x,y,z,\beta)=\mathfrak{A}_{gq}(x,y,z,\beta)\exp(-\mathrm{i}\beta z)\,; \qquad \mathfrak{A}_{gq}(x,y,z,\beta)=\mathfrak{A}_{eq}(x,y,z,\beta)\exp(\mathrm{i}\beta z)\,.$$

$\mathfrak{A}_{gq}$ are periodic functions of *z* with the period of *D*. Because of the regular eigenmodes β dependency, the generalized potential can be evaluated only using an expansion of $\mathfrak{A}_{gq}$ in the Taylor series in β, i.e.:

$$\mathfrak{A}(t,x,y,z)=\sum_q\left(\mathfrak{A}_{gq0}u_{gq}+\mathrm{i}\frac{\partial\mathfrak{A}_{gq0}}{\partial\beta}\frac{\partial u_{gq}}{\partial z}-\frac{1}{2}\frac{\partial^2\mathfrak{A}_{gq0}}{\partial\beta^2}\frac{\partial^2 u_{gq}}{\partial z^2}-\frac{\mathrm{i}}{6}\frac{\partial^3\mathfrak{A}_{gq0}}{\partial\beta^3}\frac{\partial^3 u_{gq}}{\partial z^3}+\frac{1}{24}\frac{\partial^4\mathfrak{A}_{gq0}}{\partial\beta^4}\frac{\partial^4 u_{gq}}{\partial z^4}+\ldots\right)$$

where $u_{gq}(t,z)$ is a temporal and longitudinal dependence of the regular mode instantaneous value. The subscript 0 implies that an item is taken for $\beta=0$. The function u_{gq} is a solution of the generalized wave equation, which may be derived by decomposition of so-called partial modes of the line in the Taylor series (see [2]):

$$\begin{aligned}&\frac{\partial^2 u_{gq}}{\partial t^2}+2\frac{\partial}{\partial t}\left(\delta_{eq0}u_{gq}-\frac{1}{2}\frac{d^2\delta_{eq0}}{d\beta^2}\frac{\partial^2 u_{gq}}{\partial z^2}+\frac{1}{24}\frac{d^4\delta_{eq0}}{d\beta^4}\frac{\partial^4 u_{gq}}{\partial z^4}-\ldots\right)+(\omega_{eq}^2)_0 u_{gq}-\frac{1}{2}\frac{d^2(\omega_{eq}^2)_0}{d\beta^2}\frac{\partial^2 u_{gq}}{\partial z^2}+\\&+\frac{1}{24}\frac{d^4(\omega_{eq}^2)_0}{d\beta^4}\frac{\partial^4 u_{gq}}{\partial z^4}-\ldots=\frac{1}{2D}\int\limits_{z-D/2}^{z+D/2}d\zeta\int\limits_{S_\perp}dxdy\left[\frac{\mathfrak{A}^*_{gq0}(x,y,\zeta)}{\tilde{W}_{gq0}}\mathfrak{J}(t,x,y,\zeta)-\mathrm{i}\frac{\partial}{\partial\beta}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial\mathfrak{J}}{\partial z}-\right.\\&\left.-\frac{1}{2}\frac{\partial^2}{\partial\beta^2}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial^2\mathfrak{J}}{\partial z^2}+\frac{\mathrm{i}}{6}\frac{\partial^3}{\partial\beta^3}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial^3\mathfrak{J}}{\partial z^3}+\frac{1}{24}\frac{\partial^4}{\partial\beta^4}\left(\frac{\mathfrak{A}^*_{gq}}{\tilde{W}_{gq}}\right)_0\frac{\partial^4\mathfrak{J}}{\partial z^4}-\ldots\right],\end{aligned}\qquad(1)$$

where $\mathfrak{J}(t,x,y,z)=c\rho$ or $\vec{j}$ is so-called generic current density; $\rho(t,x,y,z)$ and $\vec{j}(t,x,y,z)$ are the charge density and the current density vector respectively; $\omega_{eq}(\beta)$ and $\delta_{eq}(\beta)$ are the eigenfrequency and the damping factor of the line q-th eigenmode respectively; $\tilde{W}_{gq}(\beta)$ is so-called linear pseudoenergy (norm function) of the q-th regular mode:

$$\tilde{W}_{gq}(\beta)=\frac{\varepsilon_0}{2D}\int\limits_D dz\int\limits_{S_\perp}dxdy\,\mathfrak{A}_{gq}(x,y,z,\beta)\mathfrak{A}^*_{gq}(x,y,z,\beta).$$

For the smoothbore line, D is a value of the averaging interval defining the upper bound of considered β.

The classic wave equation, the Klein-Gorgon equation, as well as the Telegraphist's equation are subsets of (1). In spite of a formal resemblance to the 1D wave equation, this is a fully 3D expression. E.g., a gradual formation of the line field transverse structure during a transient process can be simulated with one. The right-hand side of (1) is simplified, if the energy normalization is used [$\tilde{W}_{gq}(\beta)\equiv 1$ J·s^2/m].

Let us consider possible technique of solving equation (1) for radio and video pulses propagation in a lossless delay line. Only one wave type is examined here, so q subscript is omitted below. Assuming $\delta_e\equiv 0$ and approximating the line dispersion characteristic within the operating frequency band by a second-order polynomial $\omega_e(\beta)\approx b_0+b_1\beta+b_2\beta^2$, a finite-difference equation is constructed. This uses a second-order pattern along the time coordinate and second-order and fourth-order ones in the longitudinal direction:

$$\Lambda^2 u_{gk}=(u_{gk-1}-2u_{gk}+u_{gk+1})/\Delta z^2;\qquad \Lambda^4 u_{gk}=(u_{gk-2}-4u_{gk-1}+6u_{gk}-4u_{gk+1}+u_{gk+2})/\Delta z^4$$

(the orders of the polynomial and of the equation in z direction may be increased, if needs). The equation is solved with an explicit three-layer finite-difference scheme. The right-hand part of (1) is substituted with some source function $s(t,z)$, which emulates entering the pulse into the line by an input signal.

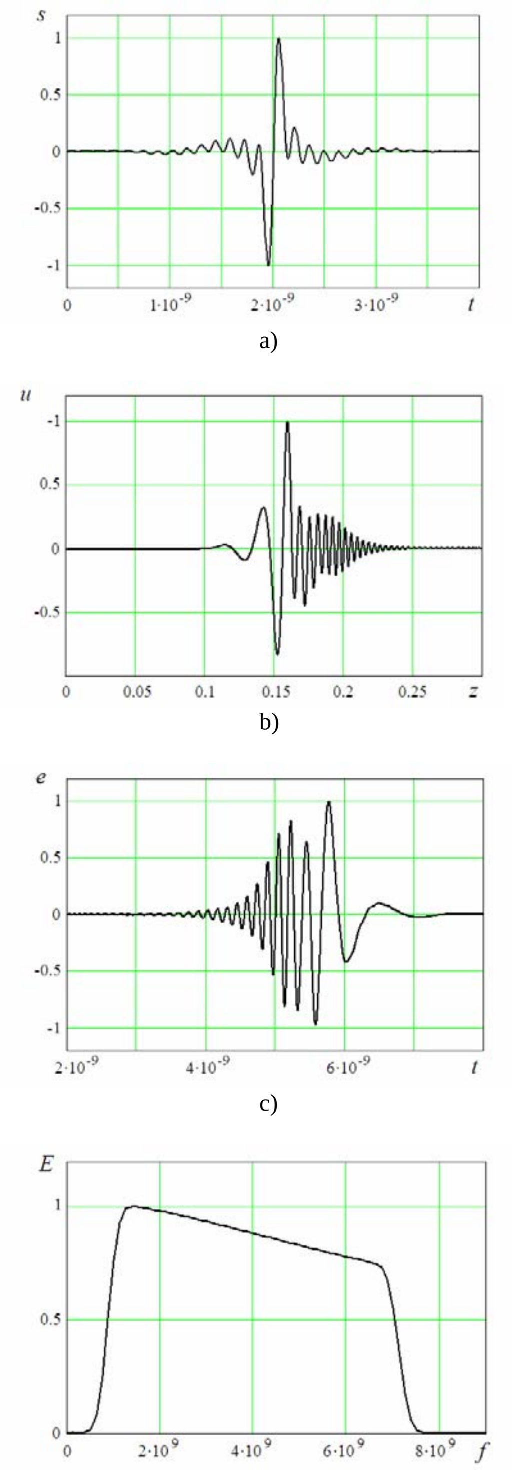
The boundary conditions at the line endpoints can be simulated like the discrete approximation [2]. Two methods of the line matching are usable for UWB signals specified in the time domain: (i) adding frequency-independent impedances of input and output couplings to the line endpoints; and (ii) expansion of the line in both directions with perfect matching loads.

As an example of the described simulation technique using, Figs. 1 and 2 show advance of short radio and video pulses respectively in a dispersive delay line with b_1 = 2.5·10^7 m/s. Both pulses have rectangular input spectrum in the frequency ranges of 1…7 GHz (the radio pulse) and of 0…7 GHz (the video pulse). In both figures, a) is the input signal $s(t,0)$ in the time domain; b) is the $u_g(t,z)$ function at t = 8 ns; c) is a certain "electric field function" $e_g(t,z)=-\partial u_g(t,z)/\partial t$ at z = 10 cm in the time domain; d) is the same function in the frequency domain. All curves are normalized to its peak values.

The line dispersion parameters for Fig. 1 is b_0 = 0; b_2 = 9·10^3 m^2/(rad·s). The same features for Fig. 2 is b_0 = 10·10^9 rad/s; b_2 = 0. Thus, the radio pulse moves in the line with strongly nonlinear dispersion curve, but without cutoff. On the contrary, the video pulse propagates in the line with almost linear dispersion characteristic at high β, but having low-frequency cutoff (like a smoothbore waveguide).

As it can be seen in Fig. 1, because the high-frequency components of the radio pulse move more rapidly than the low-frequency ones, the first overtake the latter after some time. In Fig. 2, the delay line does not transmit the low-frequency components of the video pulse at all, so this turns into a radio pulse gradually passing some distance. It is obviously that in both cases the dispersion, causing "spreading" the pulse in the time and the space, lowers the upper-frequency part of the pulse spectrum.

The offered technique is examined while investigations of passive UWB circuits yet. However, this is intended in reality for the spectral and other numerical simulations of active UWB devices (e.g., TWTs).



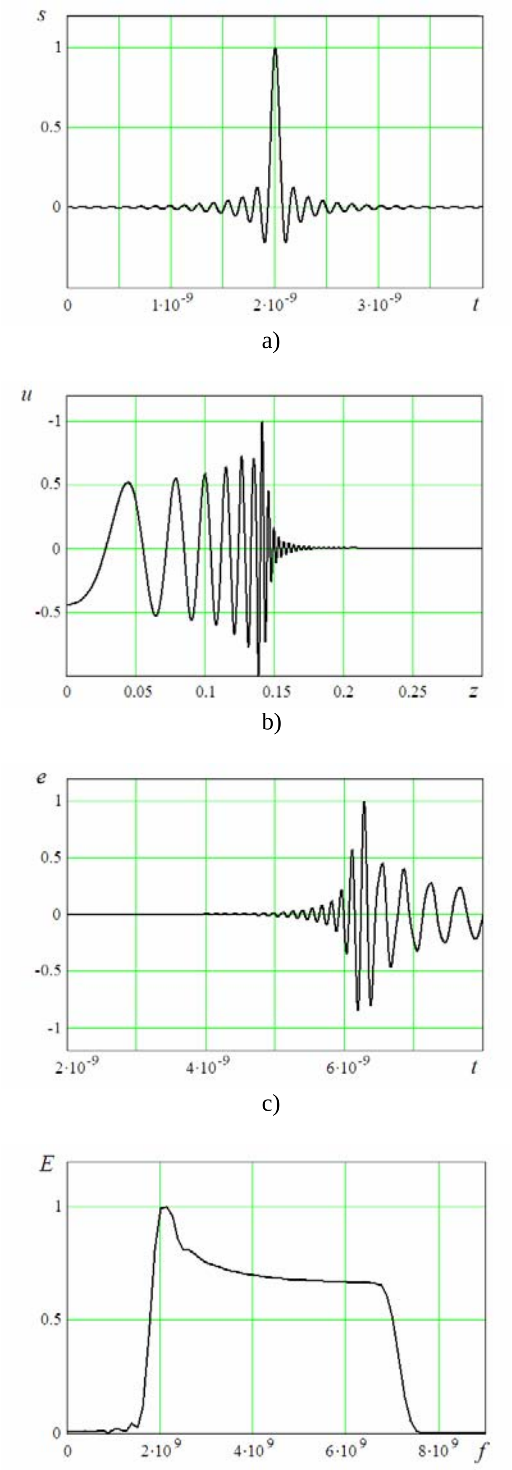
a)

b)

c)

d)

Figure 1. The UWB radio pulse propagation



a)

b)

c)

d)

Figure 2. The UWB video pulse propagation

References

1. A.V. Gritsunov, “On a Spectral Approach to Simulation of Microwave Devices”, J. Commun. Technol. and Electronics, Vol. 49, No. 7, pp. 829-832, 2004.
2. A.V. Gritsunov, “Methods of Evaluation of Unsteady Non-Harmonic Fields in Electrodynamic Lines,” J. Commun. Technol. and Electronics, Vol. 52, 2007 (to be published).