

Evaluation of Unsteady Non-Harmonic Fields in Microwave Devices.

I. Decomposition in the Partial Modes

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Introduction

A self-consistent theoretical analysis of unsteady non-harmonic electromagnetic fields excited in dispersive electrodynamic systems of microwave devices by electron beams is urgent today because of the EMC problem, the digital telecommunications systems and the UWB EMP generators development, *etc.* So-called *spectral approach* to simulation of the devices is described in [1] (new unorthodox terms are in italics). The *spectral model* is defined there as a transient algorithm, which correctly takes into account the nonlinear interaction of all time harmonics of the field with the beam in a frequency continuum, and is joined with algorithms of the Fourier synthesis of the input signals and the Fourier analysis of the output data.

A time domain evaluation of arbitrary non-harmonic fields in the tube may be performed in the best way by FDTD and FETD methods, but they are too computationally intensive, especially if the tube has complicated structure. The variable separation method (expansion of the field in a mode series) might be more effective [2]. Besides the eigenfunctions, which are not quite suitable for the spectral simulations of the matched line devices because their spectrum is continual there, other modes may be used as a base for the decomposition, depending on a preferred approximation of the field: (i) discrete, (ii) lattice, (iii) continuous.

The Discrete Approximation

This approximation [2] is based on expansion of the RF line field in the *partial modes*. A *generic potential* $\mathcal{A} = \phi/c | \vec{A}$ [$\phi(t,x,y,z)$ and $\vec{A}(t,x,y,z)$ are the scalar and the vector potentials in the Lorentz gauge respectively, V·s/m] is a solution of the inhomogeneous wave equation

$$\nabla^2 \mathcal{A} - \frac{1}{c^2} \frac{\partial^2 \mathcal{A}}{\partial t^2} = -\mu_0 j$$

with homogeneous boundary conditions. The line is assumed empty (filled with the vacuum). $j = c\rho | \vec{j}$ is a *generic current density* [$\rho(t,x,y,z)$ and $\vec{j}(t,x,y,z)$ are the charge and the current densities respectively]. Electric $\vec{E}(t,x,y,z)$ and magnetic $\vec{B}(t,x,y,z)$ fields are derived from \mathcal{A} as $\vec{E} = -\partial \vec{A} / \partial t - \vec{\nabla} \phi$; $\vec{B} = \vec{\nabla} \times \vec{A}$.

If the generic potential has a finite spectrum in the wavenumber domain, one can be evaluated as a finite series of the partial modes in the Hilbert L^2 space (generally, this is not a Fourier series):

$$\mathcal{A}(t,x,y,z) = \sum_q \mathfrak{A}_{pq}(x,y,z) \mathbf{u}_{pq}(t)$$

where $\mathfrak{A}_{pq}(x,y,z)$ is a vector of N the partial modes for a q -th wave mode (passband) of the line; $\mathbf{u}_{pq}(t)$ is a vector of N arbitrary instantaneous values of the partial modes. The vector \mathfrak{A}_{pq} is a solution of the matrix Helmholtz equation

$$\nabla^2 \mathfrak{A}_{pq} + [k_{pq}^2] \mathfrak{A}_{pq} = 0$$

with homogeneous boundary conditions, which longitudinally localizes all items of the sought vector. $[k_{pq}^2]$ is a $N \times N$ matrix of the line *intervalues* (squared mutual wavenumbers of the partial modes) in a q -th passband.

On the other hand, the vectors of the partial modes and the normal eigenmodes $\mathfrak{A}_{rq}(x,y,z)$ in the line q -th passband may be related as $\mathfrak{A}_{rq} = [F] \mathfrak{A}_{pq}$; $\mathfrak{A}_{pq} = [F]^{-1} \mathfrak{A}_{rq}$ where $[F]$ is a $N \times N$ *form-matrix* of the line eigenmodes. In contrast to lumped-element circuits, this matrix may be arbitrary nonsingular for electrodynamic lines. However, only the matrices localizing all the partial modes in the longitudinal (z) direction are of interest. E.g., for the closed-loop line and even N this may be:

$$\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \cos \Delta\varphi_1 & \cos 2\Delta\varphi_1 & \dots & \cos(N-1)\Delta\varphi_1 \\ 1 & \cos \Delta\varphi_2 & \cos 2\Delta\varphi_2 & \dots & \cos(N-1)\Delta\varphi_2 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & -1 & 1 & \dots & (-1)^{N-1} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \sin \Delta\varphi_2 & \sin 2\Delta\varphi_2 & \dots & \sin(N-1)\Delta\varphi_2 \\ 0 & \sin \Delta\varphi_1 & \sin 2\Delta\varphi_1 & \dots & \sin(N-1)\Delta\varphi_1 \end{pmatrix}$$

where $\Delta\varphi_m = 2\pi m/N$ is the m -th normal eigenmode phase shift between adjacent partial modes ($m = 0, 1, \dots, N/2$). Each of N the partial modes may be treated as a “cloud” of the field that oscillates as a single whole, i.e., in an equal phase $\varphi_n = n\Delta\varphi_m$ ($n = 0, 1, \dots, N-1$). After such definition of the partial modes, the usual theory of lumped-element oscillating systems with N degrees of freedom may be applied, producing, e.g.

$$\begin{aligned} [k_{rq}^2] &= [F][k_{pq}^2][F]^{-1}; \\ [k_{pq}^2] &= [F]^{-1}[k_{rq}^2][F] \end{aligned}$$

where $[k_{rq}^2]$ is the diagonal $N \times N$ matrix of the line eigenvalues (squared wavenumbers of the normal eigenmodes) in a q -th passband.

A $N \times N$ matrix $[\tilde{W}_{pq}]$ of the partial mode unit mutual *pseudoenergies* is defined as

$$[\tilde{W}_{pq}] = \frac{\varepsilon_0}{2} \int_{\Delta Z} dz \int_{S_{\perp}} dx dy (\mathfrak{A}_{pq})(\mathfrak{A}_{pq})^T$$

where (\mathfrak{A}_{pq}) is a column of N the partial modes; ΔZ is the device length; S_{\perp} is the device transverse (x,y) section. The term “pseudoenergy” is given as this value (in $\text{J}\cdot\text{s}^2$) is calculated for the generic potential under the formula similar to one for the electric field energy. The unit partial pseudoenergies relate to the diagonal $N \times N$ matrix $[\tilde{W}_{rq}]$ of the normal eigenmode unit pseudoenergies as

$$\begin{aligned} [\tilde{W}_{rq}] &= [F][\tilde{W}_{pq}][F]^T; \\ [\tilde{W}_{pq}] &= [F]^{-1}[\tilde{W}_{rq}][F]^{-1T}. \end{aligned}$$

The vector \mathfrak{A}_{rq} must be normalized before the transforms. The amplitude normalization (e.g., $\max|\mathfrak{A}_{rqm}| \equiv 1 \text{ V}\cdot\text{s/m}$) is the most plain. For an orthogonal form-matrix, the energy normalization ($\tilde{W}_{rqm} = 1 \text{ J}\cdot\text{s}^2$ if $m=0$ and $m=N/2$; otherwise $\tilde{W}_{rqm} \equiv 0.5 \text{ J}\cdot\text{s}^2$) orthogonalizes all partial modes (diagonalizes the matrix $[\tilde{W}_{pq}]$).

The truncated Gaussian normalization [e.g. $\max|\mathfrak{A}_{rqm}| = \exp(-4\psi m^2/N^2) \text{ V}\cdot\text{s/m}$, where $\psi > 0$] provides extra longitudinal localization of the partial modes, but may increase errors and noise in calculation of the right-hand side of the excitation equation (see also the Part II).

Thus, an expansion of a non-stationary non-harmonic field in a Fourier series in

the line eigenfunctions may be supplemented with a decomposition of one in a non-Fourier series in longitudinally localized *partial functions*. The latter seem to be like the wavelets (see Figs. 3 and 4 of [2]); however, those are not the wavelets, as (generally) non-orthogonal linear combinations of the line eigenfunctions.

The excitation equation for the vector $\mathbf{u}_{pq}(t)$ of the partial mode time factors is:

$$\frac{d^2 \mathbf{u}_{pq}}{dt^2} + 2[\delta_{pq}]^T \frac{d\mathbf{u}_{pq}}{dt} + [\omega_{pq}^2]^T \mathbf{u}_{pq} = \frac{1}{2} \cdot [\tilde{W}_{pq}]^{-1} \int_{\Delta Z} dz \int_{S_{\perp}} dx dy \mathfrak{A}_{pq}(x, y, z) j(t, x, y, z)$$

where $[\omega_{pq}^2] = c^2 [k_{pq}^2]$ and $[\delta_{pq}]$ are $N \times N$ matrices of the squared mutual frequencies and mutual damping factors respectively. The matrix $[\delta_{pq}]$ relates to the $N \times N$ matrix $[\delta_{rq}]$ of the normal eigenmode damping factors caused by the line wall losses as

$$[\delta_{rq}] = [F][\delta_{pq}][F]^{-1};$$

$$[\delta_{pq}] = [F]^{-1}[\delta_{rq}][F].$$

For regular lines, the normal eigenmodes may be treated as complex. The form-matrix also is complex in this case:

$$\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & e^{-i\Delta\varphi_1} & e^{-i2\Delta\varphi_1} & \dots & e^{-i(N-1)\Delta\varphi_1} \\ 1 & e^{-i\Delta\varphi_2} & e^{-i2\Delta\varphi_2} & \dots & e^{-i(N-1)\Delta\varphi_2} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & e^{-i\Delta\varphi_{N-1}} & e^{-i2\Delta\varphi_{N-1}} & \dots & e^{-i(N-1)\Delta\varphi_{N-1}} \end{pmatrix}.$$

Such eigenmodes are related to the real partial modes and vice versa as the discrete Fourier transforms [2].

Nevertheless, the definition of the partial modes as a linear transform of the normal eigenmodes using the form-matrix is more general and flexible. Let us consider a hypothetical longitudinally non-

uniform dispersive line shorted at its endpoints $z=0$ and $z=\Delta Z$. Supposing the longitudinal dependences of the longitudinal wavenumber $\beta_m(z)$ for different eigenmodes describing by the formula

$$\beta_m(z) = \frac{\pi m}{\Delta Z} [1 - (1 - m/N)(1 - 2z/\Delta Z)],$$

as these are shown in Fig. 1 ($N=6$ is chosen small for the clearness), the lowest eigenmode ($m=1$) is the most longitudinally irregular, while the highest one ($m=N-1$) is almost regular (see Fig. 2). When the partial modes of such line are synthesized from the normal eigenmodes using the “regular” form-matrix

$$\begin{pmatrix} \sin \Delta\varphi_1 & \sin 2\Delta\varphi_1 & \dots & \sin(N-1)\Delta\varphi_1 \\ \sin \Delta\varphi_2 & \sin 2\Delta\varphi_2 & \dots & \sin(N-1)\Delta\varphi_2 \\ \dots & \dots & \dots & \dots \\ \sin \Delta\varphi_{N-1} & \sin 2\Delta\varphi_{N-1} & \dots & \sin(N-1)\Delta\varphi_{N-1} \end{pmatrix}$$

that is, in fact, the discrete sine Fourier transform, they are poor longitudinally localized (see Fig. 3 where $N=64$). If a

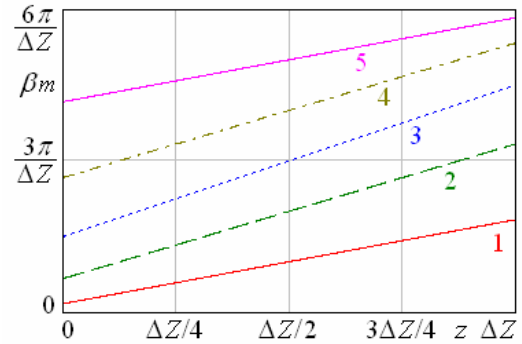


Fig. 1

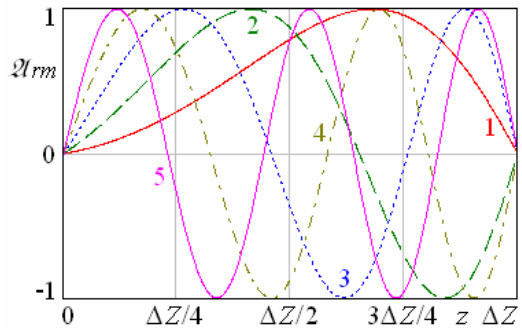


Fig. 2

non-regular form-matrix is used with

$$F_{mn} = \sin \frac{\pi mn}{N} [1 - (1 - m/N)(1 - n/N)]$$

the partial modes become better (see Fig. 4). However, the phase shifts $\Delta\varphi_m$ between different pairs of adjacent partial modes are no more equal.

Conclusion

The decomposition of the line field in the partial modes is useful for the spectral simulations of the traveling-wave devices, where decomposition in the discrete Fourier series is unsuitable to the line eigenfunctions because of their spectrum continuity. In addition to avoidance of the Fourier integral over a continuum of the normal eigenmodes in matched lines, the partial modes are effective also in simulations of the devices

with long and irregular delay lines. Due to the longitudinal localization of the partial modes, the field structure and the electrodynamic parameters of ones depend on the characteristic of a limited longitudinal part of the line. In turn, only limited number of the partial modes may be taken into account in each transverse section of the device.

References

- [1] A. V. Gritsunov, "On a spectral approach to simulation of microwave devices," *J. Comm. Technol. and Electronics*, vol. 49, no. 7, pp. 829-832, July 2004.
- [2] A. V. Gritsunov, "Non-monochromatic fields in a dispersive electrodynamic line. I. The discrete approximation," in *Proc. Fifth IEEE Int. Vacuum Electronics Conf. (IVEC 2004)*, Monterey, CA, 2004, pp. 220-221.

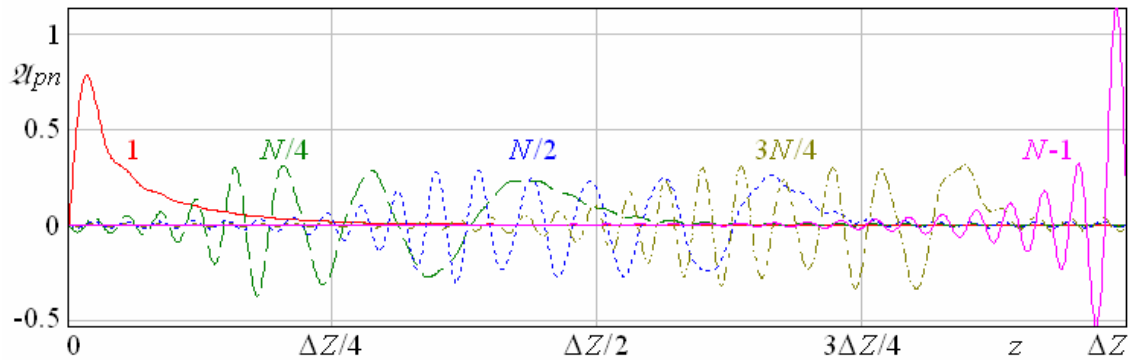


Fig. 3

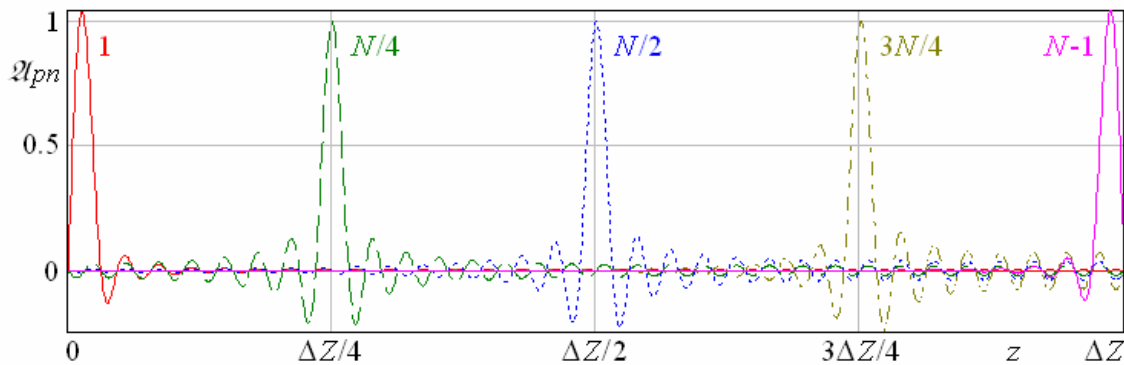


Fig. 4