Stochastic Model of Operating Modes of a Group of Artesian Wells in Water Supply Systems

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***Abstract*—This work is about development of a stochastic model and a method of modeling of the operating modes of a group of artesian wells.**

Keywords—artesian well, water supply system, optimization, stochastic model.

I. Introduction

Water supply of large cities is arranged, as a rule, from open reservoirs - lakes, water reservoirs, large rivers, that is, from sources with significant water supplies. In this case, the water supply scheme includes: pumping stations of the first lift, which take water directly from the reservoir; water treatment system (cleaning); pumping stations of the second and third lift, pumping clean water at considerable distances and supplying it to urban settlements [1].

If the urban settlement is located in an area where there are no large open water sources, or the geographical location of the urban settlement allows the use of groundwater, in this case artesian wells are used as a source of water supply.

Usually small urban settlements are supplied with water from artesian wells. Artesian wells are widely used for the water supply of villages, industrial water supply, for irigation (watering systems), in the mining industry, to reduce the level of groundwater. The advantage of using artesian wells to supply residents with drinking water is that water is taken from great depths and treated using natural filters [2]. Thus, costs of water treatment are significantly reduced. The disadvantage of using artesian wells is usually low well productivity, strong dependence of water volumes in the well on climatic and weather conditions, depletion of wells as a result of large construction or depositing natural resources, for example, shale gas [3].

II. Water intake wells

Individual water intake wells are widely used in private domestic facilities. The productivity of some wells becomes insufficient to supply water to urban settlements, so in water supply systems there used groups (clusters) of spatially distributed wells, combined with each other with sections of pipelines (common manifold), which supply water to the clean water reservoir (CWR). The group of wells together with the CWR is called the water intake unit (WIU) of the water supply system (WSS). After that water is supplied to consumers into urban settlements from the CWR with pumping stations (PS) of the second lift.

The mode of operation of the pump unit (PU), that is, the position of the working point at every point in time, in each well depends on the actual values of many internal and external factors: dynamic water level in the well; actual characteristics of the PU; position of control units (drive revolutions, degree of opening of the control valve); hydraulic resistance values of WIW pipelines; water level in the CWR; number and operating modes of PU in other WIW wells [4], [5].

The water intake well is shown on Fig. 1.

The following names are used in Fig. 1: CWR – clean water reservoir, SPU – submersible pumping unit, CP – centrifugal pump, SEM – submersible electrical motor; lSPU – SPU length, CV – control valve; geodetic marks according to: neck (ground level) of water intake well – h^g_w; CP – h^g_{CP}; CWR – h^g_R; depths of well – h_w; depth of SPU submersion (installation) – h_{SPU}; pressure values: at the output (neck) of water intake well – H_w; pressure developed by CP – H_{CP}; water levels: in CWR – H_{CWR}; static level in well (when water intake is at zero level) – H_{st}; dynamic level in well (when water intake is nominal) – H_{dyn}; CP support level – H^s_{CP}.

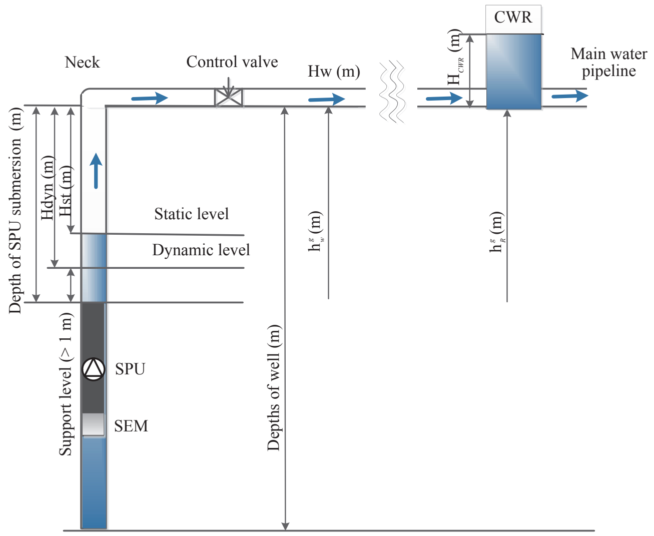

Fig. 1. Scheme of SPU installation into the water intake well

III. DETERMINISTIC MATHEMATICAL MODEL OF WATER INTAKE WELL

When creating the deterministic mathematical model of the water intake well, it is assumed that all parameters of the water intake well are a priorily known. This allows to determine the model of the water intake well operation in the form of an interdependent system of algebraic equations of the following type:

CP geodetic mark:

$$h^g_{CP} = h^g_w - h^g_{SPU} - 0.5 l_{SPU}. \tag{1}$$

CP support value:

$$H^s_{CP} = h^g_{CP} - H_{dyn}. \tag{2}$$

Pressure made by CP:

$$H_{CP}(Q) = a_0 + a_1 Q + a_2 Q^2. \tag{3}$$

CP capacity losses:

$$N_{CP}(Q) = c_0 + c_1 Q + c_2 Q^2. \tag{4}$$

C3 performance coefficient:

$$\eta_{CP}(Q) = d_0 + d_1 Q + d_2 Q^2, \tag{5}$$

where Q – water consumption via CP, $a_0, a_1, a_2, c_0, c_1, c_2, d_0, d_1, d_2$ – numerous values of CP parameters. H pressure loss in the pipeline area H=H(Q) is expressed with:

$$H(Q) = sgn(Q) \cdot \phi \cdot |Q|^{\chi}, \tag{6}$$

where Q – water consumption on the pipeline area [m^3/s]; sgn(Q) – Q symbol; $\phi > 0$ – hydraulic resistance of the pipeline area; $\chi > 1$ – non-linearity factor.

H pressure loss on CV has the following view:

$$H_v = \frac{C}{E_v^2} \operatorname{sgn}(Q) \cdot |Q|, \tag{7}$$

where C – equivalent CV hydraulic resistance in the «open» position; E_v [0,1] – degree of CV opening, where 1 is in the «open» position; 0 is in the «closed» position.

If the value of water consumption via CP is known, then the pressure value at the output (neck) of the well is expressed in the following way:

$$H_w = H^s_{CP} + H_{CP} - H - H_v. \tag{8}$$

Also ratios are present:

$$H^s_{CP} \geq 1 \text{ m}, \tag{9}$$

$$H_{st} \leq H_{dyn} \leq h_{SPU}. \tag{10}$$

Stochastic mathematical model Water consumption processes are stochastic processes, as they depend on many uncontrolled and unmanageable factors. Parameters of WIU technological equipment are also stochastic values, as they are evaluated according to experimental data of the final length [6]. To create stohastic models of technological elements of WIU we will introduce a number of symbols: let (Ω, B, P) – Cartesian product of probability spaces (Ω_i, B_i, P_i), $i = 1, 2, ..., n$, $\Omega = \Omega_1 \times \Omega_2 \times ... \times \Omega_n$, $B = B_1 \times B_2 \times ... \times B_n$, $P = P_1 \times P_2 \times ... \times P_n$, where Ω_i – probability spaces; B_i – σ-algebra of events with Ω_i; P_i – probability measures on B_i). Then the stochastic model of the water intake well has the following view:

$$H_v(\omega) = H^s_{CP}(\omega) + H_{CP}(\omega) - H(\omega) - H_v(\omega). \tag{11}$$

$$H_{CP}(Q(\omega)) = a_0(\omega) + a_1(\omega) Q(\omega) + a_2(\omega) Q^2(\omega). \tag{12}$$

$$H\left(Q(\omega)\right)=\operatorname{sgn}(Q)\cdot\phi(\omega)\cdot\left|Q(\omega)\right|^{\chi}. \tag{13}$$

$$H_v(\omega)=\frac{C(\omega)}{E_v^2}\operatorname{sgn}(Q)\cdot\left|Q(\omega)\right|. \tag{14}$$

$$H_{CP}^s(\omega)=h_{CP}^g(\omega)-H_{dyn}(\omega). \tag{15}$$

IV. WIU Stohastic mathematical model

In mathematical modeling and optimization of WIU operation modes, we will use a stochastic model of quasi-stationary WIU modes.

Let us review WIU with n artesian wells (Fig. 2).

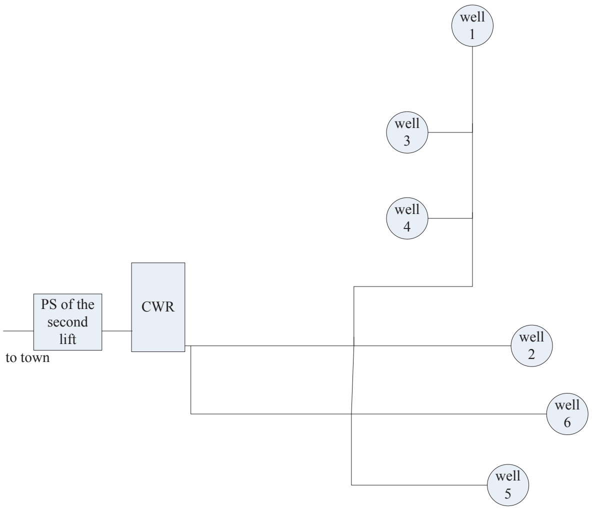

Fig. 2. WIU scheme

The real WIU is put in line with the indicative graph reflecting its structure.

We will present the WIU structure in the form of an orgraph $G(V, E)$, where V is a multiple of vertices, E is a multiple of arcs (e=Card(E), v=Card(V), the real WIU is added to the zero vertice and dummy chords connecting the zero vertice with all the entrances (wells) and its output (CWR). For mathematical formulation of the problem the following WIU encoding is made: the graph tree is chosen so that dummy areas of the network become chords. And real areas will become chords partially, and partially they become branches of tree. Tree branches with a pump are numbered as 1, other branches are numbered from 2 to v–1, chords of real areas from v to $v+\eta_2-1$, dummy areas with the given unit consumption (from $v+\eta_2$ to $v+\eta_2+\xi_1-1$, dummy areas with the given pressure values from $v+\eta_2+\xi_1$ up to e, where η_2 is the number of chords of real areas, ξ_1 is the number of outputs with the given unit consumption.

In this case the stochastic model of quasi-stationary WIU modes will have the following view:

$$\Omega : \underset{\omega}{M}\left(H_r\left(Q(\omega)\right)+\sum_{i=1}^{v-1}b_{1ri}H_i\left(Q(\omega)\right)\right)=0,$$

$$\left(r=v,\ldots,v+\eta_2-1\right), \tag{16}$$

$$\underset{\omega}{M}\left(h_r^c(\omega)-H_{v1}(\omega)+\sum_{i=1}^{v-1}b_{1ri}\left(H_r\left(Q(\omega)\right)+h_i^g\right)\right)=0,$$

$$\left(r=v+\eta_2,\ldots,v+\eta_2+\xi_1-1\right), \tag{17}$$

$$\underset{\omega}{M}\left(H_r\left(Q(\omega)\right)+h_r^g-H_{vj}(\omega)+H_{v1}(\omega)+\right.$$

$$\left.+\sum_{i=1}^{v}b_{1ri}\left(H_i\left(Q(\omega)\right)+h_i^g\right)\right)=0,$$

$$\left(r=v+\eta_2+\xi_1,\ldots,e;\ j=1,\ldots,n\right), \tag{18}$$

$$\underset{\omega}{M}\left(q_i(\omega)\right)=\underset{\omega}{M}\left(\sum_{r=v}^{v+\eta_2-1}b_{1ri}Q_r(\omega)+\sum_{r=v+\eta_2}^{e}b_{1ri}Q_r(\omega)\right),$$

$$\left(i=1,\ldots,v-1\right), \tag{19}$$

$$P\left(h_i^c(\omega)\geq h_i^+\right)\geq\alpha,\ (\alpha\cong 1),$$

$$i=\left(v+\eta_2,\ldots,\ v+\eta_2+\xi_1-1\right), \tag{20}$$

$$P\left(Q_i(\omega)>0\right)\cong\beta,\ (\beta\cong 1),\ i\in N, \tag{21}$$

$$H_{vj}(\omega)=H_{CPj}^s(\omega)+H_{CPj}(\omega)-H_j(\omega)-H_{vj}(\omega).$$

$$\left(j=1,\ldots,n\right). \tag{22}$$

$$H_{CPj}\left(Q_j(\omega)\right)=a_{0j}(\omega)+a_{1j}(\omega)Q_j(\omega)+a_{2j}(\omega)Q_j^{\,2}(\omega)$$

$$\left(j=1,\ldots,n\right). \tag{23}$$

$$H_k\left(Q_k(\omega)\right)=\operatorname{sgn}(Q_k)\cdot\phi_k(\omega)\cdot\left|Q_k(\omega)\right|^{\chi}.$$

$$k=\left(1,\ldots,v+\eta_2-1\right)\cup\left(v+\eta_2+\xi_1,\ldots,e\right). \tag{24}$$

$$H_{vj}(\omega)=\frac{C_j(\omega)}{E_{vj}^2}\operatorname{sgn}(Q_j)\cdot\left|Q_j(\omega)\right|,\ \left(j=1,\ldots,n\right). \tag{25}$$

$$H_{CPj}^s(\omega)=h_{CPj}^s(\omega)-H_{dynj}(\omega),\ \left(j=1,\ldots,n\right). \tag{26}$$

Where stochastic values characterize: $Q_j(\omega)$ is water consumption on the j section of the pipeline; $h_r^c(\omega)$ is free pressure in the r unit of WIU $(r = v + \eta_2, ..., v + \eta_2 + \xi_1 - 1)$; h_r^+ is minimum permissible pressure in the r unit of WIU; h_i^g is a geodesic mark of the *i* section of the pipeline ($i \in M$); b_{1ri} is an element of the cyclomatic matrix; $H_{v1}(\omega)$ is pressure at the output of the *j* source $(j = 1, ..., n)$; $H_{vj}(\omega)$ is pressure at the output of the *j* source.

As the above research [6] has shown, this model provides more adequate results of modelling of actual WIU operation modes.

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