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RESEARCH ON CENTRALITY MEASURES IN SCALE-FREE NETWORKS

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Understanding the behavior of centrality measures in the context of various network models is crucial for grasping the structural properties of complex systems. This study explores the constraints imposed on centrality measures by different network models, revealing the maximum achievable centrality values for individual nodes and for the network as a whole. We adopt a systematic approach, selecting specific network models and centrality metrics, modeling, and assessing centrality through empirical experiments. The study encompasses maximum centrality for nodes, the distribution of node centrality, and network centrality (Freeman's centrality) in various network models and centrality metrics. Additionally, we investigate the possibility of deriving theoretical estimates for these centrality limits. The findings of this study not only contribute to a deeper understanding of network dynamics but also pave the way for further analyses of real networks, allowing for a comparison between model-generated and real networks. This research bridges the gap between theoretical modeling and practical network analysis, offering insights into the fundamental principles governing the structure of complex networks.

1. Introduction

The modern world is becoming increasingly interconnected and complex, and key to understanding these interconnections is the study of scale-free networks. Scale-free networks are fundamental in many systems and processes, from internet communications to biological networks and social structures. In these networks, centrality measures, which help to determine the importance and influence of individual nodes in the network structure, are of particular interest.

Node centrality in scale-free networks is a critical factor affecting the network's resilience and efficiency. The study of centrality is important in various fields, including computer science, sociology, biology, and economics. In this article, we focus on several key aspects of centrality, namely: degree centrality, closeness centrality, betweenness centrality, as well as influence and other relevant metrics.

Special attention is given to the comparative analysis of these measures in scale-free networks, characterized by the presence of highly connected nodes, which significantly influence the dynamics and behavior of the network. The aim of the study is to identify the features of centrality distribution in such networks and to assess their impact on the overall structure and functionality of the network.

Using theoretical analysis and modeling, we aim to examine in detail how various centrality measures can be applied to assess the importance of individual nodes, their influence on information dissemination, network resilience to failures, and opportunities for optimizing network processes.

In this study, we aim to explore these constraints in the context of different models of scale-free networks and various centrality measures. It is known that most networks in the real world [1, 4] are scale-free networks. Both social networks and networks of a technical nature (electrical, transport), biological, etc. have this property. A network is called scale-free if the distribution of nodes by the number of connections is power-law at least asymptotically:

$$Pr(k) \sim k^{-\gamma} \quad \text{under} \quad k \to \infty .$$

Thus, the probability that a node will have exactly k links is proportional to $k^{-\gamma}$. This dependence is performed more precisely, the larger is k. Parameter γ is called the scaling exponent and, as a rule, is within $\gamma \in (2;3]$.

We propose a systematic approach to investigate the maximum achievable centrality values for individual nodes and the network as a whole through empirical experiments. The corresponding results will allow us not only to gain a deeper understanding of the dynamics of scale-free networks but also to explore the possibility of comparing model-generated and real networks in terms of their structural organization and centrality. Essentially, this study aims to combine theoretical modeling with practical network analysis to provide insights into the fundamental principles governing the structure of complex networks.

Thus, the study of centrality limits in scale-free networks plays a key role in advancing our knowledge of the properties and behavior of complex systems, which can have important implications for various fields, including social sciences, engineering, and biology.

2. Methodology

For conducting experimental calculations, the Python programming language and its NumPy matrix operations module were used. This choice is motivated by the flexibility and extensive capabilities of Python in data processing and analysis, as well as the efficiency of NumPy in working with matrices and large data volumes, which is critically important for calculations in the field of complex networks.

During these experimental calculations, we were able to obtain quantitative estimates of node importance in the network, and determine how different centrality measures behave in different network models. This allowed us to identify key nodes and understand the structural characteristics of the network, which is important for further analysis of the dynamics and resilience of scale-free networks.

3. Results

The analysis of the obtained calculation results was provided taking into account the models of scale-free networks created. The analysis allowed us to draw valuable conclusions about the behavior and characteristics of different nodes in complex network structures.

Key findings from the analysis:

Identifying Node Importance: The analysis revealed which nodes are considered the most central or important in the context of a particular network. This helped identify key elements of the network that may play a role in its development and dynamics.

Assessing Network Structural Features: Analysis of centrality measures provided information about the structural differences between different network models. For example, higher levels of degree centrality were observed in Barabási-Albert (BA) networks compared to Erdős-Rényi (ER) models.

Asymptotic and Exact Limits: Calculations of asymptotic and exact limits for centrality measures revealed the maximum and minimum possible values of these measures for the used network models. This helped understand the limitations and potential capabilities of different types of networks.

Software Analysis and Data Interpretation: The use of software applications for counting these values allowed for effective analysis of large data sets, which in turn contributed to a deeper understanding of the dynamics of scale-free networks. The analysis of centrality measures based on the obtained results allowed for conclusions not only about individual nodes but also about the overall structure and properties of the network.

Overall, the analysis results helped identify both general and unique characteristics of nodes in different types of networks and provided information for further research on the impact of these characteristics on the behavior and resilience of complex network systems.

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