ДОДАТОК А

Програмна реалізація для $n = 4$

Clasik

$a := 1 \quad b := 1 \quad c := 0.5$

$f(x,y) := -2$

$N := 4$

$k := 0..N$

$c_{k,1} := 0.15$

$l := 0..N$

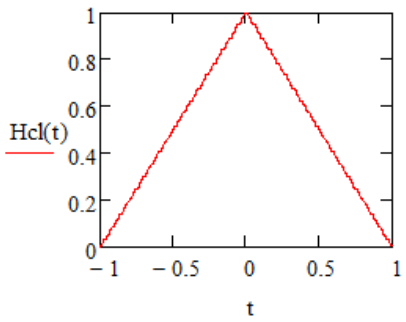
$X_k := \frac{k \cdot a}{N} \qquad X = \begin{pmatrix} 0 \\ 0.25 \\ 0.5 \\ 0.75 \\ 1 \end{pmatrix}$

$Y_l := \frac{1 \cdot b}{N} \qquad Y = \begin{pmatrix} 0 \\ 0.25 \\ 0.5 \\ 0.75 \\ 1 \end{pmatrix}$

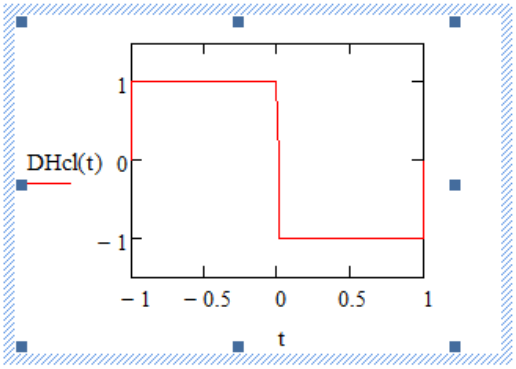
$p := 1..N - 1$

$q := 1..N - 1$

$$Hcl(t) := \begin{cases} 0 & \text{if } t \le -1 \\ (1+t) & \text{if } -1 < t \le 0 \\ (1-t) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$



$$DHcl(t) := \begin{cases} 0 & \text{if } t \le -1 \\ 1 & \text{if } -1 < t < 0 \\ 0 & \text{if } t = 0 \\ (-1) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$



$$Ucl4(x,y,N,c) := \left[\sum_{k=1}^{N-1} \left(c_{k,1} \cdot Hcl(N\cdot x - k)\, Hcl(N\cdot y - 1)\right)\right] + \sum_{l=1}^{N-1} \left(c_{1,l} \cdot Hcl(N\cdot x - 1)\, Hcl(N\cdot y - l)\right) - \sum_{k=1}^{\frac{N}{2}-1} \sum_{l=1}^{\frac{N}{2}-1} \left(c_{k,l}\, Hcl(N\cdot x - k) \cdot Hcl(N\cdot y - l)\right)$$

$$Ucl4(0.1, 0.1, N, c) = 0.024$$

$$DxUcl4(x,y,N,c) := \left[\left[\sum_{k=1}^{N-1} \left(c_{k,1} \cdot DHcl(N\cdot x - k)\, Hcl(N\cdot y - 1)\right)\right] N\right] + \left[\sum_{l=1}^{N-1} \left(c_{1,l} \cdot DHcl(N\cdot x - 1)\, Hcl(N\cdot y - l)\right)\right] N - \left[\sum_{k=1}^{\frac{N}{2}-1} \sum_{l=1}^{\frac{N}{2}-1} \left(c_{k,l}\, DHcl(N\cdot x - k) \cdot Hcl(N\cdot y - l)\right)\right] N$$

$$DxUcl4(0.1, 0.1, N, c) = 0.24$$

$$DyUcl4(x,y,N,c) := \left[\sum_{k=1}^{N-1} \left(c_{k,1} \cdot Hcl(N\cdot x - k)\, DHcl(N\cdot y - 1)\right)\right] N + \left[\sum_{l=1}^{N-1} \left(c_{1,l} \cdot Hcl(N\cdot x - 1)\, DHcl(N\cdot y - l)\right)\right] N - \left[\sum_{k=1}^{\frac{N}{2}-1} \sum_{l=1}^{\frac{N}{2}-1} \left(c_{k,l}\, Hcl(N\cdot x - k) \cdot DHcl(N\cdot y - l)\right)\right] N$$

$$DyUcl4(0.1, 0.1, N, c) = 0.24$$

$$HHcl := \begin{cases} \text{for } p \in 1..N-1 \\ \quad \text{for } i \in 1..N-1 \\ \quad\quad hh_{p,i} \leftarrow \int_0^1 Hcl(N\cdot x - p) \cdot Hcl(N\cdot x - i)\, dx \\ hh \end{cases}$$

$$HHcl = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.167 & 0.042 & 0 \\ 0 & 0.042 & 0.167 & 0.042 \\ 0 & 0 & 0.042 & 0.167 \end{pmatrix}$$

$$DHHcl := \left| \begin{array}{l} \text{for } p \in 1..N-1 \\ \quad \text{for } i \in 1..N-1 \\ \quad\quad hh_{p,i} \leftarrow \left(\int_0^1 DHcl(N\cdot x - p)\cdot DHcl(N\cdot x - i)\, dx \right) N^2 \\ hh \end{array} \right.$$

$$DHHcl = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 8 & -4 & 0 \\ 0 & -4 & 8 & -4 \\ 0 & 0 & -4 & 8 \end{pmatrix}$$

$$AAA := \left| \begin{array}{l} \text{for } p \in 1..N-1 \\ \quad \text{for } q \in 1..N-1 \\ \quad\quad \text{for } i \in 1..N-1 \\ \quad\quad\quad \text{for } j \in 1..N-1 \\ \quad\quad\quad\quad a_{(N-1)\cdot(p-1)+q,(N-1)(i-1)+j} \leftarrow HHcl_{p,i}\cdot DHHcl_{q,j} + DHHcl_{p,i}\cdot HHcl_{q,j} \\ a \end{array} \right.$$

$AAA =$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	2.667	-0.333	0	-0.333	-0.333	0	0
2	0	-0.333	2.667	-0.333	-0.333	-0.333	-0.333	0
3	0	0	-0.333	2.667	0	-0.333	-0.333	0
4	0	-0.333	-0.333	0	2.667	-0.333	0	-0.333
5	0	-0.333	-0.333	-0.333	-0.333	2.667	-0.333	-0.333
6	0	0	-0.333	-0.333	0	-0.333	2.667	0
7	0	0	0	0	-0.333	-0.333	0	2.667
8	0	0	0	0	-0.333	-0.333	-0.333	-0.333
9	0	0	0	0	0	-0.333	-0.333	...

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.667 & -0.333 & 0 & -0.333 & 0 \\ 0 & -0.333 & 2.667 & -0.333 & 0 & 0 \\ 0 & 0 & -0.333 & 2.667 & 0 & 0 \\ 0 & -0.333 & 0 & 0 & 2.667 & -0.333 \\ 0 & 0 & 0 & 0 & -0.333 & 2.667 \end{pmatrix}$$

$$Acl := \left| \begin{array}{l} \text{for } k \in 0..4 \\ \quad \text{for } l \in 0..4 \\ \quad\quad acl_{k,l} \leftarrow A_{k+1,l+1} \\ acl \end{array} \right.$$

$$Acl = \begin{pmatrix} 2.667 & -0.333 & 0 & -0.333 & 0 \\ -0.333 & 2.667 & -0.333 & 0 & 0 \\ 0 & -0.333 & 2.667 & 0 & 0 \\ -0.333 & 0 & 0 & 2.667 & -0.333 \\ 0 & 0 & 0 & -0.333 & 2.667 \end{pmatrix}$$

$$F_1 := \int_0^1 \left(\int_0^{0.5} f(x,y) \cdot Hcl(N \cdot x - 1) \cdot Hcl(N \cdot y - 1)\, dy \right) dx$$

$$F_2 := \int_0^1 \left(\int_0^{0.5} f(x,y) \cdot Hcl(N \cdot x - 2) \cdot Hcl(N \cdot y - 1)\, dy \right) dx$$

$$F_3 := \int_0^1 \left(\int_0^{0.5} f(x,y) \cdot Hcl(N \cdot x - 3) \cdot Hcl(N \cdot y - 1)\, dy \right) dx$$

$$F_4 := \int_0^{0.5} \left(\int_0^1 f(x,y) \cdot Hcl(N \cdot x - 1) \cdot Hcl(N \cdot y - 2)\, dy \right) dx$$

$$F_5 := \int_0^{0.5} \left(\int_0^1 f(x,y) \cdot Hcl(N \cdot x - 1) \cdot Hcl(N \cdot y - 3)\, dy \right) dx$$

$$F = \begin{pmatrix} 0 \\ -0.125 \\ -0.125 \\ -0.125 \\ -0.125 \\ -0.125 \end{pmatrix}$$

$$Fcl := \left| \begin{array}{l} \text{for } p \in 0..4 \\ \quad fcl_p \leftarrow F_{p+1} \\ fcl \end{array} \right.$$

$$Fcl = \begin{pmatrix} -0.125 \\ -0.125 \\ -0.125 \\ -0.125 \\ -0.125 \end{pmatrix}$$

$$Acl^{-1} \cdot Fcl = \begin{pmatrix} -0.062 \\ -0.061 \\ -0.055 \\ -0.061 \\ -0.055 \end{pmatrix}$$

$c4cl_{0,0} := -0.062$ $c4cl_{0,1} := -0.061$ $c4cl_{0,2} := -0.055$

$c4cl_{1,0} := -0.061$ $c4cl_{2,0} := -0.055$

$$c4cl = \begin{pmatrix} -0.062 & -0.061 & -0.055 \\ -0.061 & 0 & 0 \\ -0.055 & 0 & 0 \end{pmatrix}$$

OFEM4

$f(x,y) := -2$ $a := 1$ $b := 1$ $c := 0.5$

$N := 4$ $\lambda := 1.284$ $\gamma := 0.532$ $\mu := 0.409$

$k := 0..N$

$l := 0..N$

$c_{0,0} := 0$ $c_{0,1} := 1.284$ $c_{0,2} := 0.532$ $c_{0,3} := 0.409$

$c_{1,0} := 0$ $c_{1,1} := -0.062$ $c_{1,2} := -0.061$ $c_{1,3} := -0.055$

$c_{2,0} := 0$ $c_{2,1} := -0.061$ $c_{2,2} := 0$ $c_{2,3} := 0$

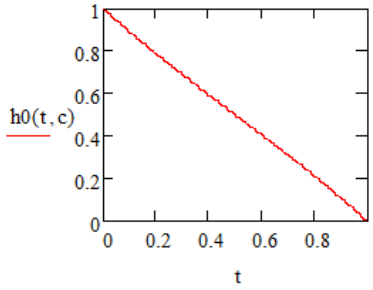
$c_{3,0} := 0$ $c_{3,1} := -0.055$ $c_{3,2} := 0$ $c_{3,3} := 0$

$$h0(t) := \frac{1 - 2\cdot\gamma}{2} \cdot \frac{\cosh[\mu\cdot(t - 0.5)]}{\cosh(\mu\cdot 0.5)} - \frac{\sinh[\lambda\cdot(t - 0.5)]}{2\cdot\sinh(\lambda\cdot 0.5)} + \gamma$$

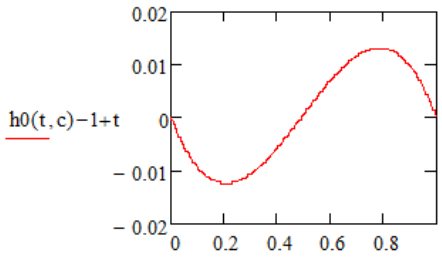
$$H(t,c) := \begin{cases} 0 & \text{if } t \le -1 \\ h0(-t,c) & \text{if } -1 < t \le 0 \\ h0(t,c) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$

$$h0(t,c) := \frac{1 - 2\cdot c_{0,2}}{2} \cdot \frac{\cosh\left[c_{0,3}\cdot(t - 0.5)\right]}{\cosh\left(c_{0,3}\cdot 0.5\right)} - \frac{\sinh\left[c_{0,1}\cdot(t - 0.5)\right]}{2\cdot\sinh\left(c_{0,1}\cdot 0.5\right)} + c_{0,2}$$

$$X_k := \frac{k \cdot a}{N} \qquad X = \begin{pmatrix} 0 \\ 0.25 \\ 0.5 \\ 0.75 \\ 1 \end{pmatrix}$$



$$Y_l := \frac{1 \cdot b}{N} \qquad Y = \begin{pmatrix} 0 \\ 0.25 \\ 0.5 \\ 0.75 \\ 1 \end{pmatrix}$$



$$Dh0(t,c) := \frac{1 - 2 \cdot c_{0,2}}{2} \cdot c_{0,3} \cdot \frac{\sinh\left[c_{0,3} \cdot (t - 0.5)\right]}{\cosh\left(c_{0,3} \cdot 0.5\right)} - \frac{c_{0,1} \cosh\left[c_{0,1} \cdot (t - 0.5)\right]}{2 \cdot \sinh\left(c_{0,1} \cdot 0.5\right)}$$

$$DH(t,c) := \begin{cases} 0 & \text{if } t \le -1 \\ -Dh0(-t,c) & \text{if } -1 < t < 0 \\ 0 & \text{if } t = 0 \\ Dh0(t,c) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$

$p := 1 .. N - 1$

$q := 1 .. N - 1$

$$U4(x,y,N,c) := \left[\sum_{k=1}^{N-1} \left(c_{k,1} \cdot H(N \cdot x - k, c)\, H(N \cdot y - 1, c)\right)\right] + \sum_{l=1}^{N-1} \left(c_{1,l} \cdot H(N \cdot x - 1, c)\, H(N \cdot y - 1, c)\right) - \sum_{k=1}^{1} \sum_{l=1}^{1} \left(c_{k,1} H(N \cdot x - k, c) \cdot H(N \cdot y - 1, c)\right)$$

$$DxU4(x,y,N,c) := \left[\sum_{k=1}^{N-1} \left(c_{k,1} \cdot DH(N \cdot x - k, c)\, H(N \cdot y - 1, c)\, N\right)\right] + \sum_{l=1}^{N-1} \left(c_{1,l} \cdot DH(N \cdot x - 1, c)\, H(N \cdot y - 1, c)\, N\right) - N \cdot \left[\sum_{k=1}^{1} \sum_{l=1}^{1} \left(c_{k,1} DH(N \cdot x - k, c) \cdot H(N \cdot y - 1, c)\right)\right]$$

$$DyU4(x,y,N,c) := \left[\sum_{k=1}^{N-1} \left(c_{k,1} \cdot H(N \cdot x - k, c)\, DH(N \cdot y - 1, c)\, N\right)\right] + \sum_{l=1}^{N-1} \left(c_{1,l} H(N \cdot x - 1, c)\, DH(N \cdot y - 1, c)\, N\right) - N \cdot \left[\sum_{k=1}^{1} \sum_{l=1}^{1} \left(c_{k,1} H(N \cdot x - k, c) \cdot DH(N \cdot y - 1, c)\right)\right]$$

$DxU4(0.125, 0.125, N, c) = -0.116$

$DyU4(0.125, 0.125, N, c) = -0.116$

$JJJ(c) = -0.037$

Given

$B := \text{Minimize}(JJJ, c)$

$$B = \begin{pmatrix} 0 & -0.894 & 4.141 & 0.547 \\ 0 & -0.051 & -0.051 & -0.049 \\ 0 & -0.051 & 0 & 0 \\ 0 & -0.049 & 0 & 0 \end{pmatrix}$$

$JJJ(B) = -0.044$

ДОДАТОК Б

Програмна реалізація для $n=8$

ClasikN8

$a := 1$ $\quad b := 1$ $\quad c := 0.5$

$f(x,y) := -2$ $\quad N := 8$

$k := 0..N$

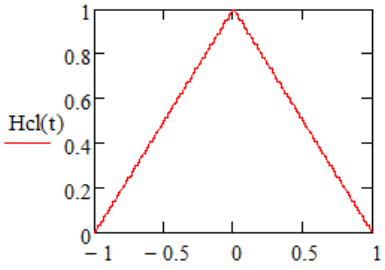
$l := 0..N$

$c_{k,l} := 0.15$ $\quad X_k := \frac{k \cdot a}{N}$ $\quad X = \begin{pmatrix} 0 \\ 0.125 \\ 0.25 \\ 0.375 \\ 0.5 \\ 0.625 \\ 0.75 \\ 0.875 \\ 1 \end{pmatrix}$ $\quad Y_l := \frac{l \cdot b}{N}$ $\quad Y = \begin{pmatrix} 0 \\ 0.125 \\ 0.25 \\ 0.375 \\ 0.5 \\ 0.625 \\ 0.75 \\ 0.875 \\ 1 \end{pmatrix}$

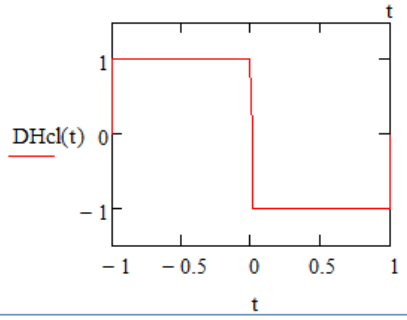
$p := 1..N-1$

$q := 1..N-1$

$$Hcl(t) := \begin{cases} 0 & \text{if } t \le -1 \\ (1+t) & \text{if } -1 < t \le 0 \\ (1-t) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$



$$DHcl(t) := \begin{cases} 0 & \text{if } t \le -1 \\ 1 & \text{if } -1 < t < 0 \\ 0 & \text{if } t = 0 \\ (-1) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$



$$Ucl8(x,y,N,c) := \left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\cdot Hcl(N\cdot x-k)\,Hcl(N\cdot y-l)\right)\right] + \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(c_{k,l}\cdot Hcl(N\cdot x-k)\,Hcl(N\cdot y-l)\right) - \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\,Hcl(N\cdot x-k)\cdot Hcl(N\cdot y-l)\right)$$

$$Ucl8(0.1,0.1,N,c) = 0.096$$

$$DxUcl8(x,y,N,c) := \left[\left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\cdot DHcl(N\cdot x-k)\,Hcl(N\cdot y-l)\right)\right] + \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(c_{k,l}\cdot DHcl(N\cdot x-k)\,Hcl(N\cdot y-l)\right) - \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\,DHcl(N\cdot x-k)\cdot Hcl(N\cdot y-l)\right)\right]\cdot N$$

$$DxUcl8(0.1,0.1,N,c) = 0.96$$

$$DyUcl8(x,y,N,c) := \left[\left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\cdot Hcl(N\cdot x-k)\,DHcl(N\cdot y-l)\right)\right] + \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(c_{k,l}\cdot Hcl(N\cdot x-k)\,DHcl(N\cdot y-l)\right) - \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\,Hcl(N\cdot x-k)\cdot DHcl(N\cdot y-l)\right)\right]\cdot N$$

$$DyUcl8(0.1,0.1,N,c) = 0.96$$

$$HHcl := \left|\begin{array}{l}\text{for } p \in 1..N-1\\ \quad \text{for } i \in 1..N-1\\ \quad\quad hh_{p,i} \leftarrow \int_0^1 Hcl(N\cdot x-p)\cdot Hcl(N\cdot x-i)\,dx\\ hh\end{array}\right.$$

$$HHcl = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.083 & 0.021 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.021 & 0.083 & 0.021 & 0 & 0 & 0 & 0\\ 0 & 0 & 0.021 & 0.083 & 0.021 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.021 & 0.083 & 0.021 & 0 & 0\\ 0 & 0 & 0 & 0 & 0.021 & 0.083 & 0.021 & 0\\ 0 & 0 & 0 & 0 & 0 & 0.021 & 0.083 & 0.021\\ 0 & 0 & 0 & 0 & 0 & 0 & 0.021 & 0.083 \end{pmatrix}$$

$$DHHcl := \left| \begin{array}{l} \text{for } p \in 1..N-1 \\ \quad \text{for } i \in 1..N-1 \\ \quad\quad hh_{p,i} \leftarrow \left(\int_0^1 DHcl(N\cdot x - p)\cdot DHcl(N\cdot x - i)\, dx \right) N^2 \\ hh \end{array} \right.$$

$$DHHcl = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16 & -8 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8 & 16 & -8 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 16 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & -8 & 16 & -8 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8 & 16 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & -8 & 16 & -8 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & 16 \end{pmatrix}$$

$$AAA := \left| \begin{array}{l} \text{for } p \in 1..N-1 \\ \quad \text{for } q \in 1..N-1 \\ \quad\quad \text{for } i \in 1..N-1 \\ \quad\quad\quad \text{for } j \in 1..N-1 \\ \quad\quad\quad\quad a_{(N-1)\cdot(p-1)+q,(N-1)(i-1)+j} \leftarrow HHcl_{p,i}\cdot DHHcl_{q,j} + DHHcl_{p,i}\cdot HHcl_{q,j} \\ a \end{array} \right.$$

$$a(p,q,i,j) := \left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j} \right)$$

$$A := \left| \begin{array}{l} \text{for } p \in 1..N-1 \\ \quad \text{for } q \in 1..N-1 \\ \quad\quad \text{for } i \in 1..N-1 \\ \quad\quad\quad \text{for } j \in 1..N-1 \\ \quad\quad\quad\quad \left| \begin{array}{l} AA_{(N-1)\cdot(p-1)+q,(N-1)\cdot(i-1)+j} \leftarrow HHcl_{p,i}\cdot DHHcl_{q,j} + DHHcl_{p,i}\cdot HHcl_{q,j} \text{ if } |p-i| \le 1 \wedge |q-j| \le 1 \\ AA_{(N-1)\cdot(p-1)+q,(N-1)\cdot(i-1)+j} \leftarrow 0 \text{ otherwise} \end{array} \right. \\ AA \end{array} \right.$$

$$Anew := \left| \begin{array}{l} \text{for } p \in 1..N-1 \\ \quad \text{for } q \in 1..\frac{N}{2}-1 \\ \quad\quad \text{for } i \in 1..N-1 \\ \quad\quad\quad \text{for } j \in 1..\frac{N}{2}-1 \\ \quad\quad\quad\quad A_{(N-1)\cdot(p-1)+q,(N-1)\cdot(i-1)+j} \leftarrow HHcl_{p,i}\cdot DHHcl_{q,j} + DHHcl_{p,i}\cdot HHcl_{q,j} \\ A \end{array} \right.$$

$$w(x,y) := [x\cdot(1-x) + y\cdot(1-y)] - \sqrt{[x\cdot(1-x)]^2 + [y\cdot(1-y)]^2}$$

$$w\left(\frac{1}{2},\frac{1}{2}\right) = 0.146 \qquad w(2,0) = -4$$

$$w1(x,y) := [(0.5 - x) + (0.5 - y)] + \sqrt{(0.5 - x)^2 + (0.5 - y)^2} \qquad w1(1,0) = 0.707$$

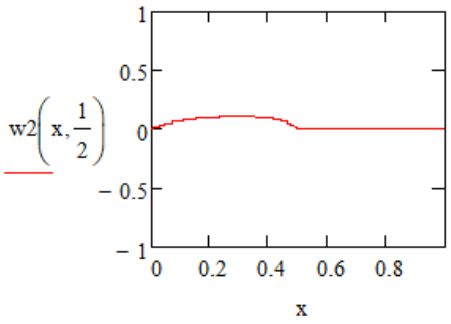
$$w2(x,y) := (w(x,y) + w1(x,y)) - \sqrt{w(x,y)^2 + w1(x,y)^2}$$

$$w2(0.5,0.5) = 0$$

$kk := 0..100$

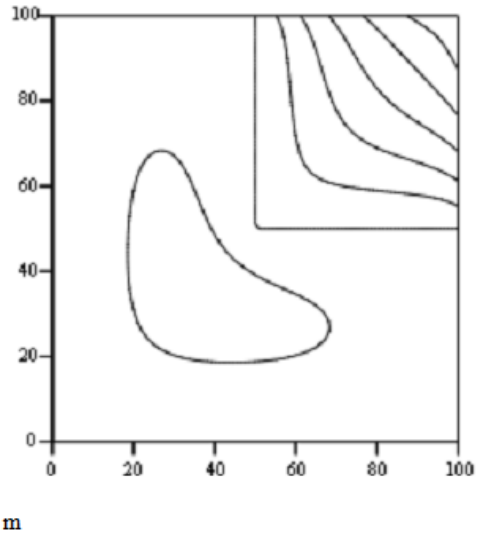
$ll := 0..100$

$$m_{kk,ll} := w2\left(\frac{ll}{100}, \frac{kk}{100}\right)$$



$$f(x,y) := \begin{cases} (-2) & \text{if } w2(x,y) > 0 \\ 0 & \text{if } w2(x,y) \le 0 \end{cases}$$

$$ww(x,y) := \begin{cases} 1 & \text{if } w2(x,y) > 0 \\ 0 & \text{if } w2(x,y) \le 0 \end{cases}$$



$$FFF := \left| \begin{array}{l} \text{for } i \in 1..N-1 \\ \quad \text{for } j \in 1..N-1 \\ \quad\quad fff_{i,j} \leftarrow \int_0^1 \left(\int_0^1 f(x,y) \cdot Hcl(N \cdot x - i) \cdot Hcl(N \cdot y - j)\, dy \right) dx \\ fff \end{array} \right.$$

$$FFF = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 \\ 0 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 \\ 0 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 & -0.031 \\ 0 & -0.031 & -0.031 & -0.031 & -0.023 & -0.016 & -0.016 & -0.016 \\ 0 & -0.031 & -0.031 & -0.031 & -0.016 & 0 & 0 & 0 \\ 0 & -0.031 & -0.031 & -0.031 & -0.016 & 0 & 0 & 0 \\ 0 & -0.031 & -0.031 & -0.031 & -0.016 & 0 & 0 & 0 \end{pmatrix}$$

$$a(p,q,i,j) := \left[\int_0^1 \left[\left[\int_0^1 (Hcl(N\cdot x - p)\cdot Hcl(N\cdot x - i))\cdot(DHcl(N\cdot y - q)\cdot DHcl(N\cdot y - j))\, N^2\cdot[(DHcl(N\cdot x - p)\cdot DHcl(N\cdot x - i))\cdot(Hcl(N\cdot y - q)\cdot Hcl(N\cdot y - j))]\cdot ww(x,y)\, dy \right] \right] dx \right]$$

$$Arf := \left| \begin{array}{l} \text{for } p \in 1..N-1 \\ \quad \text{for } q \in 1..N-1 \\ \quad\quad \text{for } i \in 1..N-1 \\ \quad\quad\quad \text{for } j \in 1..N-1 \\ \quad\quad\quad\quad arf_{(N-1)\cdot(p-1)+q,(N-1)\cdot(i-1)+j} \leftarrow a(p,q,i,j) \text{ if } |p-i| \le 1 \wedge |q-j| \le 1 \\ arf \end{array} \right.$$

$$Acl := \left| \begin{array}{l} \text{for } pp \in 0..(N-1)^2 - 1 \\ \quad \text{for } qq \in 0..(N-1)^2 - 1 \\ \quad\quad acl_{pp,qq} \leftarrow Arf_{pp+1,qq+1} \\ acl \end{array} \right.$$

Acl =

	0	1	2	3	4	5	6	7
0	0.444	-0.111	0	0	0	0	0	-0.111
1	-0.111	0.444	-0.111	0	0	0	0	0.028
2	0	-0.111	0.444	-0.111	0	0	0	0
3	0	0	-0.111	0.444	-0.111	0	0	0
4	0	0	0	-0.111	0.444	-0.111	0	0
5	0	0	0	0	-0.111	0.444	-0.111	0
6	0	0	0	0	0	-0.111	0.444	0
7	-0.111	0.028	0	0	0	0	0	0.444
8	0.028	-0.111	0.028	0	0	0	0	-0.111
9	0	0.028	-0.111	0.028	0	0	0	0
10	0	0	0.028	-0.111	0.028	0	0	0
11	0	0	0	0.028	-0.111	0.028	0	0
12	0	0	0	0	0.028	-0.111	0.028	0
13	0	0	0	0	0	0.028	-0.111	0
14	0	0	0	0	0	0	0	-0.111
15	0	0	0	0	0	0	0	...

$$Fcl := \left| \begin{array}{l} \text{for } i \in 0..N-2 \\ \quad \text{for } j \in 0..N-2 \\ \quad\quad fcl_{(N-1)\cdot i+j} \leftarrow FFF_{i+1,j+1} \\ fcl \end{array} \right.$$

Fcl =

	0
0	-0.031
1	-0.031
2	-0.031
3	-0.031
4	-0.031
5	-0.031
6	-0.031
7	-0.031
8	-0.031
9	-0.031
10	-0.031
11	-0.031
12	-0.031
13	-0.031
14	-0.031
15	...

$$epc := 0.01$$

$$EM := \left| \begin{array}{l} \text{for } pp \in 0..(N-1)^2-1 \\ \quad \text{for } qq \in 0..(N-1)^2-1 \\ \quad\quad \left| em_{pp,qq} \leftarrow 1 \text{ if } pp = qq \right. \\ \quad\quad \left| em_{pp,qq} \leftarrow 0 \text{ if } pp \neq qq \right. \\ em \end{array} \right.$$

$$jj := 1..N-1$$

$$ccc_{ii,jj} := 0.15$$

$$fi(c)_{i,j} \leftarrow \left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left[c_{k,l}\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right]\right] + \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left[c_{k,l}\cdot\left(\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right)\right] - \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left[c_{k,l}\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right]$$

$$pci(c) := \left| \begin{array}{l} \text{for } i \in 1..\frac{N}{2}-1 \\ \quad \text{for } j \in 1..N-1 \\ \quad (pci)_{i,j} \leftarrow \left[\left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left[c_{k,l}\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right]\right] + \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right) - \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right)\right] \\ pci \end{array} \right.$$

$$pci(c,i,j) := \left[\left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left[c_{k,l}\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right]\right] + \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right) - \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(\left(DHHcl_{p,i}\cdot HHcl_{q,j} + HHcl_{p,i}\cdot DHHcl_{q,j}\right)\right)\right]$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\left(DHHcl_{p,1}\cdot HHcl_{q,1} + HHcl_{p,1}\cdot DHHcl_{q,1}\right)\right]\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left[ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,1} + HHcl_{p,1}\cdot DHHcl_{q,1}\right)\right] - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,1} + HHcl_{p,1}\cdot DHHcl_{q,1}\right)\right] = FFF_{1,1}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,1}\cdot HHcl_{q,2} + HHcl_{p,1}\cdot DHHcl_{q,2}\right)\right|\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,2} + HHcl_{p,1}\cdot DHHcl_{q,2}\right)\right| - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,2} + HHcl_{p,1}\cdot DHHcl_{q,2}\right)\right| = FFF_{1,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,1}\cdot HHcl_{q,3} + HHcl_{p,1}\cdot DHHcl_{q,3}\right)\right|\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,3} + HHcl_{p,1}\cdot DHHcl_{q,3}\right)\right| - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,3} + HHcl_{p,1}\cdot DHHcl_{q,3}\right)\right| = FFF_{1,3}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,1}\cdot HHcl_{q,4} + HHcl_{p,1}\cdot DHHcl_{q,4}\right)\right|\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,4} + HHcl_{p,1}\cdot DHHcl_{q,4}\right)\right| - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,4} + HHcl_{p,1}\cdot DHHcl_{q,4}\right)\right| = FFF_{1,4}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,1}\cdot HHcl_{q,5} + HHcl_{p,1}\cdot DHHcl_{q,5}\right)\right|\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,5} + HHcl_{p,1}\cdot DHHcl_{q,5}\right)\right| - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,5} + HHcl_{p,1}\cdot DHHcl_{q,5}\right)\right| = FFF_{1,5}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,1}\cdot HHcl_{q,6} + HHcl_{p,1}\cdot DHHcl_{q,6}\right)\right|\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,6} + HHcl_{p,1}\cdot DHHcl_{q,6}\right)\right| - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,6} + HHcl_{p,1}\cdot DHHcl_{q,6}\right)\right| = FFF_{1,6}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,1}\cdot HHcl_{q,7} + HHcl_{p,1}\cdot DHHcl_{q,7}\right)\right|\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,7} + HHcl_{p,1}\cdot DHHcl_{q,7}\right)\right| - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,1}\cdot HHcl_{q,7} + HHcl_{p,1}\cdot DHHcl_{q,7}\right)\right| = FFF_{1,7}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,2}\cdot HHcl_{q,1} + HHcl_{p,2}\cdot DHHcl_{q,1}\right)\right|\right] + \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,1} + HHcl_{p,2}\cdot DHHcl_{q,1}\right)\right| - \sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,1} + HHcl_{p,2}\cdot DHHcl_{q,1}\right)\right| = FFF_{2,1}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,2}+HHcl_{p,2}\cdot DHHcl_{q,2}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,2}+HHcl_{p,2}\cdot DHHcl_{q,2}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,2}+HHcl_{p,2}\cdot DHHcl_{q,2}\right)\right|=FFF_{2,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,3}+HHcl_{p,2}\cdot DHHcl_{q,3}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,3}+HHcl_{p,2}\cdot DHHcl_{q,3}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,3}+HHcl_{p,2}\cdot DHHcl_{q,3}\right)\right|=FFF_{2,3}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,4}+HHcl_{p,2}\cdot DHHcl_{q,4}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,4}+HHcl_{p,2}\cdot DHHcl_{q,4}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,4}+HHcl_{p,2}\cdot DHHcl_{q,4}\right)\right|=FFF_{2,4}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,5}+HHcl_{p,2}\cdot DHHcl_{q,5}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,5}+HHcl_{p,2}\cdot DHHcl_{q,5}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,2}\cdot HHcl_{q,5}+HHcl_{p,2}\cdot DHHcl_{q,5}\right)\right|=FFF_{2,5}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,6}+HHcl_{p,3}\cdot DHHcl_{q,6}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,6}+HHcl_{p,3}\cdot DHHcl_{q,6}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,6}+HHcl_{p,3}\cdot DHHcl_{q,6}\right)\right|=FFF_{3,6}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,7}+HHcl_{p,3}\cdot DHHcl_{q,7}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,7}+HHcl_{p,3}\cdot DHHcl_{q,7}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,7}+HHcl_{p,3}\cdot DHHcl_{q,7}\right)\right|=FFF_{3,7}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,1}+HHcl_{p,4}\cdot DHHcl_{q,1}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,1}+HHcl_{p,4}\cdot DHHcl_{q,1}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,1}+HHcl_{p,4}\cdot DHHcl_{q,1}\right)\right|=FFF_{4,1}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,2}+HHcl_{p,4}\cdot DHHcl_{q,2}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,2}+HHcl_{p,4}\cdot DHHcl_{q,2}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,2}+HHcl_{p,4}\cdot DHHcl_{q,2}\right)\right|=FFF_{4,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,3}+HHcl_{p,4}\cdot DHHcl_{q,3}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,3}+HHcl_{p,4}\cdot DHHcl_{q,3}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,4}\cdot HHcl_{q,3}+HHcl_{p,4}\cdot DHHcl_{q,3}\right)\right|=FFF_{4,3}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,1}+HHcl_{p,3}\cdot DHHcl_{q,1}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,1}+HHcl_{p,3}\cdot DHHcl_{q,1}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,1}+HHcl_{p,3}\cdot DHHcl_{q,1}\right)\right|=FFF_{3,1}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,2}+HHcl_{p,3}\cdot DHHcl_{q,2}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,2}+HHcl_{p,3}\cdot DHHcl_{q,2}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,2}+HHcl_{p,3}\cdot DHHcl_{q,2}\right)\right|=FFF_{3,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,2}+HHcl_{p,3}\cdot DHHcl_{q,2}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,2}+HHcl_{p,3}\cdot DHHcl_{q,2}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,2}+HHcl_{p,3}\cdot DHHcl_{q,2}\right)\right|=FFF_{3,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,5}\cdot HHcl_{q,1}+HHcl_{p,5}\cdot DHHcl_{q,1}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,5}\cdot HHcl_{q,1}+HHcl_{p,5}\cdot DHHcl_{q,1}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,5}\cdot HHcl_{q,1}+HHcl_{p,5}\cdot DHHcl_{q,1}\right)\right|=FFF_{5,1}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,5}\cdot HHcl_{q,2}+HHcl_{p,5}\cdot DHHcl_{q,2}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,5}\cdot HHcl_{q,2}+HHcl_{p,5}\cdot DHHcl_{q,2}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,5}\cdot HHcl_{q,2}+HHcl_{p,5}\cdot DHHcl_{q,2}\right)\right|=FFF_{5,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,3}+HHcl_{p,3}\cdot DHHcl_{q,3}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,3}+HHcl_{p,3}\cdot DHHcl_{q,3}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\cdot\left(DHHcl_{p,3}\cdot HHcl_{q,3}+HHcl_{p,3}\cdot DHHcl_{q,3}\right)\right|=FFF_{3,3}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,5}\cdot HHcl_{q,3}+HHcl_{p,5}\cdot DHHcl_{q,3}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,5}\cdot HHcl_{q,3}+HHcl_{p,5}\cdot DHHcl_{q,3}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,5}\cdot HHcl_{q,3}+HHcl_{p,5}\cdot DHHcl_{q,3}\right)\right|=FFF_{5,3}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,2}\cdot HHcl_{q,6}+HHcl_{p,2}\cdot DHHcl_{q,6}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,2}\cdot HHcl_{q,6}+HHcl_{p,2}\cdot DHHcl_{q,6}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,2}\cdot HHcl_{q,6}+HHcl_{p,2}\cdot DHHcl_{q,6}\right)\right|=FFF_{2,6}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\left(DHHcl_{p,3}\cdot HHcl_{q,4}+HHcl_{p,3}\cdot DHHcl_{q,4}\right)\right]\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left[ccc_{p,q}\left(DHHcl_{p,3}\cdot HHcl_{q,4}+HHcl_{p,3}\cdot DHHcl_{q,4}\right)\right]-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\left(DHHcl_{p,3}\cdot HHcl_{q,4}+HHcl_{p,3}\cdot DHHcl_{q,4}\right)\right]=FFF_{3,4}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,3}\cdot HHcl_{q,5}+HHcl_{p,3}\cdot DHHcl_{q,5}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,3}\cdot HHcl_{q,5}+HHcl_{p,3}\cdot DHHcl_{q,5}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,3}\cdot HHcl_{q,5}+HHcl_{p,3}\cdot DHHcl_{q,5}\right)\right|=FFF_{3,5}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,1}+HHcl_{p,6}\cdot DHHcl_{q,1}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,1}+HHcl_{p,6}\cdot DHHcl_{q,1}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,1}+HHcl_{p,6}\cdot DHHcl_{q,1}\right)\right|=FFF_{6,1}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,2}+HHcl_{p,6}\cdot DHHcl_{q,2}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,2}+HHcl_{p,6}\cdot DHHcl_{q,2}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,2}+HHcl_{p,6}\cdot DHHcl_{q,2}\right)\right|=FFF_{6,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,3}+HHcl_{p,6}\cdot DHHcl_{q,3}\right)\right]\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left[ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,3}+HHcl_{p,6}\cdot DHHcl_{q,3}\right)\right]-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,3}+HHcl_{p,6}\cdot DHHcl_{q,3}\right)\right]=FFF_{6,3}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,3}+HHcl_{p,6}\cdot DHHcl_{q,3}\right)\right]\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left[ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,3}+HHcl_{p,6}\cdot DHHcl_{q,3}\right)\right]-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left[ccc_{p,q}\left(DHHcl_{p,6}\cdot HHcl_{q,3}+HHcl_{p,6}\cdot DHHcl_{q,3}\right)\right]=FFF_{6,3}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,1}+HHcl_{p,7}\cdot DHHcl_{q,1}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,1}+HHcl_{p,7}\cdot DHHcl_{q,1}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,1}+HHcl_{p,7}\cdot DHHcl_{q,1}\right)\right|=FFF_{7,1}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,2}+HHcl_{p,7}\cdot DHHcl_{q,2}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,2}+HHcl_{p,7}\cdot DHHcl_{q,2}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,2}+HHcl_{p,7}\cdot DHHcl_{q,2}\right)\right|=FFF_{7,2}$$

$$\left[\sum_{p=1}^{N-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,3}+HHcl_{p,7}\cdot DHHcl_{q,3}\right)\right|\right]+\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{N-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,3}+HHcl_{p,7}\cdot DHHcl_{q,3}\right)\right|-\sum_{p=1}^{\frac{N}{2}-1}\sum_{q=1}^{\frac{N}{2}-1}\left|ccc_{p,q}\left(DHHcl_{p,7}\cdot HHcl_{q,3}+HHcl_{p,7}\cdot DHHcl_{q,3}\right)\right|=FFF_{7,3}$$

$B := \text{Minerr}(ccc)$

$$B=\begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -0.031 & -0.044 & -0.049 & -0.049 & -0.045 & -0.039 & -0.028\\ 0 & -0.044 & -0.065 & -0.071 & -0.068 & -0.061 & -0.052 & -0.035\\ 0 & -0.049 & -0.071 & -0.069 & -0.059 & -0.047 & -0.04 & -0.028\\ 0 & -0.049 & -0.068 & -0.059 & 0.15 & 0.15 & 0.15 & 0.15\\ 0 & -0.045 & -0.061 & -0.047 & 0.15 & 0.15 & 0.15 & 0.15\\ 0 & -0.039 & -0.052 & -0.04 & 0.15 & 0.15 & 0.15 & 0.15\\ 0 & -0.028 & -0.035 & -0.028 & 0.15 & 0.15 & 0.15 & 0.15\end{pmatrix}$$

OFEM8

$f(x,y) := -2 \qquad a := 1 \qquad b := 1 \qquad c := 0.5$

$N := 8 \qquad \lambda := -0.894 \quad \gamma := 4.141 \quad \mu := 0.547$

$k := 0..N$

$l := 0..N$

$$c := \begin{pmatrix} 0 & -0.894 & 4.141 & 0.547 & 0 & 0 & 0 & 0 \\ 0 & -0.031 & -0.044 & -0.049 & -0.049 & -0.045 & -0.039 & -0.028 \\ 0 & -0.044 & -0.065 & -0.071 & -0.068 & -0.061 & -0.052 & -0.035 \\ 0 & -0.049 & -0.071 & -0.069 & -0.059 & -0.047 & -0.04 & -0.028 \\ 0 & -0.049 & -0.068 & -0.059 & 0 & 0 & 0 & 0 \\ 0 & -0.045 & -0.061 & -0.047 & 0 & 0 & 0 & 0 \\ 0 & -0.039 & -0.052 & -0.04 & 0 & 0 & 0 & 0 \\ 0 & -0.028 & -0.035 & -0.028 & 0 & 0 & 0 & 0 \end{pmatrix}$$

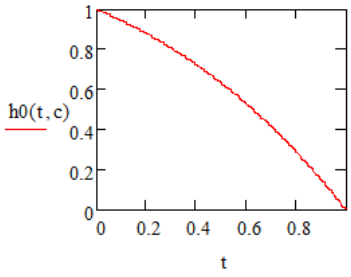
$$h0(t) := \frac{1 - 2\cdot\gamma}{2}\cdot\frac{\cosh[\mu\cdot(t-0.5)]}{\cosh(\mu\cdot 0.5)} - \frac{\sinh[\lambda\cdot(t-0.5)]}{2\cdot\sinh(\lambda\cdot 0.5)} + \gamma$$

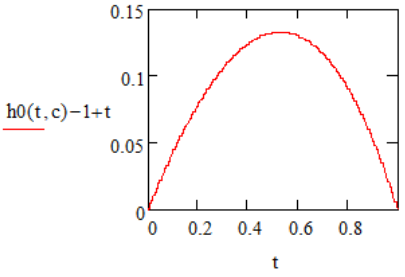
$$h0(t,c) := \frac{1 - 2\cdot c_{0,2}}{2}\cdot\frac{\cosh\left[c_{0,3}\cdot(t-0.5)\right]}{\cosh\left(c_{0,3}\cdot 0.5\right)} - \frac{\sinh\left[c_{0,1}\cdot(t-0.5)\right]}{2\cdot\sinh\left(c_{0,1}\cdot 0.5\right)} + c_{0,2}$$

$$X_k := \frac{k\cdot a}{N} \qquad X = \begin{pmatrix} 0 \\ 0.125 \\ 0.25 \\ 0.375 \\ 0.5 \\ 0.625 \\ 0.75 \\ 0.875 \\ 1 \end{pmatrix}$$

$$H(t,c) := \begin{cases} 0 & \text{if } t \le -1 \\ h0(-t,c) & \text{if } -1 < t \le 0 \\ h0(t,c) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$

$$Y_l := \frac{l\cdot b}{N} \qquad Y = \begin{pmatrix} 0 \\ 0.125 \\ 0.25 \\ 0.375 \\ 0.5 \\ 0.625 \\ 0.75 \\ 0.875 \\ 1 \end{pmatrix}$$





$p := 1..N-1$

$q := 1..N-1$

$$Dh0(t,c) := \frac{1-2\cdot c_{0,2}}{2}\cdot c_{0,3}\cdot\frac{\sinh\left|c_{0,3}\cdot(t-0.5)\right|}{\cosh\left(c_{0,3}\cdot 0.5\right)} - \frac{c_{0,1}\cosh\left|c_{0,1}\cdot(t-0.5)\right|}{2\cdot\sinh\left(c_{0,1}\cdot 0.5\right)}$$

$$DH(t,c) := \begin{cases} 0 & \text{if } t \le -1 \\ -Dh0(-t,c) & \text{if } -1 < t < 0 \\ 0 & \text{if } t = 0 \\ Dh0(t,c) & \text{if } 0 < t < 1 \\ 0 & \text{if } t \ge 1 \end{cases}$$

$$U8(x,y,N,c) := \left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\cdot H(N\cdot x-k,c)\,H(N\cdot y-1,c)\right)\right] + \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(c_{k,l}\cdot H(N\cdot x-k,c)\,H(N\cdot y-1,c)\right) - \sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}H(N\cdot x-k,c)\cdot H(N\cdot y-1,c)\right)$$

$U8(0.1,0.1,N,c) = -0.024$

$$DxU8(x,y,N,c) := N\cdot\left[\left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\cdot DH(N\cdot x-k,c)\,H(N\cdot y-1,c)\right)\right]\right] + N\cdot\left[\sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(c_{k,l}\cdot DH(N\cdot x-k,c)\,H(N\cdot y-1,c)\right)\right] - N\cdot\left[\sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}DH(N\cdot x-k,c)\cdot H(N\cdot y-1,c)\right)\right]$$

$DxU8(0.1,0.1,N,c) = -0.149$

$$DyU8(x,y,N,c) := N\cdot\left[\sum_{k=1}^{N-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}\cdot H(N\cdot x-k,c)\,DH(N\cdot y-1,c)\right)\right] + N\cdot\left[\sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{N-1}\left(c_{k,l}\cdot H(N\cdot x-k,c)\,DH(N\cdot y-1,c)\right)\right] - N\cdot\left[\sum_{k=1}^{\frac{N}{2}-1}\sum_{l=1}^{\frac{N}{2}-1}\left(c_{k,l}H(N\cdot x-k,c)\cdot DH(N\cdot y-1,c)\right)\right]$$

$JJJ(c) = 8.554 \times 10^{-3}$

Given

$c_{1,2} = c_{2,1}$

$c_{1,3} = c_{3,1}$ $c_{2,3} = c_{3,2}$

$c_{1,4} = c_{4,1}$ $c_{2,4} = c_{4,2}$ $c_{3,4} = c_{4,3}$

$c_{1,5} = c_{5,1}$ $c_{2,5} = c_{5,2}$ $c_{3,5} = c_{5,3}$ $c_{4,5} = c_{5,4}$

$c_{1,6} = c_{6,1}$ $c_{2,6} = c_{6,2}$ $c_{3,6} = c_{6,3}$ $c_{4,6} = c_{6,4}$

$c_{1,7} = c_{7,1}$ $c_{2,7} = c_{7,2}$ $c_{3,7} = c_{7,3}$ $c_{4,7} = c_{7,4}$

$B := \text{Minimize}(JJJ, c)$

$$B = \begin{pmatrix} 0 & -0.859 & 2.887 & 0.288 & 0 & 0 & 0 & 0 \\ 0 & -0.03 & -0.042 & -0.046 & -0.046 & -0.043 & -0.038 & -0.027 \\ 0 & -0.042 & -0.061 & -0.065 & -0.063 & -0.057 & -0.05 & -0.034 \\ 0 & -0.046 & -0.065 & -0.062 & -0.047 & -0.043 & -0.038 & -0.027 \\ 0 & -0.046 & -0.063 & -0.047 & 0 & 0 & 0 & 0 \\ 0 & -0.043 & -0.057 & -0.043 & 0 & 0 & 0 & 0 \\ 0 & -0.038 & -0.05 & -0.038 & 0 & 0 & 0 & 0 \\ 0 & -0.027 & -0.034 & -0.027 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$JJJ(B) = -0.049$

ВІДОМІСТЬ АТЕСТАЦІЙНОЇ РОБОТИ

Позначення	Найменування	Дод. відомості
	Текстові документи	
1	Пояснювальна записка	95 с.
2	Презентаційний матеріал	21 с.
	Інші документи	
3	Роздруківки програм	2 с.
4	Рецензія	2 с.
5	Відгук керівника	1 с.

					Дослідження оптимального методу скінченних елементів при розв'язанні крайової задачі кручення стержня			
Змін	**Арк.**	**Номер докум.**	**Підп.**	**Дата**				
Розроб.		**Кобозєв В.К.**			**(Тема роботи) Відомість атестаційної роботи**		**Аркуш**	**Аркушів**
Перевір.		**Литвин О.М.**						
Н. контр.		**Сидоров М.В.**				**ХНУРЕ Кафедра ПМ**		
Затв.		**Тевяшев А.Д.**						