Some Hypotheses in the Theory of the Azimuthally Magnetized Circular Ferrite Waveguides

Georgi Nikolov Georgiev and Mariana Nikolova Georgieva-Grosse

Abstract — **Hypotheses for the eigenvalue spectrum, phase characteristics and differential phase shift of the circular waveguides that comprise azimuthally magnetized ferrite are advanced, generalizing results from the theory of these transmission lines, worked out in terms of the confluent hypergeometric functions. Some corollaries of the proposed statements are considered. Numerical substantiation of the assumptions is made for two configurations, using iterative methods. Computational problems which arise in the analysis are discussed.**

I. Introduction

The circular waveguides with azimuthally magnetized remanent ferrite and dielectric layers appear to be microwave structures, eligible for nonreciprocal digital phase shifters with possible application in electronically scanned antenna arrays [1-16]. The reason for this is their ability to produce differential phase shift $\Delta\beta$ when latching the ferrite remanent magnetization [1-16]. It has been shown that the theory of the normal TE_{0n} modes in these configurations may successfully be built by the complex Kummer and Tricomi confluent hypergeometric functions (CHFs) $\Phi(a,c;x)$ and $\Psi(a,c;x)$, resp. [17] for the ferrite and the real cylindrical ones for the dielectric media [10-16]. Until now three geometries have been examined mainly: the completely filled with ferrite circular and coaxial waveguides, and the one that contains coaxially positioned ferrite rod and dielectric toroid, applying the boundary-value problem approach [10-16]. The principal aims of the study can be summarized as follows: i) to specify the configuration which provides maximum $\Delta\beta$ per unit length in the widest possible frequency band with minimum dependence on frequency; ii) to determine the conditions for propagation and the ones for operation of the waveguides as phasers; iii) to find formulae for direct computation of the differential phase shift from the structure and material parameters.

Manuscript received January 31, 2008.
Georgi Nikolov Georgiev is with the Faculty of Mathematics and Informatics, University of Veliko Tirnovo "St. St. Cyril and Methodius", BG–5000 Veliko Tirnovo, Bulgaria (corresponding author e-mail: gngeorgiev@yahoo.com).
Mariana Nikolova Georgieva–Grosse, Meterstrasse 4, D–70839 Gerlingen, Germany

An essential component of the analysis performed was the investigation of the influence of imaginary part k (k – real) of the complex first parameter $a = c/2 - jk$ of CHFs on the purely imaginary roots $\xi_{k,n}^{(c)}$ ($n = 1,2,3,...$), of the characteristic equations of geometries explored. The interval of variation of k was accepted finite, symmetrically singled out with regard to the point $k = 0$, assuming $c = 3$ and $x = jz$ (z – real, positive) [10]. It has recently been established that due to this fact (the incomplete knowledge of the eigenvalue spectrum), the phase behaviour even of the simplest of above configurations is not fully known [14].

Here, generalizing results of the research of two basic structures, obtained by means of specially developed numerical techniques, several hypotheses are formulated which reveal important features of the eigenvalue spectrum, phase curves and differential phase shift provided. The techniques involve iterative procedures, based on the repeated numerical solution of complex transcendental equations that take in CHFs for varying parameters. Graphs and numerical data are adduced as testimony of the truthfulness of the assertions. The significance of the hypotheses and some of their corollaries are debated. The ways of calculating the L numbers and factors A, B, C which come into being in the formulations, are described. Some issues in substantiating the statements are threshed out.

II. Normal TE_{0n} Modes in the Circular Waveguides with Azimuthally Magnetized Ferrite

A. Propagation Problem

We focus for simplicity on two geometries: the circular waveguide, entirely filled with azimuthally magnetized ferrite and the one, loaded with ferrite cylinder and dielectric toroid. The waveguide and cylinder radii are r_0 and r_1, resp. The anisotropic medium has a permeability tensor with off-diagonal element $\alpha = \gamma M_r / \omega$, $-1 < \alpha < 1$, (γ – gyromagnetic ratio, M_r – remanent magnetization, ω – angular frequency of the wave). The relative permittivities of inner and outer strata are ε_r and ε_d, resp. Both fillings are lossless, the structure wall is perfectly conducting and the thickness of central switching conductor is ignored. The equations [10-16]:

$$\Phi(a,c;x_0)=0, \qquad (1)$$

$$\frac{1-\alpha^2}{2}\frac{\Phi(a,c;\rho x_0)}{\Phi(a-1,c-2;\rho x_0)}=$$

$$\begin{cases} -\dfrac{1}{v_0}\dfrac{\partial}{\partial v_0}\ln\left|\dfrac{\partial}{\partial y_0}bsn_m(y_0-v_0)\right|, & q>0 \\ -\dfrac{1-\rho^2}{2\rho^2}, & q\equiv p\equiv 0 \\ \dfrac{1}{w_0}\dfrac{\partial}{\partial w_0}\ln\left|\dfrac{\partial}{\partial u_0}bsh_m(u_0-w_0)\right|, & p=jq>0 \end{cases} \qquad (2)$$

control the normal TE_{0n} modes propagation in the first and second configuration, resp. Eqn. (1) and the left-hand side of eqn. (2) are expressed by means of the complex Kummer CHF [17]. The first and third forms of the right-hand side of eqn. (2) are written in terms of ordinary and modified derivative difference Bessel functions [18]. The following notations are used: $a=1.5-jk$, $c=3$, $x_0=jz_0$, $k=\alpha\overline{\beta}/(2\overline{\beta}_2)$, ($k$ – real, $-\infty<k<+\infty$), $\overline{\beta}_2=\left(1-\alpha^2-\overline{\beta}^2\right)^{1/2}$, $z_0=2\overline{\beta}_2\overline{r}_0$, ($z_0$ – real, $z_0>0$), $\rho=\overline{r}_1/\overline{r}_0$, ($0<\rho\le 1$), $y_0=qz_0$, $v_0=\rho y_0$, $u_0=pz_0$, $w_0=\rho u_0$, $p=jq$, $q=$ $=0.5\left\{(\varepsilon_d/\varepsilon_r)\left[1+(2k/\alpha)^2\right]/\left(1-\alpha^2\right)-(2k/\alpha)^2\right\}^{1/2}$, $m=0$. The phase constant, radial wavenumber, guide and cylinder radii are normalized by the relations $\overline{\beta}=\beta/(\beta_0\sqrt{\varepsilon_r})$, $\overline{\beta}_2=\beta_2/(\beta_0\sqrt{\varepsilon_r})$, $\overline{r}_0=\beta_0 r_0\sqrt{\varepsilon_r}$, $\overline{r}_1=\beta_0 r_1\sqrt{\varepsilon_r}$ where $\beta_0=\omega\sqrt{\varepsilon_0\mu_0}$. The condition q (p) – real, positive or $q\equiv p\equiv 0$ needs application of the first (third) or second form of eqn. (2). If $\xi_{k,n}^{(c)}$ is the nth positive purely imaginary root in x_0 ($n=1,2,3...$) of any of above equalities, the eigenvalue spectrum of the fields examined is given by $\overline{\beta}_2=\xi_{k,n}^{(c)}/(2\overline{r}_0)$ [10-16].

B. Phase Portrait

To figure the $\overline{\beta}(\overline{r}_0)$ characteristics with α as parameter for normal TE_{01} mode in the ferrite-dielectric waveguide, we extend the approach, described lately for the ferrite case [14]. First the ratio ρ is specified and the permittivities ε_r and ε_d are fixed. Then, discrete α's are selected. For each set $\{\varepsilon_r,\varepsilon_d,\rho,\alpha,k\}$, considering k as altering parameter with varying step, an iterative technique is harnessed which yields the first roots $\xi_{k,1}^{(c)}$ of eqns. (1), (2) in x_0 for both signs of k. In the calculations a finite number of terms of the infinite power series, defining the wave functions involved [17,18], is used. The numbers $\xi_{k,1}^{(c)}$ are determined, utilizing the method of halving, applied with respect to the real parts of the equations. The results are checked also in relation to their imaginary parts. Thirty to forty iterations provide an accuracy of at least ten decimal places. Computations are performed in the interval $-10^5<k<10^5$. At each cycle the values of α, k and $\xi_{k,1}^{(c)}$ ($c=3$), are put in [14]:

$$\overline{r}_0=\left(k\xi_{k,1}^{(c)}/\alpha\right)\sqrt{\left\{1+[\alpha/(2k)]^2\right\}/\left(1-\alpha^2\right)}, \qquad (3)$$

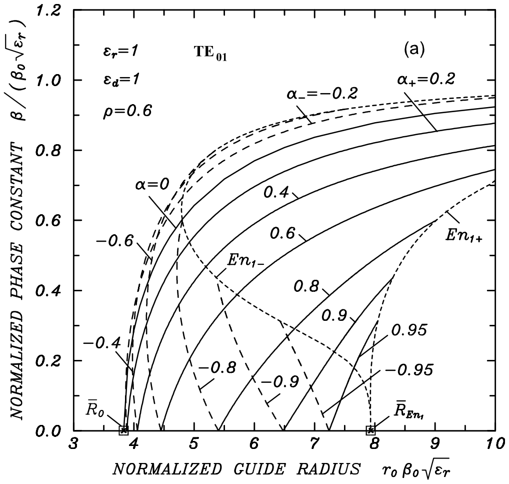
$$\overline{\beta}=\sqrt{\left(1-\alpha^2\right)/\left\{1+[\alpha/(2k)]^2\right\}}. \qquad (4)$$

Since the phase chart of first configuration for normal TE_{01} mode has been analyzed in detail [14], results for the second one are plotted here only in Figs. 1a–c in case $\rho=0.6$, $\varepsilon_r=1$ and $\varepsilon_d=1,5,10$, resp. (In view of the formula for q the ratio $\varepsilon_d/\varepsilon_r$ is of importance and not the actual values of ε_r and ε_d.) The solid (dashed) lines correspond to $\alpha_+>0$ ($\alpha_-<0$). (The subscripts "+","–" indicate quantities, correlated to positive or negative magnetization, resp.) The curve, labelled $\alpha=0$ is relevant to the dielectric case. All characteristics are of finite length in contrast to the ferrite waveguide where the ones for $\alpha_+>0$ are infinitely long [14]. The common (starting) points of curves for the same $|\alpha|$ at the horizontal axis give the cutoff frequencies $\overline{r}_{0cr}=\xi_{0,1}^{(c)}/\left(2\sqrt{1-\alpha^2}\right)$ of the geometry. Their ends $\overline{r}_{0en-}$ and $\overline{r}_{0en+}$ shape envelopes (dotted lines), designated by the symbols En_{1-} and En_{1+}, resp. (The subscripts "$en\pm$" are used to denote the parameters of the envelopes.) The quantities $\overline{r}_{0cr}$ are distributed between the beginning of $\alpha=0$ – characteristic, marked by $\overline{R}_0$ and the bifurcation point of the envelopes $\overline{R}_{En1}$. There are areas of partial overlapping of the intervals of propagation for $\alpha_+>0$ and $\alpha_-<0$ (areas of phaser operation in which for each $\overline{r}_0$ transmission is possible for both signs of M_r and $\Delta\overline{\beta}=\overline{\beta}_- - \overline{\beta}_+$ is obtained).

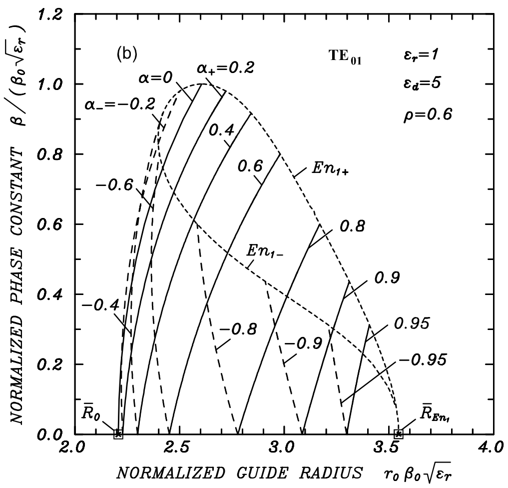
C. Prerequisites for Advancing Some Hypotheses

If $k_\pm\to\pm\infty$, expressions (3), (4) are simplified as $\overline{r}_{0\pm}=\left(k_\pm\xi_{k_\pm,1}^{(c)}/\alpha_\pm\right)/\sqrt{1-\alpha_\pm^2}$, $\overline{\beta}_\pm=\sqrt{1-\alpha_\pm^2}$. The numerical analysis shows that in this case $\overline{r}_{0-}$, $\overline{r}_{0+}$ or both $\overline{r}_{0\pm}$ tend to certain finite values $\overline{r}_{0en\pm}$, the pertinent $\overline{\beta}(\overline{r}_0)$ – curves become finite and their ends constitute the En_{1-} and En_{1+} – lines. This is possible, if the products

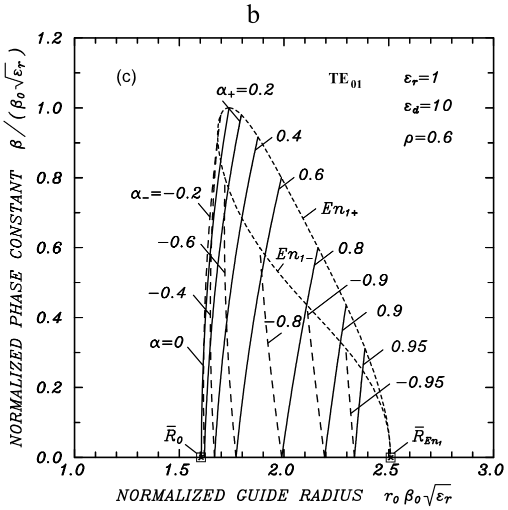
$\left|k_{\pm}\right|\xi_{k_{\pm},1}^{(c)}$ possess finite limits $L_{\pm}$, when $k_{\pm} \to \pm\infty$.



a



b



c

Fig. 1. Phase curves $\overline{\beta}(\overline{r}_0)$ of the normal TE_{01} mode in the circular ferrite-dielectric waveguide with α as parameter for $\rho = 0.6$ in case: (a) $\varepsilon_r = 1$, $\varepsilon_d = 1$; (b) $\varepsilon_r = 1$, $\varepsilon_d = 5$; (c) $\varepsilon_r = 1$, $\varepsilon_d = 10$

Moreover, a proportionality among $\Delta\overline{\beta}$, α and $\overline{r}_0$ is observed in the ferrite waveguide [10,11]. These facts lead us to the conclusion that there are some general properties, inherent to all configurations with azimuthally magnetized ferrite. In view of this we advance three hypotheses for: i) the eigenvalue spectrum; ii) the phase characteristics and iii) the differential phase shift and strive to substantiate them.

III. HYPOTHESES

A. Hypothesis for the Eigenvalue Spectrum

Hypothesis 1: *If* $\xi_{k,n}^{(c)}\big(\varepsilon_{r1},\varepsilon_{r2},...,\varepsilon_{rl},\varepsilon_{d1},\varepsilon_{d2},...,\varepsilon_{dg},$ $\rho_1,\rho_2,..., \rho_s,\alpha_1,\alpha_2,...,\alpha_l\big)$ *is the* n*th positive purely imaginary root in* x_0 *(*$n=1,2,3...$*) of the characteristic equation for normal* TE_{0n} *modes of any stratified circular waveguide of radius* r_0*, containing arbitrary positioned* l *coaxial azimuthally magnetized ferrite layers (*$l \geq 1$*), having off-diagonal ferrite tensor elements* α_l *and relative permittivities* ε_{rl}*, and* g *coaxial isotropic dielectric layers (*$g \geq 0$*) of relative permittivities* ε_{dg}*, of outer and inner radii ratios* ρ_t *and* ρ_{t+1} *where* $\rho_t = r_t / r_0$ *and* $\rho_t > \rho_{t+1}$ *[*r_t *is the radius of the interface, separating the* t *th and* $(t+1)$ *th layers,* $t=1,2,...,s$*,* $s = l + g$*;* $r_{s+1} = 0$ *or* $r_{s+1} > 0$*, if the thickness of central switching conductor is or is not neglected, (*r_{s+1}*− inner radius of the inmost layer)]:*

$$F\big(c,\varepsilon_{r1},\varepsilon_{r2},...,\varepsilon_{rl},\varepsilon_{d1},\varepsilon_{d2},...,\varepsilon_{dg},$$
$$\rho_1,\rho_2,...,\rho_s,\alpha_1,\alpha_2,...,\alpha_l,n,k;x_0\big)=0, \qquad (5)$$

derived in terms of confluent hypergeometric $\mathcal{F}(a,c;x)$, $\mathcal{F}(a-1,c-2;x)$ *and cylindrical* $\mathcal{C}_{\nu}(y)$*,* $\mathcal{C}_{\nu-1}(y)$ *functions, provided* $a = c/2 - jk$ *– complex,* $c = 3$ *– real,*

$$x = x_t = \rho_t q_t x_0,\ y = y_t = \rho_t q_{dt} z_0,$$
$$q_t = q_t(\varepsilon_{rt},\varepsilon_r,k,\alpha_t,\alpha),\ q_{dt} = q_{dt}(\varepsilon_{dt},\varepsilon_r,k,\alpha),$$

$x_0 = jz_0$ *– positive purely imaginary,* z_0 *– real, positive,* α*,* ε_r*,* $k = j(a - c/2)$ *– parameters, relevant to anyone of the ferrite layers, (*k *– real),* $\nu = 1$*,* $\varepsilon_{rl} > 0$*,* $\varepsilon_{dg} > 0$*,* $0 < \rho_t \leq 1$*, then the infinite sequences of numbers* $\left\{\xi_{k,n}^{(c)}\right\}$*,* $\left\{|k|\xi_{k,n}^{(c)}\right\}$ *and* $\left\{|a|\xi_{k,n}^{(c)}\right\}$ *are convergent for* $|k| \to +\infty$ *at least for the one of the (or for both) signs of* k *(*n *– fixed). The limit of the first sequence is zero and the limit of the second and third ones is the same. It equals the finite positive real number* L *where*

$$L\big(c,\varepsilon_{r1},\varepsilon_{r2},...,\varepsilon_{rl},\varepsilon_{d1},\varepsilon_{d2},...,\varepsilon_{dg},$$

$\rho_1, \rho_2, ..., \rho_s, \alpha_1, \alpha_2, ..., \alpha_l, n)$.

If L *exists for* $k > 0$ *and* $k < 0$*, it holds:*

$$\lim_{k \to \pm\infty} |k| \xi_{k,n}^{(c)} (\varepsilon_{r1}, \varepsilon_{r2}, ..., \varepsilon_{rl}, \varepsilon_{d1}, \varepsilon_{d2}, ..., \varepsilon_{dg}, \rho_1, \rho_2, ..., \rho_s, \alpha_1, \alpha_2, ..., \alpha_l) = L_{\pm}(c, \varepsilon_{r1}, \varepsilon_{r2}, ..., \varepsilon_{rl}, \varepsilon_{d1}, \varepsilon_{d2}, ..., \varepsilon_{dg}, \rho_1, \rho_2, ..., \rho_s, \alpha_1, \alpha_2, ..., \alpha_l, n), \quad (6)$$

$$\lim_{k \to \pm\infty} |a| \xi_{k,n}^{(c)} (\varepsilon_{r1}, \varepsilon_{r2}, ..., \varepsilon_{rl}, \varepsilon_{d1}, \varepsilon_{d2}, ..., \varepsilon_{dg}, \rho_1, \rho_2, ..., \rho_s, \alpha_1, \alpha_2, ..., \alpha_l) = L_{\pm}(c, \varepsilon_{r1}, \varepsilon_{r2}, ..., \varepsilon_{rl}, \varepsilon_{d1}, \varepsilon_{d2}, ..., \varepsilon_{dg}, \rho_1, \rho_2, ..., \rho_s, \alpha_1, \alpha_2, ..., \alpha_l, n). \quad (7)$$

Corollaries: Let k_1 and k_2 be any two positive or negative real numbers such that $|k_1|$ and $|k_2|$ are large and $\operatorname{sgn} k_1 \equiv \operatorname{sgn} k_2$. Let $\xi_{k_1,n}^{(c)}$ and $\xi_{k_2,n}^{(c)}$ be the respective roots of eqn. (5) for given set of parameters $\varepsilon_{r1}, \varepsilon_{r2}, ..., \varepsilon_{rl}, \varepsilon_{d1}, \varepsilon_{d2}, ..., \varepsilon_{dg}, \rho_1, \rho_2, ..., \rho_s, \alpha_1, \alpha_2, ..., \alpha_l$ and n, $(c = 3)$. Then at least for one of the (or for both) signs of k it holds: i) $\xi_{k_1,n}^{(c)} \approx 10^{-h} \xi_{k_2,n}^{(c)}$ (if $|k_1| = 10^h |k_2|$ and h is a positive or negative integer or zero); ii) $k_1 / k_2 \approx \xi_{k_2,n}^{(c)} / \xi_{k_1,n}^{(c)}$; iii) $\xi_{k_1,n}^{(c)} / k_2 \approx \xi_{k_2,n}^{(c)} / k_1$; iv) $|k_1| \xi_{k_1,n}^{(c)} \approx |k_2| \xi_{k_2,n}^{(c)} \approx L$; v) $\xi_{k,n}^{(c)} \approx L / |k|$; and vi) $\xi_{k,n}^{(c)} / |k| \approx L / |k|^2$, ($|k| \to +\infty$ and L is a finite positive real number). Similar properties possess also the moduli $|a_1|$ and $|a_2|$ of parameter a, corresponding to k_1 and k_2, introduced above.

B. Hypothesis for the Phase Characteristics

<u>Hypothesis 2</u>: *For any stratified circular structure, containing at least one azimuthally magnetized ferrite layer at least one (or two) envelope curve(s), relevant to the one of the (or to both) signs of* M_r*, restricts (restrict) the* $\bar{\beta}(\bar{r}_0)$*– phase characteristics of normal* TE_{01} *mode with* α *as parameter from the one of their sides. The other side of characteristics is fixed by the cut-off frequencies* $\bar{r}_{0cr}$.

Corollaries: Transmission takes place in a restricted frequency band(s) at least for the one of the (or for both) signs of M_r. The co-ordinates of the points, forming the envelopes are $\bar{r}_{0en\pm} = L_{\pm} / \left(|\alpha_{en\pm}| \sqrt{1 - \alpha_{en\pm}^2} \right)$ and $\bar{\beta}_{en\pm} = \sqrt{1 - \alpha_{en\pm}^2}$.

C. Hypothesis for the Differential Phase Shift

<u>Hypothesis 3</u>: *The differential phase shift* $\Delta\bar{\beta} = \bar{\beta}_- - \bar{\beta}_+$ *produced in each area of phaser operation by any stratified circular waveguide, containing at least one azimuthally magnetized ferrite layer which sustains normal* TE_{01} *mode, can directly be computed in normalized form from the structure and material parameters, using anyone of the formulae:*

$$\Delta\bar{\beta} = A|\alpha|, \quad (8)$$

$$\Delta\bar{\beta} = B / \bar{r}_0, \quad (9)$$

$$\Delta\bar{\beta} = (C / \bar{r}_0)|\alpha|, \quad (10)$$

in which $\Delta\bar{\beta} = \Delta\beta / \left(\beta_0 \sqrt{\varepsilon_r}\right)$.

The factors A*,* B*,* C*, being a function of pa-rameters*

$$(\varepsilon_{r1}, \varepsilon_{r2}, ..., \varepsilon_{rl}, \ \varepsilon_{d1}, \varepsilon_{d2}, ..., \varepsilon_{dg}, \rho_1, \rho_2, ..., \rho_s, \ \alpha_1, \alpha_2, ..., \ \alpha_l, \ \bar{r}_0)$$

are determined by an iterative method, consisting in a repeated numerical solution of the complex characteristic equation of the configuration, followed by a high-accuracy calculation of the phase constant for altered parameters and both sings of M_r.

Corollaries: If the factors A, B, C and the areas of application of the formulae are known in each case, it is enough to predict the phaser operation of any structure. The accuracy of results obtained, applying the formulae, depends on the density of the network of parameters for which the factors are tabulated and deviates from the exact ones with a few percents.

IV. Substantiation and Computational Problems

The truthfulness of statements III.A and III.B assigned to eqn. (1) has recently been proved for $k < 0$ [14] and that of III.C has been discussed earlier [11]. In the case considered affirmation III.A is valid also for c – restricted positive integer [14], as well as for c – arbitrary real number [16]. Table I manifests the effect of k on $\xi_{k,1}^{(c)}$, $|k|\xi_{k,1}^{(c)}$ and $|a|\xi_{k,1}^{(c)}$, $\xi_{k,n}^{(c)}$ – roots of eqn. (1), for c = 1,5,10. If $\lim_{k \to -\infty} |k| \xi_{k,n}^{(c)} = L(c,n)$, provided c = 1,2,…10 and $n = 1$, then $L(c,n)$ = 1.4457964906, 3.6704926603, 6.5936541063, 10.1766164533, 14.3957352232, 19.2347320785, 24.6815681106, 30.7269000369, 37.3632201944, 44.5843352790. Fig. 2 illustrates the dependence of $L(c,n)$ on c with n as parameter. The numerical analysis of eqn. (2) shows that assumptions III.A and III.B hold both for $k < 0$ and $k > 0$ (cf. Table II and Fig. 1a–c). Table III and Fig. 3 demon-strate the influence of parameters involved on L_+ and L_- numbers where $\lim_{k \to \pm\infty} |k| \xi_{k,n}^{(c)} (\varepsilon_r, \varepsilon_d, \rho, \alpha) = L_{\pm}(c, \varepsilon_r, \varepsilon_d, \rho, \alpha, n)$, $\xi_{k,n}^{(c)}(\varepsilon_r, \varepsilon_d, \rho, \alpha)$ – roots of eqn. (2). Assertion III.C is true in two areas whose specification needs special investigation [13]. Table IV lists the factors

A, B, C for normal TE_{01} mode as a function of ρ and $\bar{r}_0$ in case $\varepsilon_r = 1$, $\varepsilon_d = 1,5,10$ and $|\alpha| = 0.01$.

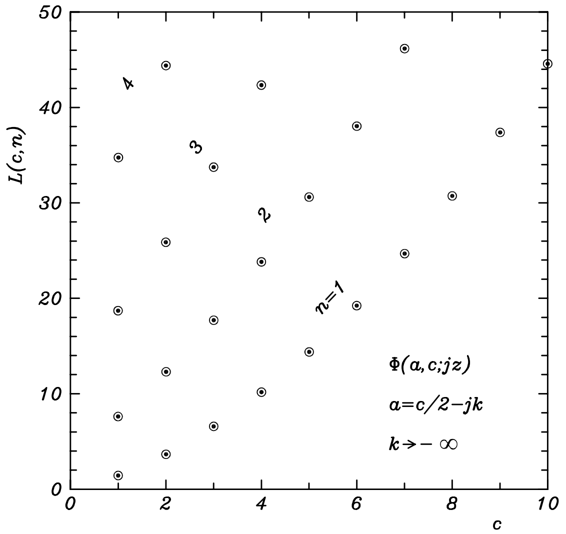


Fig. 2. $L(c,n)$ numbers versus c for values of the natural number $n = 1$ to 4

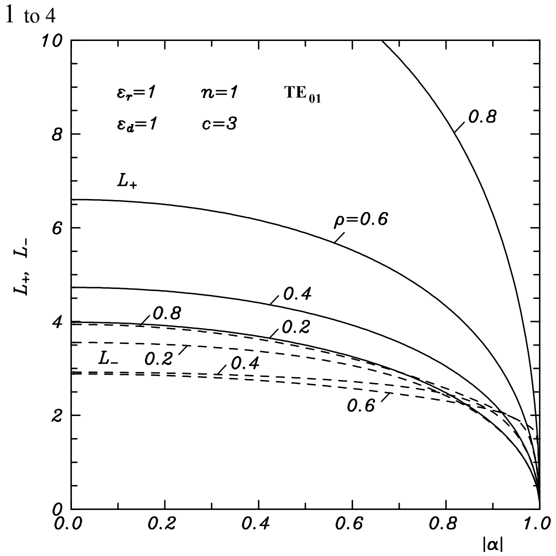


Fig. 3. L_+ and L_- numbers for eqn. (2) versus $|\alpha|$ with ρ as parameter, assuming $\varepsilon_r = 1$, $\varepsilon_d = 1$, $c = 3$ and $n = 1$.

The L numbers are found, employing the procedure, described in Section II.B with $|k| = 10^5$. To determine the quantities A, B, C for fixed $\{|\alpha|, \bar{r}_0\}$, resp. $\{\varepsilon_r, \varepsilon_d, \rho, |\alpha|, \bar{r}_0\}$, we extend it. First, eqns. (1), (2) are solved with respect to x_0 (z_0) for the set $\{\alpha, k\}$, $\{\varepsilon_r, \varepsilon_d, \rho, \alpha, k\}$, resp., varying k in an arbitrary interval $\Delta k^{(0)} = \left[k_{left}^{(0)}, k_{right}^{(0)}\right]$ with an arbitrary step $\delta k^{(0)}$ (e.g. $k_{left}^{(0)} = 1.10^{-3}$, $k_{right}^{(0)} = 2.10^{-3}$, $\delta k^{(0)} = 10^{-4}$), yielding the relevant values $\bar{r}_0^{(0)}$ and $\bar{\beta}^{(0)}$ from expressions (3), (4). The limits of $\Delta k^{(0)}$ are changed till we get $\bar{r}_0 \in \Delta\bar{r}_0^{(0)}$, $\Delta\bar{r}_0^{(0)} = \left[\bar{r}_{0left}^{(0)}, \bar{r}_{0right}^{(0)}\right]$. (The endpoints of $\Delta\bar{r}_0^{(0)}$ correspond to the ones of $\Delta k^{(0)}$). The closest larger and smaller computed values of guide radius are accepted as first approximations $\bar{r}_{0left}^{(1)}$, $\bar{r}_{0right}^{(1)}$ to $\bar{r}_0$ ($\bar{r}_0 \in \Delta\bar{r}_0^{(1)} \in \Delta\bar{r}_0^{(0)}$, $\Delta\bar{r}_0^{(1)} = \left[\bar{r}_{0left}^{(1)}, \bar{r}_{0right}^{(1)}\right]$ and the respective parameters k are taken as left and right ends of the new interval $\Delta k^{(1)} = \left[k_{left}^{(1)}, k_{right}^{(1)}\right]$. Next we fix $\delta k^{(1)} = \delta k^{(0)}/10$. This is reiterated until for the nth approximation it holds $\bar{r}_{0right}^{(n)} - \bar{r}_{0left}^{(n)} < \varepsilon$ (ε – prescribed accuracy, e.g. $\varepsilon = 10^{-10}$). Any of the computed values $\bar{\beta}^{(n)}$ from the interval $\left[\bar{\beta}_{left}^{(n)}, \bar{\beta}_{right}^{(n)}\right]$, pertinent to $\Delta\bar{r}_0^{(n)} = \left[\bar{r}_{0left}^{(n)}, \bar{r}_{0right}^{(n)}\right]$, may be adopted as the one of $\bar{\beta}$ looked for. The procedure is repeated twice for both signs of k, resulting in $\bar{\beta}_-$ and $\bar{\beta}_+$, resp. $\Delta\bar{\beta} = \bar{\beta}_- - \bar{\beta}_+$ for the set $\{|\alpha|, \bar{r}_0\}$, resp. $\{\varepsilon_r, \varepsilon_d, \rho, |\alpha|, \bar{r}_0\}$ chosen. The expressions:

$$A = \Delta\bar{\beta}/|\alpha|, \tag{11}$$

$$B = \Delta\bar{\beta}\bar{r}_0, \tag{12}$$

$$C = \Delta\bar{\beta}\bar{r}_0/|\alpha|, \tag{13}$$

give the factors of interest. The dependence of the latter on certain characteristics of the first structure is slight [11]. Neglecting it permits to develop in this case a straightforward approximate technique for counting up $\Delta\bar{\beta}$ which brings in an insignificant error in the results [16].

The ever increasing number of parameters is a serious obstacle for an exhaustive analysis of the eigenvalue spectrum and phase behaviour of the multilayered configurations (which as expected would be of practical importance in the applications). When the number of strata s (of wave functions involved in the characteristic equations) grows, computational problems appear, especially if $|k|$ gets large. For very large s this could lead eventually to the necessity of leaving off the CHFs based boundary-value approach and searching for new more sophisticated numerical methods for solution of the corresponding propagation tasks.

TABLE I

VALUES OF THE FIRST POSITIVE PURELY IMAGINARY ROOTS $\xi_{k,1}^{(c)}$ OF EQN. (1) AND OF THE PRODUCTS $|k|\xi_{k,1}^{(c)}$ AND $|a|\xi_{k,1}^{(c)}$ FOR LARGE NEGATIVE k IN CASE $a = c/2 - jk$ AND $c = 1,5,10$

k	$\xi_{k,1}^{(c)}$	$\|k\|\xi_{k,1}^{(c)}$	$\|a\|$	$\|a\|\xi_{k,1}^{(c)}$
$c = 1$				
-10 000	(-4) 1.44579 64895	**1.44579 64**896	10000.00001 25000	**1.44579 649**14
-20 000	(-5) 7.22898 24521	**1.44579 6490**4	20000.00000 62500	**1.44579 6490**9
-40 000	(-5) 3.61449 12266	**1.44579 64906**	40000.00000 31250	**1.44579 6490**7
-60 000	(-5) 2.40966 08177	**1.44579 64906**	60000.00000 20833	**1.44579 6490**7
-80 000	(-5) 1.80724 56133	**1.44579 64906**	80000.00000 15625	**1.44579 64906**
-100 000	(-5) 1.44579 64906	**1.44579 64906**	100000.00000 12500	**1.44579 64906**
$c = 5$				
-10 000	(-3) 1.43957 34963	**14.39573** 49631	10000.00031 24999	**14.39573 5**4130
-20 000	(-4) 7.19786 75800	**14.39573 5**1602	20000.00015 62499	**14.39573 52**726
-40 000	(-4) 3.59893 38023	**14.39573 52**094	40000.00007 81249	**14.39573 52**375
-60 000	(-4) 2.39928 92030	**14.39573 52**185	60000.00005 20833	**14.39573 52**310
-80 000	(-4) 1.79946 69027	**14.39573 522**17	80000.00003 90625	**14.39573 522**87
-100 000	(-4) 1.43957 35223	**14.39573 522**32	100000.00003 12500	**14.39573 522**77
$c = 10$				
-10 000	(-3) 4.45843 32167	**44.58433** 21678	10000.00124 99999	**44.58433** 77408
-20 000	(-3) 2.22921 67262	**44.58433** 45247	20000.00062 49999	**44.58433 5**9179
-40 000	(-3) 1.11460 83778	**44.58433 5**1140	40000.00031 24999	**44.58433 5**4623
-60 000	(-4) 7.43072 25371	**44.58433 5**2231	60000.00020 83333	**44.58433 5**3779
-80 000	(-4) 5.57304 19076	**44.58433 5**2613	80000.00015 62499	**44.58433 5**3484
-100 000	(-4) 4.45843 35278	**44.58433 5**2790	100000.00012 50000	**44.58433 5**3347

TABLE II

VALUES OF THE FIRST POSITIVE PURELY IMAGINARY ROOTS $\xi_{k,1}^{(c)}$ OF EQN. (2) AND OF THE PRODUCTS $|k|\xi_{k,1}^{(c)}$ AND $|a|\xi_{k,1}^{(c)}$ FOR LARGE $|k|$ IN CASE $a = c/2 - jk$, $c = 3$, ASSUMING $\rho = 0.6$, $|\alpha| = 0.4$, $\varepsilon_r = 1$ AND $\varepsilon_d = 1,5,10$

k	$\xi_{k,1}^{(c)}$	$\|k\|\xi_{k,1}^{(c)}$	$\|a\|\xi_{k,1}^{(c)}$	k	$\xi_{k,1}^{(c)}$	$\|k\|\xi_{k,1}^{(c)}$	$\|a\|\xi_{k,1}^{(c)}$
$\varepsilon_r = 1$, $\varepsilon_d = 1$							
-10 000	(-4) 2.74742 70808	**2.74742 708**08	**2.74742** 71117	10 000	(-4) 6.16470 15118	**6.16470 15**118	**6.16470 15**811
-20 000	(-4) 1.37371 35415	**2.74742 708**29	**2.74742 70**906	20 000	(-4) 3.08235 07594	**6.16470 15**188	**6.16470 15**361
-40 000	(-5) 6.86856 77086	**2.74742 7083**4	**2.74742 708**54	40 000	(-4) 1.54117 53801	**6.16470 152**05	**6.16470 152**49
-60 000	(-5) 4.57904 51392	**2.74742 7083**5	**2.74742 708**44	60 000	(-4) 1.02745 02535	**6.16470 152**09	**6.16470 152**28
-80 000	(-5) 3.43428 38545	**2.74742 7083**6	**2.74742 708**41	80 000	(-5) 7.70587 69012	**6.16470 152**10	**6.16470 152**20

k	$\xi_{k,1}^{(c)}$	$\lvert k\rvert \xi_{k,1}^{(c)}$	$\lvert a\rvert \xi_{k,1}^{(c)}$	k	$\xi_{k,1}^{(c)}$	$\lvert k\rvert \xi_{k,1}^{(c)}$	$\lvert a\rvert \xi_{k,1}^{(c)}$
-100 000	(-5) 2.74742 70836	**2.74742 7083**6	**2.74742 7083**9	100 000	(-5) 6.16470 15211	**6.16470 1521**1	**6.16470 1521**8
$\varepsilon_r = 1,\ \varepsilon_d = 5$							
-10 000	(-5) 8.87089 22715	**0.88708 9227**1	**0.88708 92**371	10 000	(-4) 1.04184 56590	**1.04184 565**90	**1.04184 56**707
-20 000	(-5) 4.43544 61366	**0.88708 92273**	**0.88708 922**98	20 000	(-5) 5.20922 82959	**1.04184 56592**	**1.04184 56**621
-40 000	(-5) 2.21772 30684	**0.88708 92273**	**0.88708 922**80	40 000	(-5) 2.60461 41481	**1.04184 56592**	**1.04184 565**99
-60 000	(-5) 1.47848 20456	**0.88708 92273**	**0.88708 9227**7	60 000	(-5) 1.73640 94321	**1.04184 56592**	**1.04184 5659**6
-80 000	(-5) 1.10886 15342	**0.88708 92273**	**0.88708 9227**5	80 000	(-5) 1.30230 70740	**1.04184 56592**	**1.04184 5659**4
-100 000	(-6) 8.87089 22727	**0.88708 92273**	**0.88708 9227**4	100 000	(-5) 1.04184 56592	**1.04184 56592**	**1.04184 5659**3
$\varepsilon_r = 1,\ \varepsilon_d = 10$							
-10 000	(-5) 6.17114 70323	**0.61711 4703**2	**0.61711 47**102	10 000	(-5) 6.87626 32753	**0.68762 6327**5	**0.68762 63**353
-20 000	(-5) 3.08557 35167	**0.61711 4703**3	**0.61711 470**51	20 000	(-5) 3.43813 16383	**0.68762 63277**	**0.68762 632**96
-40 000	(-5) 1.54278 67585	**0.61711 47034**	**0.61711 4703**8	40 000	(-5) 1.71906 58192	**0.68762 63277**	**0.68762 632**82
-60 000	(-5) 1.02852 45056	**0.61711 47034**	**0.61711 4703**6	60 000	(-5) 1.14604 38795	**0.68762 63277**	**0.68762 6327**9
-80 000	(-6) 7.71393 37925	**0.61711 47034**	**0.61711 4703**5	80 000	(-6) 8.59532 90963	**0.68762 63277**	**0.68762 6327**8
-100 000	(-6) 6.17114 70340	**0.61711 47034**	**0.61711 47034**	100 000	(-6) 6.87626 32767	**0.68762 63277**	**0.68762 63277**

TABLE III

VALUES OF QUANTITIES L_- AND L_+ FIGURED FROM EQN. (2) AS A FUNCTION OF ρ AND $\lvert\alpha\rvert$ IN CASE $a = c/2 - jk$, $c = 3$ AND $n = 1$, ASSUMING $\varepsilon_r = 1$ AND $\varepsilon_d = 1,5,10$

$\lvert\alpha\rvert$ \ ρ	0.2		0.4		0.6		0.8	
	L_-	L_+	L_-	L_+	L_-	L_+	L_-	L_+
$\varepsilon_r = 1,\ \varepsilon_d = 1$								
0.2	3.5111 0846	3.9142 5972	2.9014 2227	4.6501 4191	2.8496 4052	6.4984 3976	3.8628 6509	12.3814 0254
0.4	3.3675 7877	3.6848 9190	2.8397 8279	4.4014 6650	2.7474 2708	6.1647 0152	3.6252 3420	11.7515 3632
0.6	3.0623 3010	3.2520 3135	2.7202 5279	3.9241 2510	2.5715 9875	5.5228 1073	3.2075 2038	10.5456 6688
0.8	2.4194 9043	2.4794 1859	2.4554 5605	3.0444 9929	2.3061 2597	4.3292 5482	2.5741 7211	8.3109 3878
$\varepsilon_r = 1,\ \varepsilon_d = 5$								
0.2	0.3760 1710	0.3794 21099	0.4002 1981	0.4174 4054	0.4900 1483	0.5331 4437	0.8195 9956	0.9317 2318
0.4	0.6986 8964	0.7091 39418	0.7415 5971	0.7995 4995	0.8870 8923	1.0418 4566	1.4260 6290	1.8440 5258
0.6	0.9102 3957	0.9238 75270	0.9830 4444	1.0730 7612	1.1536 7481	1.4305 5163	1.7466 0085	2.5627 8540
0.8	0.9090 6543	0.9166 08585	1.0398 3181	1.1051 8240	1.2403 3095	1.5214 1909	1.7231 1356	2.7809 8414
$\varepsilon_r = 1,\ \varepsilon_d = 10$								
0.2	0.2517 7596	0.2532 9581	0.2696 1968	0.2773 1223	0.3326 6207	0.3519 5582	0.5609 1908	0.6111 2579
0.4	0.4721 6990	0.4768 9920	0.5072 6151	0.5335 2923	0.6171 1470	0.6876 2633	1.0109 9368	1.2023 9029
0.6	0.6230 2805	0.6293 3803	0.6836 1652	0.7251 7233	0.8246 0230	0.9540 1844	1.2920 3463	1.6796 4170
0.8	0.6316 6883	0.6352 7296	0.7313 9685	0.7621 5469	0.9044 0197	1.0378 1324	1.3349 3228	1.8597 5780

TABLE IV

VALUES OF FACTORS A , B , C AS A FUNCTION OF ρ AND $\bar{r}_0$ IN CASE $a = c/2 - jk$, $c = 3$ AND $n = 1$, ASSUMING $\varepsilon_r = 1$ AND $\varepsilon_d = 1{,}5{,}10$ AND $|\alpha| = 0.01$

ρ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\varepsilon_r = 1$, $\varepsilon_d = 1$										
$\bar{r}_0$	4									
A	0.01459	0.10562	0.30093	0.55953	0.79194	0.91006	0.87830	0.73651	0.58102	0.51350
B	0.00058	0.00422	0.01203	0.02238	0.03167	0.03640	0.03513	0.02946	0.02324	0.02054
C	0.05836	0.42249	1.20373	2.23813	3.16777	3.64026	3.51322	2.94606	2.32408	2.05402
$\varepsilon_r = 1$, $\varepsilon_d = 5$										
$\bar{r}_0$	1.75		1.8	1.85	2	2.25	3	4		
A	0.03222	0.21329	0.53396	0.92266	1.23424	1.40669	1.21346	0.97252	0.59889	0.51350
B	0.00056	0.00373	0.00961	0.01706	0.02468	0.03165	0.03640	0.03890	0.02395	0.02054
C	0.05639	0.37326	0.96113	1.70692	2.46848	3.16505	3.64038	3.89011	2.39557	2.05402
$\varepsilon_r = 1$, $\varepsilon_d = 10$										
$\bar{r}_0$	1.25			1.35	1.5	1.61	1.95	2.6	3.8	4
A	0.04489	0.29344	0.75537	1.20270	1.53906	1.91185	1.87055	1.47050	0.67514	0.51350
B	0.00056	0.00366	0.00944	0.01623	0.02308	0.03078	0.03647	0.03823	0.02565	0.02054
C	0.05611	0.36680	0.94421	1.62365	2.30860	3.07807	3.64757	3.82331	2.56554	2.05402

V. CONCLUSION

A guess is made on the existence of some properties of the stratified circular waveguides, containing an arbitrary number of coaxial azimuthally magnetized ferrite layers which sustain normal TE_{0n} modes. A numerical and graphical substantiation of the statements is performed for two simple geometries of the class considered, utilizing CHFs and iterative schemes. An extension of the work would reveal further interesting features of geometries treated and would allow to find the criteria for their operation as phase shifters.

REFERENCES

[1] D.M. Bolle and G.S. Heller, "Theoretical considerations on the use of circularly symmetric TE modes for digital ferrite phase shifters," *IEEE Trans. Microwave Theory Tech*., vol. MTT-13, no. 4, pp. 421-426, July 1965. See also D.M. Bolle and N. Mohsenian: Correction, *IEEE Trans. Microwave Theory Tech*., vol. MTT-34, no. 4, p. 427, Apr. 1986.

[2] P.J.B. Clarricoats and A.D. Olver, "Propagation in aniso-tropic radially stratified circular waveguides," *Electron. Lett.,* vol. 2, no. 1, pp.37-38, Jan. 1966.

[3] W.J. Ince and D.H. Temme, "Phasers and time delay elements", in *Advances in Microwaves*, Young L., Ed., vol. 4, New York, London: Academic Press, pp. 1-189, 1969.

[4] W.J. Ince and G.N. Tsandoulas, "Modal inversion in cir-cular waveguides." Part II. "Application to latching non-reciprocal phasers," *IEEE Trans. Microwave Theory Tech*., vol. MTT-19, no. 4, pp. 393-400, Apr. 1971.

[5] F.J. Bernues and D.M. Bolle, "The digital twin-ferrite-toroid circular waveguide phaser," *IEEE Trans. Microwave Theory Tech*., vol. MTT-21, no. 12 pp. 842-845, Dec. 1973.

[6] O. Parriaux and F.E. Gardiol, "Propagation in circular waveguide loaded with an azimuthally magnetized ferrite tube," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-25, no. 3, pp. 221-224, Mar. 1977.

[7] S.N. Samaddar, "Special functions associated with azimuthally magnetized ferrite rod phase shifters," *J. Appl. Phys.*, vol. 50, no. 1, pp. 518-520, Jan. 1979.

[8] G.A. Red'kin, A.E. Mudrov and V.A. Meshcheriakov, "Phase shifter with azimuthally magnetized ferrite samples," in *Proc. Vth Int. Conf. Microwave Ferrites*, Vilnius, vol. 4, pp. 170-175, Oct. 5-10, 1980 (in Russian).

[9] A.J. Baden-Fuller, *Ferrites at Microwave Frequencies.* IEE Electromagnetic Waves Series 23. London: Peter Peregrinus, 1987.

[10] K.P. Ivanov and G.N. Georgiev, "Azimuthally magnetiz-ed circular ferrite waveguides," in *Ferrite Phase Shifters and Control Devices*, J. Helszajn, Ed. London, U.K.: McGraw-Hill, 1989, ch. 14, pp. 262-288.

[11] G.N. Georgiev and M.N. Georgieva-Grosse, "Formulae for differential phase shift computation in an azimuthally magnetized circular ferrite waveguide," in *Proc. Millennium Conf. Antennas Propagation AP2000*, Davos, Switzerland, Apr. 9-14, 2000, paper 1002, in CD.

[12] G.N. Georgiev and M.N. Georgieva-Grosse, "Some new properties of the circular waveguides with azimuthally

magnetized ferrite," in *Proc. 25th ESA Antenna Workshop on Satellite Antenna Technology,* ESA/ESTEC, Noordwijk, The Netherlands, Sept. 18-20, 2002, pp. 601-608.

[13] G.N. Georgiev and M.N. Georgieva-Grosse, "Conditions for phaser operation of the circular waveguides with azimuthally magnetized ferrite", in *Proc. 26th ESA Antenna Technol. Worksh. Satell. Antenna Modell. Des. Tools,* ESA/ESTEC, Noordwijk, The Netherlands, Nov. 12-14, 2003, pp. 359-366.

[14] G.N. Georgiev and M.N. Georgieva-Grosse, "A new property of the complex Kummer function and its applica-tion to waveguide propagation," *IEEE Antennas Wireless Propagation Lett.*, vol. AWPL-2, pp. 306-309, Dec. 2003.

[15] G.N. Georgiev and M.N. Georgieva-Grosse, "Several hypotheses in the confluent hypergeometric functions based theory of the azimuthally magnetized circular ferrite waveguides", in *Proc. East-West Worksh. Adv. Techn. Electromagn.*, Warsaw, Poland, May 20-21, 2004, pp.197-204.

[16] G.N. Georgiev and M.N. Georgieva-Grosse, "An approxi-mate method for analysis of azimuthally magnetized circular ferrite waveguide phase shifter", in *Proc. 2006 1st Europ. Conf. Anten. Propagat. EUCAP 2006,* Nice, France, Nov. 6-10, 2006, in CD.

[17] F.G. Tricomi, *Fonctions Hypergéométrique Confluentes*, Paris, France: Gauthier-Villars, 1960.

[18] N.M. Sovetov and M.E. Averbuch, *Difference Bessel Functions and Their Application in Technics,* Saratov, USSR: Saratov Univ. Publ. House, 1968, (in Russian).