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MATHEMATICAL METHODS IN ELECTROMAGNETIC THEORY

Structure Functions of Electromagnetic Field Coupling in a Cavity Filled by Resonance-Size Magnetodielectric Spheres*

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ABSTRACT: A technique is suggested to effectively construct the structure functions of electromagnetic fields in a cavity filled by resonance-size magnetodielectric spheres. Expressions have been derived for the tensorial functions representing electromagnetic interactions of the magnetic and electric type in the cavity with spheres. The expressions obtained for the structure functions have been given a theoretical analysis.

The spatially ordered structures involving small homogeneous resonance-size magnetodielectric spheres are characterized by internal magnetic and electric resonances of the spheres and also by structural resonances resulting from sphere-to-sphere coupling, both of magnetic and electric type.

The structural resonances can influence the internal resonances of the spheres and their fine structure. Also, a physical effect is possible where the resonances of the kind are combined.

If such a resonant set of spheres is placed in a metal cavity, then a resonance effect can arise, consisting of the cavity-to-spheres electromagnetic interaction.

The resonance effects in such a structure can be described by introducing the notion of structure functions for the electromagnetic interaction of the cavity and the spheres.

The present paper is aimed at developing a technique for constructing the functions to describe the effects of electromagnetic interaction in a rectangular metal cavity containing a set of small homogeneous resonance-size magnetodielectric spheres, and for analyzing the properties of these functions. It is allowable for this problem that the wavelength were comparable with the spacing between the spheres in the cavity.

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1. PROBLEM FORMULATION AND SOLUTION

Let a rectangular metal cavity contain N spheres of radius a_c, characterized by the permittivity ε_c and permeability μ_c. The spheres and their related values will be labeled by the subscript c, with the 'prime' for selected spheres, i.e. c', with $c, c' \in N$. The cavity walls are specified as the planes $x = 0, x = d;$ $y = 0, y = h;$ and $z = 0, z = l$. Consider the case where the inequality $a_c/\lambda << 1$ holds in the cavity outside the spheres, whereas the resonance condition, $a_c/\lambda_g \sim 1$, might occur inside the sphere (here λ and λ_g are the wavelengths outside and inside the spheres, respectively). The field vectors in the cavity will be represented as $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{i\omega t}$ and $\vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{i\omega t}$. To solve the problem, we need to find the internal fields of the spheres induced by the cavity field. Then, the desired structure functions of the cavity-to-spheres electromagnetic interaction can be constructed, using the expressions obtained.

1.1. Topology of the model cavity with spheres

Let us apply the method of images to represent the metal rectangular cavity filled by magnetodielectric spheres in the form of a spatial lattice of spheres and their own mirror images in the cavity walls, which model will be used to solve the problem. Each of the N spheres in the cavity form its own 3D lattice of mirror images in the cavity walls, which will be referred to below as sublattice c. As a result, the cavity with the spheres can be represented as a complex spatial lattice consisting of N sublattices c. The sublattices c are generated by a configurational representation which in the Cartesian frame takes the form

$$\begin{aligned}
x_{c,s} &= \left[s - 0.5\left\{(-1)^s - 1\right\}\right] d - (-1)^{s-1} x_{c,s=0} \quad \left(s = 0, \pm 1, \pm 2, ..., \pm\infty\right), \\
y_{c,t} &= \left[t - 0.5\left\{(-1)^t - 1\right\}\right] h - (-1)^{t-1} y_{c,t=0} \quad \left(t = 0, \pm 1, \pm 2, ..., \pm\infty\right), \\
z_{c,p} &= \left[p - 0.5\left\{(-1)^p - 1\right\}\right] l - (-1)^{p-1} z_{c,p=0} \quad \left(p = 0, \pm 1, \pm 2, ..., \pm\infty\right),
\end{aligned} \tag{1}$$

where $x_{c,s=0}$, $y_{c,t=0}$ and $z_{c,p=0}$ are coordinates of the node generating the sublattice c and lying inside the domain Eq. (2) which is the internal domain of the cavity,

$$\begin{aligned}
&0 < x_{c,s=0} < d, \\
&0 < y_{c,t=0} < h, \\
&0 < z_{c,p=0} < l.
\end{aligned} \tag{2}$$

The coordinates $x_{c,s}, y_{c,t}$ and $z_{c,p}$ specify positions of the nodes of sublattice c outside the domain Eq. (2), being functions of the coordinates $x_{c,s=0}, y_{c,t=0}$ and $z_{c,p=0}$. The configurational representation Eq. (1) may be regarded as time dependent if $x_{c,s=0}, y_{c,t=0}$ and $z_{c,p=0}$ are certain functions of time. Each node of the spatial sublattice c Eq. (1) associated with an ordered triad of numbers $u = c(p,s,t)$. An isolated node of the sublattic will be denoted $u' = c'(p',s',t')$; a node inside the domain Eq. (2) as $c(p=0, s=0, t=0)$, and an arbitrary node as $c(p,s,t)$.

If the node coordinates within the domain Eq. (2) are varied, then the nodes outside this domain change their positions accordingly, which results in rearrangement of the cells and formation of the spatial configuration of a complex lattice. When the generating node is located at the center of the domain Eq. (2), then d, h and l are the constants of a regular rectangular sublattice along the x, y and z axes, respectively.

With account of Eq. (1) the node separation can be determined as

$$r_{c'(p',s',t'),c(p,s,t)} = r_{u'u} = \sqrt{\left(x_{c',s'} - x_{c,s}\right)^2 + \left(y_{c',t'} - y_{c,t}\right)^2 + \left(z_{c',p'} - z_{c,p}\right)^2}. \qquad (3)$$

The centers of the spheres and of their own mirror images are placed at the nodes of the sublattices Eq. (1) (see Fig.1).

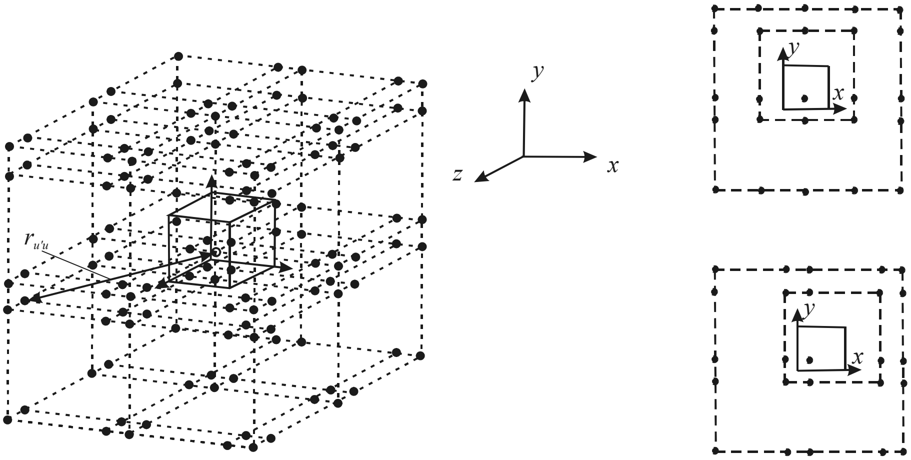

FIGURE 1. The problem geometry and model cavity with a sphere.

Shown in Fig.1 are a lattice model of a metal rectangular cavity with a sphere, which represents a spatial array (a) and several cross-sections of the lattice by the (x, y) plane for different locations of the sphere in the cavity (Here $p = 0, \pm 1, \pm 2;\quad s = 0, \pm 1, \pm 2;$ and $t = 0, \pm 1, \pm 2$).

1.2. Internal field due to the spheres in the cavity

The internal field of the spheres in the cavity can be found from an equation set involving inhomogeneous equations for the spheres and homogeneous equations for their mirror images. The equation set will be constructed, based on the integral equations suggested by Khizhnyak [2] and the solutions obtained in papers [3 and 4].

The inhomogeneous and homogeneous equations for an arbitrary isolated sphere and its image, participating in the algebraic set, a are as follows

$$\begin{aligned}
&\vec{E}_{0c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right)=\left(\frac{\left(\varepsilon_{c'eff}+2\varepsilon_0\right)+\theta_{1c'}^2\varepsilon_{c'eff}+i\theta_{1c'}\left(\varepsilon_{c'eff}+2\varepsilon_0\right)}{3\varepsilon_0 e^{i\theta_{1c'}}}\times\right.\\
&\times\vec{E}^0_{c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right)-\\
&-\sum_{\substack{-\infty\\p}}^{\infty}\sum_{\substack{-\infty\\s}}^{\infty}\sum_{\substack{-\infty\\t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\varepsilon_{c'eff}}{\varepsilon_0}-1\right)W^E_{c'\left(p,s,t\right)}\left(\vec{r}\right)\vec{E}^0_{c'\left(p,s,t\right)}\left(\vec{r}',t\right)-\right.\\
&c'\left(p,s,t\right)\neq c'\left(p'=0,s'=0,t'=0\right)\\
&\left.\left.-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^M_{c'(p,s,t)}\left(\vec{r}\right)\vec{H}^0_{c'(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right)-\\
&-\sum_{\substack{c=1\\(c\neq c')}}^{N}\left(\sum_{\substack{-\infty\\p}}^{\infty}\sum_{\substack{-\infty\\s}}^{\infty}\sum_{\substack{-\infty\\t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^E_{c\left(p,s,t\right)}\left(\vec{r}\right)\vec{E}^0_{c\left(p,s,t\right)}\left(\vec{r}',t\right)-\right.\right.\\
&\left.\left.-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^M_{c(p,s,t)}\left(\vec{r}\right)\vec{H}^0_{c(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right),
\end{aligned}\tag{4}$$

$$\vec{H}_{0c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right)=\left(\frac{\left(\mu_{c'eff}+2\mu_0\right)+\theta_{1c'}^2\mu_{c'eff}+i\theta_{1c'}\left(\mu_{c'eff}+2\mu_0\right)}{3\mu_0 e^{i\theta_{1c'}}}\times\right.$$

$$\times\vec{H}^{0}_{c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right)-$$

$$-\sum_{\substack{-\infty \\ p}}^{\infty}\sum_{\substack{-\infty \\ s}}^{\infty}\sum_{\substack{-\infty \\ t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^{M}_{c'\left(p,s,t\right)}\left(\vec{r}\right)\vec{H}^{0}_{c'\left(p,s,t\right)}\left(\vec{r}',t\right)+\right.$$

$$c'\left(p,s,t\right)\neq c'\left(p'=0,s'=0,t'=0\right)$$

$$\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{c'eff}}{\varepsilon_0}-1\right)W^{E}_{c'(p,s,t)}\left(\vec{r}\right)\vec{E}^{0}_{c'(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right)-$$

$$-\sum_{\substack{c=1 \\ (c\neq c')}}^{N}\left(\sum_{\substack{-\infty \\ p}}^{\infty}\sum_{\substack{-\infty \\ s}}^{\infty}\sum_{\substack{-\infty \\ t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^{M}_{c\left(p,s,t\right)}\left(\vec{r}\right)\vec{H}^{0}_{c\left(p,s,t\right)}\left(\vec{r}',t\right)+\right.\right. \tag{5}$$

$$\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^{E}_{c(p,s,t)}\left(\vec{r}\right)\vec{E}^{0}_{c(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right),$$

$$0=\left(\frac{\left(\varepsilon_{c'eff}+2\varepsilon_0\right)+\theta_{1c'}^2\varepsilon_{c'eff}+i\theta_{1c'}\left(\varepsilon_{c'eff}+2\varepsilon_0\right)}{3\varepsilon_0 e^{i\theta_{1c'}}}\vec{E}^{0}{}_{c'\left(p',s',t'\right)}\left(\vec{r}',t\right)-\right.$$

$$-\sum_{\substack{-\infty \\ p}}^{\infty}\sum_{\substack{-\infty \\ s}}^{\infty}\sum_{\substack{-\infty \\ t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\varepsilon_{c'eff}}{\varepsilon_0}-1\right)W^{E}_{c'\left(p,s,t\right)}\left(\vec{r}\right)\vec{E}^{0}_{c'\left(p,s,t\right)}\left(\vec{r}',t\right)-\right.$$

$$c'\left(p,s,t\right)\neq c'\left(p',s',t'\right)$$

$$\left.\left.-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^{M}_{c'(p,s,t)}\left(\vec{r}\right)\vec{H}^{0}_{c'(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right)-$$

$$-\sum_{\substack{c=1 \\ (c\neq c')}}^{N}\left(\sum_{\substack{-\infty \\ p}}^{\infty}\sum_{\substack{-\infty \\ s}}^{\infty}\sum_{\substack{-\infty \\ t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^{E}_{c\left(p,s,t\right)}\left(\vec{r}\right)\vec{E}^{0}_{c\left(p,s,t\right)}\left(\vec{r}',t\right)-\right.\right.$$

$$\left.\left.-ik\mu_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^{M}_{c(p,s,t)}\left(\vec{r}\right)\vec{H}^{0}_{c(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right),$$

$$0=\left(\frac{\left(\mu_{c'eff}+2\mu_0\right)+\theta_{1c'}^2\mu_{c'eff}+i\theta_{1c'}\left(\mu_{c'eff}+2\mu_0\right)}{3\mu_0 e^{i\theta_{1c'}}}\vec{H}^0{}_{c'\left(p',s',t'\right)}\left(\vec{r}',t\right)-\right.$$

$$-\sum_{\substack{-\infty\\p}}^{\infty}\sum_{\substack{-\infty\\s}}^{\infty}\sum_{\substack{-\infty\\t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\mu_{c'eff}}{\mu_0}-1\right)W^M_{c'\left(p,s,t\right)}\left(\vec{r}\right)\vec{H}^0_{c'\left(p,s,t\right)}\left(\vec{r}',t\right)+\right.$$

$$c'\left(p,s,t\right)\neq c'\left(p',s',t'\right)$$

$$\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{c'eff}}{\varepsilon_0}-1\right)W^E_{c'\left(p,s,t\right)}\left(\vec{r}\right)\vec{E}^0_{c'\left(p,s,t\right)}\left(\vec{r}',t\right)\right]\right\}\right)-$$

$$-\sum_{\substack{c=1\\(c\neq c')}}^{N}\left(\sum_{\substack{-\infty\\p}}^{\infty}\sum_{\substack{-\infty\\s}}^{\infty}\sum_{\substack{-\infty\\t}}^{\infty}\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\mu_{ceff}}{\mu_0}-1\right)W^M_{c\left(p,s,t\right)}\left(\vec{r}\right)\vec{H}^0_{c\left(p,s,t\right)}\left(\vec{r}',t\right)+\right.\right.$$

$$\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)W^E_{c\left(p,s,t\right)}\left(\vec{r}\right)\vec{E}^0_{c\left(p,s,t\right)}\left(\vec{r}',t\right)\right]\right\}\right),$$

Here $\vec{E}_{0c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right),\vec{H}_{0c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right)$, and $\vec{E}^0_{c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right)$, $\vec{H}^0_{c'\left(p'=0,s'=0,t'=0\right)}\left(\vec{r}',t\right)$ are, respectively, the field of an unloaded cavity in sphere c' and the internal field of sphere c'; $\vec{E}^0_{c\left(p,s,t\right)}\left(\vec{r}',t\right)$ and $\vec{H}^0_{c\left(p,s,t\right)}\left(\vec{r}',t\right)$ are the internal fields of the rest of the spheres and their images in the cavity walls; $k=2\pi/\lambda$, and $\theta_{1c}^2=k^2a_c^2\varepsilon_0\mu_0$, with $\varepsilon_{0,}$ μ_0 being, respectively, the permittivity and permeability of the cavity filling outside the spheres.

The expressions for $W^E_{c\left(p,s,t\right)}\left(\vec{r}'\right)$; $W^M_{c\left(p,s,t\right)}\left(\vec{r}'\right)$, ε_{ceff} and μ_{ceff} are as follows

$$W^E_{c\left(p,s,t\right)}\left(\vec{r}'\right)=\frac{4\pi}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\frac{e^{-ik_1r_{c'\left(p',s',t'\right),c\left(p,s,t\right)}}}{r_{c'\left(p',s',t'\right),c\left(p,s,t\right)}},$$

$$W^M_{c\left(p,s,t\right)}\left(\vec{r}'\right)=-\frac{4\pi}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\frac{e^{-ik_1r_{c'\left(p',s',t'\right),c\left(p,s,t\right)}}}{r_{c'\left(p',s',t'\right),c\left(p,s,t\right)}},\qquad(6)$$

$$\varepsilon_{ceff} = \varepsilon_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) \ ,$$

$$\mu_{ceff} = \mu_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) \ ,$$

where, according to paper [5] and Fig.2,

$$F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = \frac{2\left(\sin ka_c\sqrt{\varepsilon_c\mu_c} - ka_c\sqrt{\varepsilon_c\mu_c}\cos ka_c\sqrt{\varepsilon_c\mu_c}\right)}{\left(k^2a_c^2\varepsilon_c\mu_c - 1\right)\sin ka_c\sqrt{\varepsilon_c\mu_c} + ka_c\sqrt{\varepsilon_c\mu_c}\cos ka_c\sqrt{\varepsilon_c\mu_c}}.$$

The first terms in the right-hand parts of Eqs. (4) and (5) represent the contribution from the internal field of sphere c'. If the effect of the rest of the spheres (and their mirror images) were neglected, then the remaining terms would make allowance for he effect of all other spheres and their images upon the scatterer c'. The equation set Eqs. (4) and (5) takes into account the electromagnetic cross-interaction effect of the spheres and their mirror images. The basic matrix of this set of algebraic equations (4) and (5) contains information on the features of electromagnetic interaction of the magnetodielectric spheres in the cavity.

Figure 2 shows the behavior of $\operatorname{Re}F(\theta)$ (solid curve) and $\operatorname{Im}F(\theta)$ (dotted curve) in dependence on $\operatorname{Re}\theta$ for several values of the loss tangent $\tan\delta_\varepsilon$, namely, $\tan\delta_\varepsilon = 0$ (curves 1); $\tan\delta_\varepsilon = 0.05$ (curves 2) and $\tan\delta_\varepsilon = 0.1$ (curves 3), with $\mu_c = 1$ and $\theta = ka_c\sqrt{\varepsilon_c\mu_c}$.

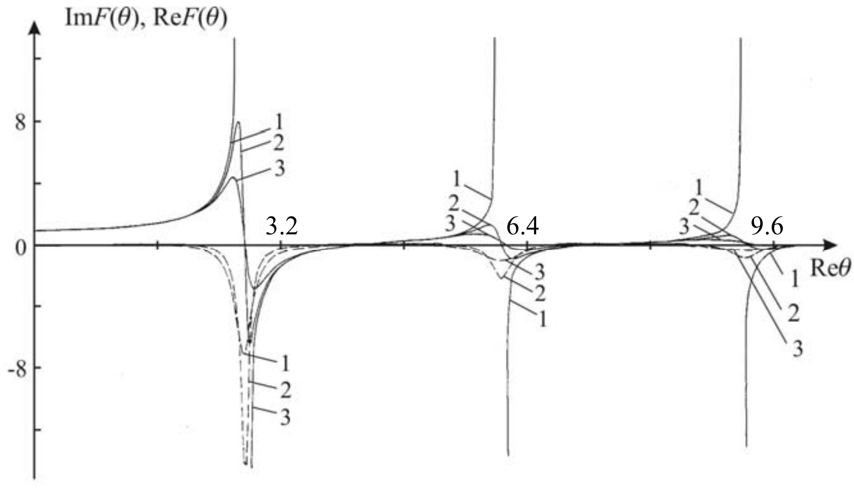

FIGURE 2. The $F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ function.

The algebraic equation set Eqs. (4) and (5) consists of $2N$ inhomogeneous vectorial equations for the spheres in the form of Eq. (4), with N being the total number of spheres, plus an infinite number of vectorial homogeneous equations for the images in the form of Eq. (5). The solution of the equation set for an individual sphere or image is

$$\begin{aligned}
&\vec{E}^{0}_{c'(p',s',t')}\left(\vec{r}',t\right)=\frac{1}{\Delta^{EM}}\sum_{c=1}^{N}\left(\sum_{u}\left[\hat{g}_{u}^{Eu'}\vec{E}_{0c(p,s,t)}\left(\vec{r}',t\right)+\hat{\beta}_{u}^{Eu'}\vec{H}_{0c(p,s,t)}\left(\vec{r}',t\right)\right]\right),\\
&\vec{H}^{0}_{c'(p',s',t')}\left(\vec{r}',t\right)=\frac{1}{\Delta^{EM}}\sum_{c=1}^{N}\left(\sum_{u}\left[\hat{\beta}_{u}^{Mu'}\vec{H}_{0c(p,s,t)}\left(\vec{r}',t\right)+\hat{g}_{u}^{Mu'}\vec{E}_{0c(p,s,t)}\left(\vec{r}',t\right)\right]\right),
\end{aligned}\tag{7}$$

where

$$\hat{g}_{u}^{Eu'}=\begin{bmatrix} g_{xxu}^{Eu'} & g_{xyu}^{Eu'} & g_{xzu}^{Eu'}\\ g_{yxu}^{Eu'} & g_{yyu}^{Eu'} & g_{yzu}^{Eu'}\\ g_{zxu}^{Eu'} & g_{zyu}^{Eu'} & g_{zzu}^{Eu'} \end{bmatrix},\quad \hat{\beta}_{u}^{Eu'}=\begin{bmatrix} \beta_{xxu}^{Eu'} & \beta_{xyu}^{Eu'} & \beta_{xzu}^{Eu'}\\ \beta_{yxu}^{Eu'} & \beta_{yyu}^{Eu'} & \beta_{yzu}^{Eu'}\\ \beta_{zxu}^{Eu'} & \beta_{zyu}^{Eu'} & \beta_{zzu}^{Eu'} \end{bmatrix},$$

$$\hat{\beta}_{u}^{Mu'}=\begin{bmatrix} \beta_{xxu}^{Mu'} & \beta_{xyu}^{Mu'} & \beta_{xzu}^{Mu'}\\ \beta_{yxu}^{Mu'} & \beta_{yyu}^{Mu'} & \beta_{yzu}^{Mu'}\\ \beta_{zxu}^{Mu'} & \beta_{zyu}^{Mu'} & \beta_{zzu}^{Mu'} \end{bmatrix},\quad \hat{g}_{u}^{Mu'}=\begin{bmatrix} g_{xxu}^{Mu'} & g_{xyu}^{Mu'} & g_{xzu}^{Mu'}\\ g_{yxu}^{Mu'} & g_{yyu}^{Mu'} & g_{yzu}^{Mu'}\\ g_{zxu}^{Mu'} & g_{zyu}^{Mu'} & g_{zzu}^{Mu'} \end{bmatrix},$$

and Δ^{EM} is the determinant of the basic matrix of the equation set Eqs. (4) and (5).

The internal field component Eq. (7) for a sphere or image can be represented as

$$\begin{aligned}
E_{xu'}^{0}\left(\vec{r}',t\right)=\frac{1}{\Delta^{EM}}\sum_{c=1}^{N}\Bigg(\sum_{u}\Big[&g_{xxu}^{Eu'}\vec{E}_{0xu}\left(\vec{r}',t\right)+g_{xyu}^{Eu'}E_{0yu}\left(\vec{r}',t\right)+g_{xzu}^{Eu'}E_{0zu}\left(\vec{r}',t\right)+\\
&+\beta_{xxu}^{Eu'}H_{0xu}\left(\vec{r}',t\right)+\beta_{xyu}^{Eu'}H_{0yu}\left(\vec{r}',t\right)+\beta_{xzu}^{Eu'}H_{0zu}\left(\vec{r}',t\right)\Big]\Bigg).
\end{aligned}$$

The rest of components of the internal field in the sphere or image can be obtained from Eq. (7) in a similar way.

If the electromagnetic field coupling in the sphere-containing cavity were neglected, then the solution for the internal field Eq. (7) of an arbitrary sphere in the cavity would take the form

$$\vec{E}^{0}_{c(p,s,t)}\left(\vec{r}',t\right)=\frac{3\varepsilon_0 e^{i\theta_{1c}}}{\left(\varepsilon_{ceff}+2\varepsilon_0\right)+\theta_{1c}^2\varepsilon_{ceff}+i\theta_{1c}\left(\varepsilon_{ceff}+2\varepsilon_0\right)}\vec{E}_{0c(p,s,t)}\left(\vec{r}',t\right),$$

(8)

$$\vec{H}^{0}_{c(p,s,t)}\left(\vec{r}',t\right)=\frac{3\mu_0 e^{i\theta_{1c}}}{\left(\mu_{ceff}+2\mu_0\right)+\theta_{1c}^2\mu_{ceff}+i\theta_{1c}\left(\mu_{ceff}+2\mu_0\right)}\vec{H}_{0c(p,s,t)}\left(\vec{r}',t\right).$$

The curves shown in Fig.3 are the absolute value, $\left|\eta^M\right|$ and phase, φ^M and real, Re; and imaginary, Im, parts of the internal magnetic field Eq. (8) of a sphere in dependence on the wavelength λ. The data have been computed for $a_c = 0.1145$ cm, $\varepsilon_c = 174$, $\mu_c = \mu_0 = \varepsilon_0 = 1$, and the loss tangent $\tan\delta_\varepsilon = 0$, with λ varying within the range of the second-order internal magnetic resonance of the sphere (the resonance wavelength is shown as λ_r).

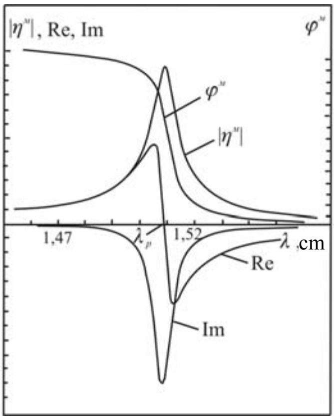

FIGURE 3. The second-order internal magnetic resonance of a sphere.

As can be seen from Fig.3, the real part of the expression Eq. (8) for the internal magnetic field of an arbitrary sphere in the cavity can turn to zero (the point λ_r in Fig.3) at the resonance, viz.

$$\mathrm{Re}\,\vec{H}^{0}_{c(p,s,t)}\left(\vec{r}',t\right)=0, \tag{9}$$

in the case where the electromagnetic interaction of the spheres in the cavity can be neglected. The condition Eq. (9) is met for the internal electric field of the spheres. If the electromagnetic interaction for the sphere containing cavity is taken into account, then the condition Eq. (9) is valid again. The resonance conditions for the spheres can be found from the determinant of the equation set Eqs. (4) and (5). While analyzing Eq. (9) for the case of identical spheres with

$a_c/\lambda_g \sim 1$ characterized by real-valued electrical parameters ε_c, μ_c, we can find that the condition holds [3, 4],

$$\det \mathrm{Re}\left\| \alpha_{ij}^{EM} \right\| = 0. \tag{10}$$

By resolving the condition Eq. (10) with respect to the function $F\left(ka_c\sqrt{\varepsilon_c, \mu_c}\right)$ (see Eq. (6)), we can obtain resonance conditions for the internal field components of the spheres and their mirror images along the axes x, y, z (see Fig.1), which are the roots of Eq. (10). Here $\left\| \alpha_{ij}^{EM} \right\|$ of Eq. (10) is the basic matrix of the algebraic equation set Eqs.(4) and (5).

The matrix $\mathrm{Re}\left\| \alpha_{ij}^{EM} \right\|$ in the left-hand part of Eq. (10) contains information on the electromagnetic interaction in the cavity filled by magnetodielectric spheres.

The field at an arbitrary point of the cavity lying outside the spheres is determined as

$$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) + \vec{E}_{sc}(\vec{r},t),$$

where $\vec{E}_0(\vec{r},t)$ is the field of the unloaded cavity.

The boundary condition for the tangential component of the total electric field in the sphere-containing cavity is met at the internal perfectly conducting surface S of the cavity in the case where

$$\vec{E}_{tsc}(\vec{r},t)\Big|_S = 0. \tag{11}$$

Proceeding from Eq. (11) we can assume the internal fields of the spheres and those of their mirror images to be equal in the rectangular cavity.

In the case where all the spheres in the cavity are identical, the order of equation (10) is determined by the order of the matrix $\mathrm{Re}\left\| \alpha_{ij}^{EM} \right\|$ which is equal to $6N$, with N being the number of the spheres in the cavity.

The cavity may accommodate spheres characterized by different resonance conditions. If the electric parameters $\varepsilon_{c,}\ \mu_c$ are real-valued, then the resonance conditions in that case can be found from Eq. (10) by resolving it with respect to the function $F\left(ka_c\sqrt{\varepsilon_c \mu_c}\right)$ Eq.(6), which is related to the resonating spheres, under the assumption that the rest of the spheres are not at resonance.

The order of Eq. (10) can be reduced by setting to zero those elements of the matrix $\mathrm{Re}\left\| \alpha_{ij}^{EM} \right\|$, Eq. (10), which are related to the curl of Eqs. (4) and (5), since they are small at the resonance. Then Eq.(10) splits into two independent equations, viz.

$$\det \operatorname{Re}\left\|\alpha_{ij}^{M}\right\| = 0, \quad \det \operatorname{Re}\left\|\alpha_{ij}^{E}\right\| = 0, \tag{12}$$

where the matrices $\left\|\alpha_{ij}^{M}\right\|$ and $\left\|\alpha_{ij}^{E}\right\|$ correspond, respectively, to the magnetic and electric internal fields of the spheres. Knowledge of these fields allows determining the sought for resonance conditions. This simplification of Eq. (10) may result in the loss of some information concerning the fine resonance structure of the internal field in the spheres.

If the electromagnetic interaction in the sphere-containing cavity is neglected, then Eq. (10) yields the following resonance condition for the internal electric and magnetic fields of an arbitrary sphere in the cavity,

$$\begin{gathered} F_0^E\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = -\frac{2\varepsilon_0\left(\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right)}{\varepsilon_c\left[\left(1+\theta_{1c}^2\right)\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right]}, \\ F_0^M\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) = -\frac{2\mu_0\left(\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right)}{\mu_c\left[\left(1+\theta_{1c}^2\right)\cos\theta_{1c}+\theta_{1c}\sin\theta_{1c}\right]}. \end{gathered} \tag{13}$$

Following paper [4], we will represent the Hertz potentials of the scattered field in the cavity as a superposition of Hertzian potentials of individual spheres and their mirror images, viz.

$$\begin{gathered} \vec{\Pi}^E\left(\vec{r},t\right) = \sum_{c=1}^{N}\left[\sum_{\substack{-\infty\\p}}^{\infty}\sum_{\substack{-\infty\\s}}^{\infty}\sum_{\substack{-\infty\\t}}^{\infty}\frac{1}{k_1^3}\left(\sin k_1a_c - k_1a_c\cos k_1a_c\right)\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)\vec{E}^0_{c(p,s,t)}\left(\vec{r}',t\right)\frac{e^{-ik_1r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right], \\ \vec{\Pi}^M\left(\vec{r},t\right) = -\sum_{c=1}^{N}\left[\sum_{\substack{-\infty\\p}}^{\infty}\sum_{\substack{-\infty\\s}}^{\infty}\sum_{\substack{-\infty\\t}}^{\infty}\frac{1}{k_1^3}\left(\sin k_1a_c - k_1a_c\cos k_1a_c\right)\left(\frac{\mu_{ceff}}{\mu_0}-1\right)\vec{H}^0_{c(p,s,t)}\left(\vec{r}',t\right)\frac{e^{-ik_1r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right]. \end{gathered} \tag{14}$$

Here $r_{c(p,s,t)} = \sqrt{\left(x-x_{c,s}\right)^2+\left(y-y_{c,t}\right)^2+\left(z-z_{c,p}\right)^2}$, where the coordinates x, y and z correspond to the point of observation of the scattered field out of the spheres in the cavity, and $\left(x_{c,s}, y_{c,t}, z_{c,p}\right)$ are coordinates of the center of the scattering sphere (see Eq. (1)).

The expressions Eq. (14) for the Hertz potentials can be used to analyze the cavity accommodating a set of magnetodielectric and metal spheres.

1.3. Resonance structure functions of electromagnetic interaction

We will construct the structure functions of electromagnetic interaction for a cavity filled by identical spheres with real-values of the permittivity, ε_c and permeability, μ_c.

To derive the structure functions we will use the resonance conditions for the internal fields of the cavity spheres. Upon finding the roots of Eq. (10) we can represent the resonance conditions for the $x-, y-,$ and z -components of the internal magnetic-type $\left(E\left(M\right)\right)$ and electric type $\left(E\left(M\right)\right)$ fields of the spheres in the cavity as

$$F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=f_{cik}^{E(M)}\left(\vec{r}_{u'u}\right),$$

$$F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=f_{cik}^{(E)M}\left(\vec{r}_{u'u}\right), \tag{15}$$

with $i,k=x,y,z$.

Here $f_{cik}^{E(M)}\left(\vec{r}_{u'u}\right), f_{cik}^{(E)M}\left(\vec{r}_{u'u}\right)$ are functions dependent on the topological structure of the set of spheres and their images, sphere radius, electric parameters of the sphere material, and the scattered wavelength; $F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ and $F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ are magnitudes of the function Eq. (6) at resonances.

The resonance conditions are the same for the $x-, y-,$ and z -components of the internal fields Eq. (8) of a free sphere. Proceeding from Eq. (13) with $\theta_{1c}<<1$, they can be brought to the form

$$F_0^M\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=-\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)},$$

$$F_0^E\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=-\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}. \tag{16}$$

By subtracting the left and right-hand parts of Eqs. (16) from the left and right-hand parts, respectively, of Eq. (15), we arrive at the following formula for the internal magnetic-type resonances of the spheres

$$F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)-F_0^M\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}+f_{cik}^{E(M)}\left(\vec{r}_{u'u}\right). \tag{17}$$

The similar expression for the electric-type internal resonances of the spheres is

$$F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)-F_0^E\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}+f_{cik}^{(E)M}\left(\vec{r}_{u'u}\right). \tag{18}$$

As can be seen, the subtraction procedure has allowed separating the components which are responsible for the shift of the resonance conditions Eq. (15) with respect to those of Eq. (16).

The functions

$$\begin{aligned}
\Phi_{cik}^{E(M)}\left(\vec{r}_{u'u}\right)&=\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}+f_{cik}^{E(M)}\left(\vec{r}_{u'u}\right),\\
\Phi_{cik}^{(E)M}\left(\vec{r}_{u'u}\right)&=\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}+f_{cik}^{(E)M}\left(\vec{r}_{u'u}\right)
\end{aligned} \tag{19}$$

will be referred to as components of the resonance structure functions for the electric- and magnetic-type electromagnetic interaction.

Now, let us introduce the structure functions of the magnetic- and electric-type electromagnetic interactions for an isolated sphere in the cavity in the form of tensorial functions, viz.

$$\begin{aligned}
\hat{\Phi}_c^{E(M)}\left(\vec{r}_{u'u}\right)&=\frac{2\mu_0}{\mu_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}\hat{I}+\hat{f}_c^{E(M)}\left(\vec{r}_{u'u}\right)=\begin{pmatrix}\Phi_{cxx}^{E(M)}\left(\vec{r}_{u'u}\right) & 0 & 0\\ 0 & \Phi_{cyy}^{E(M)}\left(\vec{r}_{u'u}\right) & 0\\ 0 & 0 & \Phi_{czz}^{E(M)}\left(\vec{r}_{u'u}\right)\end{pmatrix},\\
\hat{\Phi}_c^{(E)M}\left(\vec{r}_{u'u}\right)&=\frac{2\varepsilon_0}{\varepsilon_c}\frac{\left(1+\theta_{1c}^2\right)}{\left(1+2\theta_{1c}^2\right)}\hat{I}+\hat{f}_c^{(E)M}\left(\vec{r}_{u'u}\right)=\begin{pmatrix}\Phi_{cxx}^{(E)M}\left(\vec{r}_{u'u}\right) & 0 & 0\\ 0 & \Phi_{cyy}^{(E)M}\left(\vec{r}_{u'u}\right) & 0\\ 0 & 0 & \Phi_{czz}^{(E)M}\left(\vec{r}_{u'u}\right)\end{pmatrix}.
\end{aligned} \tag{20}$$

The constructed resonance functions Eq. (20) depends on the electric parameters, ε_c, μ_c, of the resonance-size spheres. Meanwhile it is possible to introduce resonance structure functions which would be independent of the electric parameters of the spheres. Such functions can be obtained with the use of

Eq.(12), provided that in Eq. (10) the terms involving the curl of Eqs. (4) and (5) can be neglected as contributing only slightly at the resonance. Then, the conditions Eqs. (17) and (18) can be brought to the form

$$F_{cik}^{M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)-F_0^{M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=\frac{\mu_0}{\mu_c}\left[\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{M}\left(\vec{r}_{u'u}\right)\right],$$

$$F_{cik}^{E}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)-F_0^{E}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)=\frac{\varepsilon_0}{\varepsilon_c}\left[\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{E}\left(\vec{r}_{u'u}\right)\right],\qquad(21)$$

while the functions Eq. (19) become

$$\Phi_{cik}^{M}\left(\vec{r}_{u'u}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{M}\left(\vec{r}_{u'u}\right),$$

$$\Phi_{cik}^{E}\left(\vec{r}_{u'u}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cik}^{E}\left(\vec{r}_{u'u}\right).\qquad(22)$$

These will be the components of the magnetic- and electric-type resonance structure functions of the interaction.

As a result, the functions Eq. (20) can be represented as

$$\widehat{\Phi}_c^{M}\left(\vec{r}_{u'u}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}\widehat{I}+\widehat{f}_c^{M}\left(\vec{r}_{u'u}\right)=\begin{pmatrix}\Phi_{cxx}^{M}\left(\vec{r}_{u'u}\right) & 0 & 0\\ 0 & \Phi_{cyy}^{M}\left(\vec{r}_{u'u}\right) & 0\\ 0 & 0 & \Phi_{czz}^{M}\left(\vec{r}_{u'u}\right)\end{pmatrix},$$

$$\widehat{\Phi}_c^{E}\left(\vec{r}_{u'u}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}\widehat{I}+\widehat{f}_c^{E}\left(\vec{r}_{u'u}\right)=\begin{pmatrix}\Phi_{cxx}^{E}\left(\vec{r}_{u'u}\right) & 0 & 0\\ 0 & \Phi_{cyy}^{E}\left(\vec{r}_{u'u}\right) & 0\\ 0 & 0 & \Phi_{czz}^{E}\left(\vec{r}_{u'u}\right)\end{pmatrix}.\qquad(23)$$

In contrast to the functions Eq. (20) those of Eq. (23) will be referred to as the electric-(E) and magnetic-type (M) structure functions of interaction for an isolated sphere in the cavity.

Note that Eqs.(20) and (23) associated with alternating triple lattice sums with infinite limits of summation which are analogous to the Ewald triple lattice sums for infinite crystal lattices [6]. Calculating and estimating such triple sums represent certain difficulties. Levine [5] replaced these sums by integrals, which procedure resulted in an uncontrollable error. The present paper suggests an experimental method for estimating triple lattice sums with infinite limits of summation.

1.4. Magnetic and electric resonance functions of interaction for a sphere

Let us construct components of the tensorial function $\hat{\Phi}_c^E\left(\vec{r}_{u'u}\right)$ Eq. (23). To that end we will determine the resonance conditions for the internal electric field of an isolated sphere in the cavity from a solution of Eq. (12) which can be represented in the explicit form as follows

$$\det \operatorname{Re}\left\|\alpha_{ij}^E\right\| = \det\begin{bmatrix} \psi_{cxx}^{E0'}+\psi_{uxx}^{E'} & \psi_{uxy}^{E'} & \psi_{uxz}^{E'} \\ \psi_{uyx}^{E'} & \psi_{cyy}^{E0'}+\psi_{uyy}^{E'} & \psi_{uyz}^{E'} \\ \psi_{uzx}^{E'} & \psi_{uzy}^{E'} & \psi_{czz}^{E0'}+\psi_{uzz}^{E'} \end{bmatrix} = 0. \qquad (24)$$

The elements figuring in the matrix $\operatorname{Re}\left\|\alpha_{ij}^E\right\|$ Eq. (24) are

$\left(\psi_{cxx}^{E0'}+\psi_{uxx}^{E'}\right) = A_{c\varepsilon}^{0'} - A_{c\varepsilon}\tau_{uxx}^{E'}$; $\left(\psi_{cyy}^{E0'}+\psi_{uyy}^{E'}\right) = \left(\psi_{czz}^{E0'}+\psi_{uzz}^{E'}\right) = A_{c\varepsilon}^{0'} - A_{c\varepsilon}\tau_{uyy}^{E'}$;

$\left(\psi_{czz}^{E0'}+\psi_{uzz}^{E'}\right) = A_{c\varepsilon}^{0'} - A_{c\varepsilon}\tau_{uzz}^{E'}$; $\psi_{uxy}^{E'} = \psi_{uyx}^{E'} = -A_{c\varepsilon}\tau_{uxy}^{E'}$; $\psi_{uxz}^{E'} = \psi_{uzx}^{E'} = -A_{c\varepsilon}\tau_{uxz}^{E'}$; and $\psi_{uyz}^{E'} = \psi_{uzy}^{E'} = -A_{c\varepsilon}\tau_{uyz}^{E'}$,

where

$$\begin{gathered} \tau_{uxx}^{E'} = \sum_{c=1}^{N}\left[B_c \sum_{\substack{-\infty \\ p}}^{\infty}\sum_{\substack{-\infty \\ s}}^{\infty}\sum_{\substack{-\infty \\ t}}^{\infty}\left(c_{uxx}\cos k_1 r_{u'u} + a_{uxx}\sin k_1 r_{u'u}\right)\right], \\ c'(p,s,t) \neq c'(p'=0,s'=0,t'=0) \\ \tau_{uxy}^{E'} = \sum_{c=1}^{N}\left[B_c \sum_{\substack{-\infty \\ p}}^{\infty}\sum_{\substack{-\infty \\ s}}^{\infty}\sum_{\substack{-\infty \\ t}}^{\infty}\left(c_{uxy}\cos k_1 r_{u'u} + a_{uxy}\sin k_1 r_{u'u}\right)\right], \\ c'(p,s,t) \neq c'(p'=0,s'=0,t'=0) \end{gathered} \qquad (25)$$

$$A_{c\varepsilon}^{0'} = \frac{(\varepsilon_{ceff}+2\varepsilon_0) + \theta_{1c}^2\varepsilon_{ceff} + \theta_{1c}^2(\varepsilon_{ceff}+2\varepsilon_0)}{3\varepsilon_0},$$

$$A_{c\varepsilon} = \left(\frac{\varepsilon_{ceff}}{\varepsilon_0} - 1 \right) \text{ and } B_c = \frac{1}{k_1^3}(\sin k_1 a_c - k_1 a_c \cos k_1 a_c),$$

with (sf. Eqs. (1) and (3))

$$c_{uxx} = \frac{1}{r_{u'u}} k_1^2 + \left| \frac{3\left(x_{c',s'=0} - x_{c,s}\right)^2 - r_{u'u}^2}{r_{u'u}^5} - k_1^2 \frac{\left(x_{c',s'=0} - x_{c,s}\right)^2}{r_{u'u}^3} \right|,$$

$$a_{uxx} = k_1 \frac{3\left(x_{c',s'=0} - x_{c,s}\right) - r_{u'u}^2}{r_{u'u}^4},$$

$$c_{uxy} = \left| \frac{3\left(x_{c',s'=0} - x_{c,s}\right)\left(y_{c',t'=0} - y_{c,t}\right)}{r_{u'u}^5} - k_1^2 \frac{\left(x_{c',s'=0} - x_{c,s}\right)\left(y_{c',t'=0} - y_{c,t}\right)}{r_{u'u}^3} \right|,$$

$$a_{uxy} = k_1 \frac{3\left(x_{c',s'=0} - x_{c,s}\right)\left(y_{c',t'=0} - y_{c,t}\right)}{r_{u'u}^4}.$$

If the wavelength tends to infinity, $\lambda \to \infty$, then

$$\tau_{uxx}^{E'} = \sum_{c=1}^{N} \left[B_c \sum_{\substack{-\infty \\ p}}^{\infty} \sum_{\substack{-\infty \\ s}}^{\infty} \sum_{\substack{-\infty \\ t}}^{\infty} \frac{\left| 3\left(x_{c',s'=0} - x_{c,s}\right)^2 - r_{u'u}^2 \right|}{r_{u'u}^5} \right],$$
$$c'\left(p,s,t\right) \neq c'\left(p'=0, s'=0, t'=0\right)$$

$$\tau_{uxy}^{E'} = \sum_{c=1}^{N} \left[B_c \sum_{\substack{-\infty \\ p}}^{\infty} \sum_{\substack{-\infty \\ s}}^{\infty} \sum_{\substack{-\infty \\ t}}^{\infty} \frac{3\left(x_{c',s'=0} - x_{c,s}\right)\left(y_{c',t'=0} - y_{c,t}\right)}{r_{u'u}^5} \right].$$
$$c'\left(p,s,t\right) \neq c'\left(p'=0, s'=0, t'=0\right)$$

Expression for the values $\tau_{uyy}^{E'}, \tau_{uzz}^{E'}, \tau_{uxz}^{E'}, \tau_{uyz}^{E'}$ can be obtained from Eq. (25) by changing the subscripts and coordinates of the spheres and images in Eq. (25) according to the subscripts of these values.

By solving the cubic equation Eq. (24) with respect to $F\left(ka_c\sqrt{\varepsilon_c,\mu_c}\right)$ (Eq. (6)) with an assumption of real-valued electric parameters of the sphere [3, 4], the components of the tensorial function $\widehat{\Phi}_c^E\left(\vec{r}_{u'u}\right)$ Eq. (23) can be represented as

$$\Phi_{cxx}^E\left(\vec{r}_{u'u}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cxx}^E\left(\vec{r}_{u'u}\right),$$

$$\Phi_{cyy}^E\left(\vec{r}_{u'u}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{cyy}^E\left(\vec{r}_{u'u}\right), \tag{26}$$

$$\Phi_{czz}^E\left(\vec{r}_{u'u}\right)=\frac{3+4\theta_{1c}^2}{1+2\theta_{1c}^2}+f_{czz}^E\left(\vec{r}_{u'u}\right),$$

where $f_{cxx}^E\left(\vec{r}_{u'u}\right), f_{cyy}^E\left(\vec{r}_{u'u}\right), f_{czz}^E\left(\vec{r}_{u'u}\right)$ are the roots of the cubic Equation (24).

The components of the tensorial function $\widehat{\Phi}_c^M\left(\vec{r}_{u'u}\right)$ Eq. (23) can be found from Eq. (26) by reversing the sign in front of $\tau_{uxx}^{E'}, \tau_{uxy}^{E'}, \tau_{uyy}^{E'}, \tau_{uyz}^{E'}$, etc.

Analysis of the expressions Eq. (26) shows that if the field wavelength in a complex spatial lattice representing a cavity with spheres is commensurate with the lattice constant, then structural magnetic- and electric-type resonances of electromagnetic interaction may appear in the cavity filled by resonance-size magnetodielectric spheres.

The structure resonances of the magnetic and electric type can exist simultaneously in the cavity at practically equal resonance wavelengths whose value depends on the cavity geometry, positions of the spheres and their radius. Structure resonances of each cavity type can affect only analogous internal resonances of the magnetodielectric sphere in the cavity. A physical effect is possible of combining the cavity structural resonances with internal resonances of the spheres, which serves to increase their impact on the fine structure of the latter.

1.5. Analyzing the possibility to experimentally estimate magnitudes of the cavity structure functions

The resonance conditions for the $x-, y-$ and z-components Eq. (7) of the internal fields of an isolated sphere can be expressed via the structure functions Eqs. (22) and (23) as

$$
\begin{aligned}
F_{ci\kappa}^{M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) &= -\frac{\mu_0}{\mu_c}\left[\frac{2\left(1+\theta_{1c}^2\right)}{1+2\theta_{lc}^2}-\Phi_{cik}^{M}\left(\vec{r}_{u'u}\right)\right],\\
F_{ci\kappa}^{E}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right) &= -\frac{\varepsilon_0}{\varepsilon_c}\left[\frac{2\left(1+\theta_{1c}^2\right)}{1+2\theta_{1c}^2}-\Phi_{cik}^{E}\left(\vec{r}_{u'u}\right)\right].
\end{aligned}
\qquad (27)
$$

As can be seen from Eqs. (26) and (27), the resonance conditions for the internal field components of the spheres in the cavity may assume different values. This results in the appearance of a fine structure of the internal field of the spheres in the cavity and splitting of the resonance curves which characterize this field. For a given geometry of the sphere-containing cavity, the conditions for the internal resonances of the spheres can be varied using, for example, temperature dependences [1, 7] of the electric parameters of the material which the spheres are made of. The effect allows determining the resonance values of the functions $F_{cik}^{E(M)}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ Eq. (17) and $F_{cik}^{(E)M}\left(ka_c\sqrt{\varepsilon_c\mu_c}\right)$ Eq. (18) (see Fig.2) through measuring the fine structure of the internal field [1] of the spheres, and estimating experimentally the magnitudes of the components of the cavity structure functions $\Phi_{ci\kappa}^{E(M)}\left(\vec{r}_{u'u}\right)$ and $\Phi_{ci\kappa}^{(E)M}\left(\vec{r}_{u'u}\right)$ as given Eqs. (19) and (20), respectively. The latter functions involve alternating triple lattice sums with infinite limits of summation.

CONCLUSION

The paper suggests a new technique for constructing functions to describe the structural electromagnetic interaction in the cavity filled by magnetodielectric spheres.

The notion of resonance structure functions of the electric- and magnetic-type electromagnetic interaction has been in troduced.

The possibility is discussed of estimating experimentally the magnitudes of the resonance structure functions of the electromagnetic interaction.

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