Developing software for solving some combinatorial generation and optimization problems

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Abstract. Software for solving various problems of combinatorial generation and combinatorial optimization is described. Firstly, programmatic implementation of algorithm for generating classical combinatorial sets (combinations, permutations, arrangements) and algorithm for generation of k-compositions of combinatorial sets were described. Then, implementation of algorithm for generation of permutations with partially fixed order of elements was described. Also article describes applications for solving three different combinatorial optimization problems – pickup and delivery problems with 3D loading constraints, problem of scheduling freight trains in rail-rail transshipment yards with train arrangement and problem of optimization of linear function on a set of cyclic permutations.

Keywords. Software, combinatorial generation, combinatorial optimization, combinatorial set, k-set, permutations with partially fixed order of elements, cyclic permutations, pickup and delivery problem, train scheduling.

1. Introduction

Combinatorial optimization is important scientific area which results are used in developing and implementing solutions for many scientific and applied problems [1]–[4]. Combinatorial generation is widely used to solve problems of modeling, combinatorial optimization, etc.

Despite of wide use of combinatorial generation and combinatorial optimization results in real world, existing software solving for such problems use different algorithms for solving different combinatorial problems.

This article is dedicated to describing software that uses the same method of generating combinatorial sets [5]. Software solves next problems:

- generation of various classic combinatorial sets (permutations, combinations etc.) and compositional k-images of combinatorial sets (k-sets) [5], [6]
- generation of permutations with partially fixed order of elements [7]
- solving of one-to-one pickup and delivery problem with 3D loading constraints [8]
- solving problem of scheduling freight trains in rail-rail transshipment yards with train arrangement [9]
- optimization of linear function on a set of cyclic permutations [10], [11]

The remainder of this paper is organized as follows. Section 2 describes generation software for generation various classic combinatorial sets and k-sets. In section 3, additional module to software from section 2 is described which allows generating of permutations with

partially fixed order of elements. Section 4 presents a description of software for solving of one-to-one pickup and delivery problem with 3D loading constraints. Section 5 is dedicated to description of a program for solving problem of scheduling freight trains in rail-rail transshipment yards. In section 6, software for solving problem of combinatorial optimization of linear function on a set of cyclic permutations is described. At the end of the paper, conclusions are given.

2. Software for generating classic combinatorial sets and k-sets

Article [5] describes method of generation of various classic combinatorial sets (permutations, combinations etc.) and compositional k-images of combinatorial sets (k-sets). Algorithms *GenBase* and *Gen_k-set* from [5] were implemented in Delphi 7. Let's describe how software works on example. Suppose we have k-set with below structure [5].

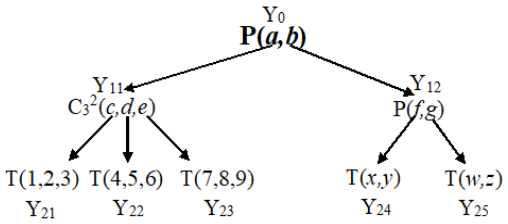


Fig.1 – Example of k-set

Here $P(a,b)$ is a set of permutation of elements a and b, $C_3^2(c,d,e)$ – 2-combinations from set (c,d,e) of 3 elements, $T(1,2,3)$ – tuple of 3 elements (123) etc. Screenshot of developed software with results of generation of k-set from Fig.1 is given below.

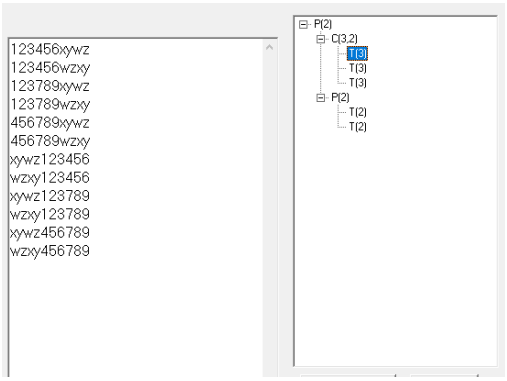


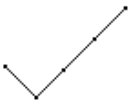
Fig. 2 – Screenshot of software with results of generating k-set from Fig. 1.

3. Module for generation of permutations with partially fixed order of elements

Article [7] describes permutations with partially fixed order of elements which generalize well-known class of permutations with given ups and downs [12], [13]. For generating such class of permutations, algorithm *PartOrderedPerm* was used. This algorithm was implemented as a module to software for generating k-sets described in section 2.

Let's demonstrate developed software module work on example of generating permutations with concrete partially fixed order of elements (example 2 from [7]).

On Figure 3, screenshot with example of generating permutations from elements $A = \{1,2,3,4,5\}$ with partially fixed order of elements which is represented by sets $D'(\pi) = \{1\}, \overleftarrow{D}'(\pi) = \{3,4\}$ is given. Note that order of elements on position 2 is not fixed, so order of elements in result permutations can correspond to diagrams



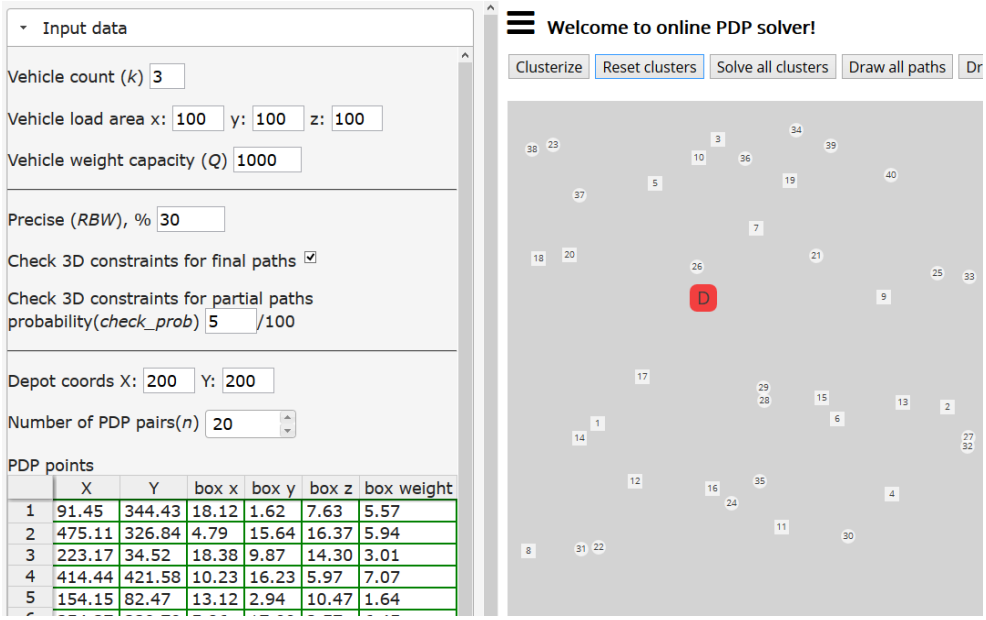
and

Fig. 3 – Screenshot of software with results of generating of permutations with partially fixed order.

4. Web application for solving pickup and delivery problem

Article [14] gives mathematical model and heuristic solution algorithm for one-to-one pickup and delivery problem with 3D loading constrains. On the basis of solution algorithm from [14], web application that solves the described problem and has a user-friendly interface was developed. Application is available online at http://rebrand.ly/pdp-app.

Screenshots of developed application are depicted below.



	X	Y	box x	box y	box z	box weight
1	91.45	344.43	18.12	1.62	7.63	5.57
2	475.11	326.84	4.79	15.64	16.37	5.94
3	223.17	34.52	18.38	9.87	14.30	3.01
4	414.44	421.58	10.23	16.23	5.97	7.07
5	154.15	82.47	13.12	2.94	10.47	1.64

Fig. 4 – A screenshot with all input parameters and PDP points

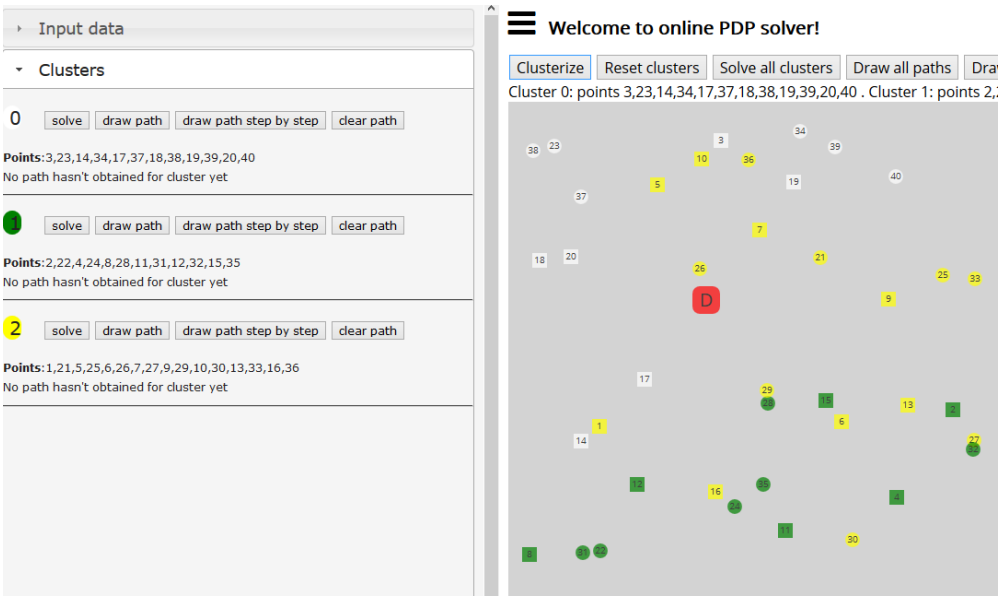


Fig. 5 – A screenshot of clustering of 20 pickup-delivery pairs (squares are pickups and circles are deliveries; the big rounded red square is a depot)

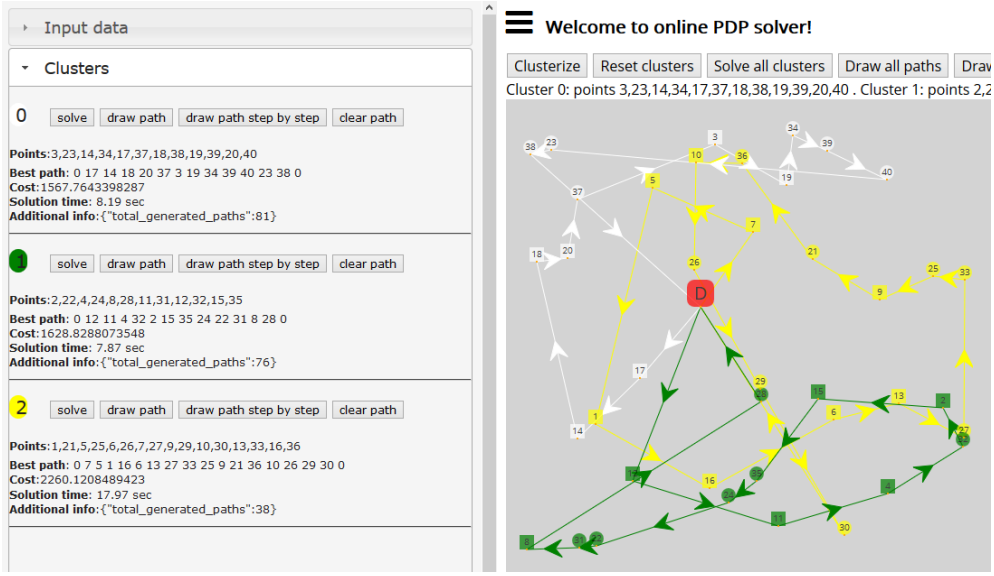


Fig. 6 – A screenshot of path generated for each cluster from the example above

5. Web application for solving problem of scheduling freight trains in rail-rail transshipment yards with train arrangement

Article [15] considers problem of scheduling freight trains in rail-rail transshipment yards. In addition to solving problem of scheduling the service slots of trains considered in original paper [16], article [15] additionally solves the problem of train arrangement, i.e. assigning each train to a certain railway track. Mathematical model which uses apparatus of combinatorial sets and solution method which uses beam search heuristic are given there.

Solution algorithm from [15] was implemented in Python 2.7 as a web application which is available online at http://rebrand.ly/tsy-app. Screenshots of example input data and solution results are given on Fig. 7-10.

6. Application for optimization of linear function on set of cyclic permutations

In article [10], problem of optimization of linear function on set of cyclic permutations was described. Mathematical model and two solution methods of considered problem were given: exact method which uses branch and bound method and heuristic method which modifies original branch and bound method to obtain feasible solution in less period of time.

Both exact and heuristic algorithms was implemented using Java 7.

In table 1, results of comparing solutions obtained by exact and heuristic methods are given.

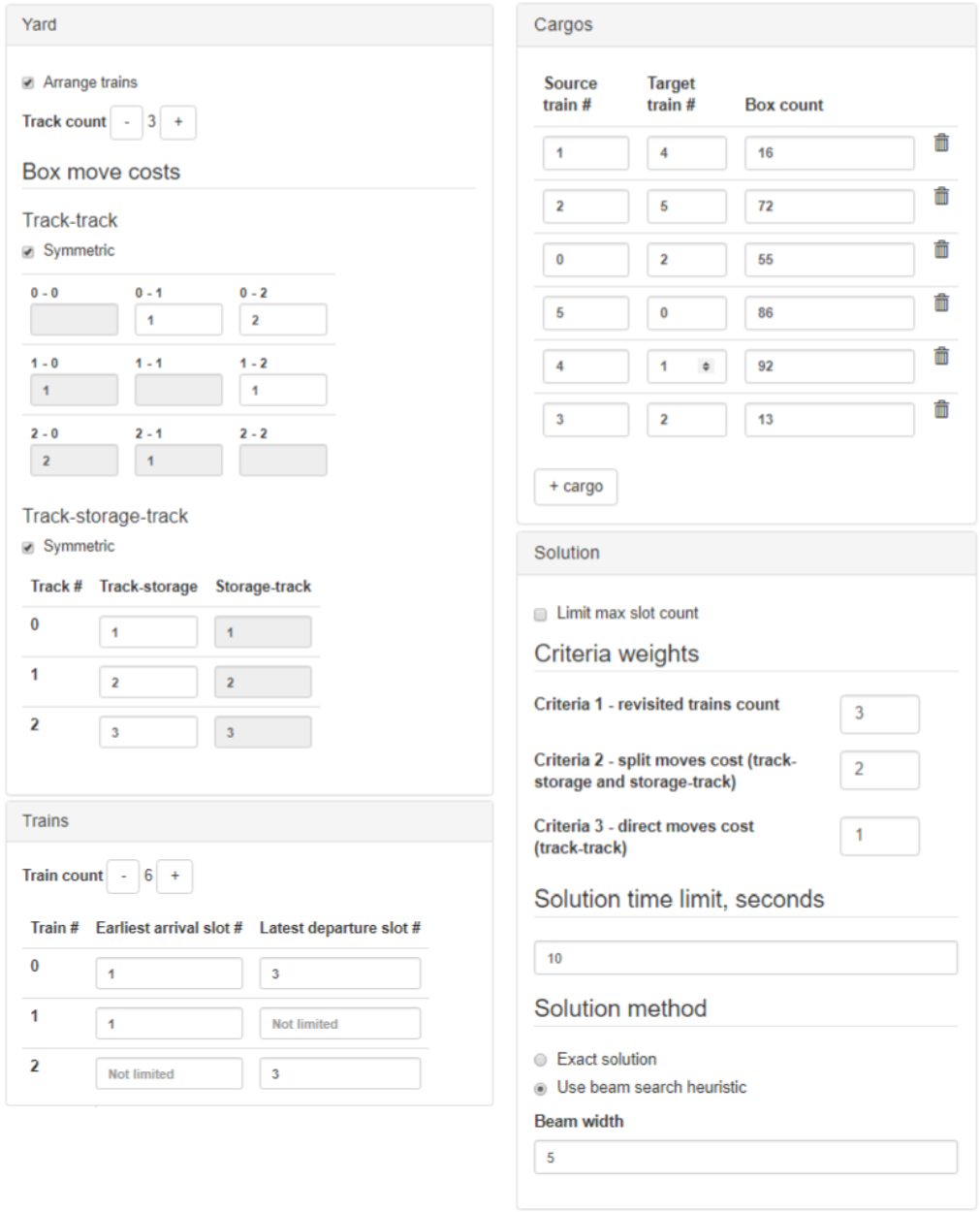


Fig. 7. Screenshot of developed application (input data)

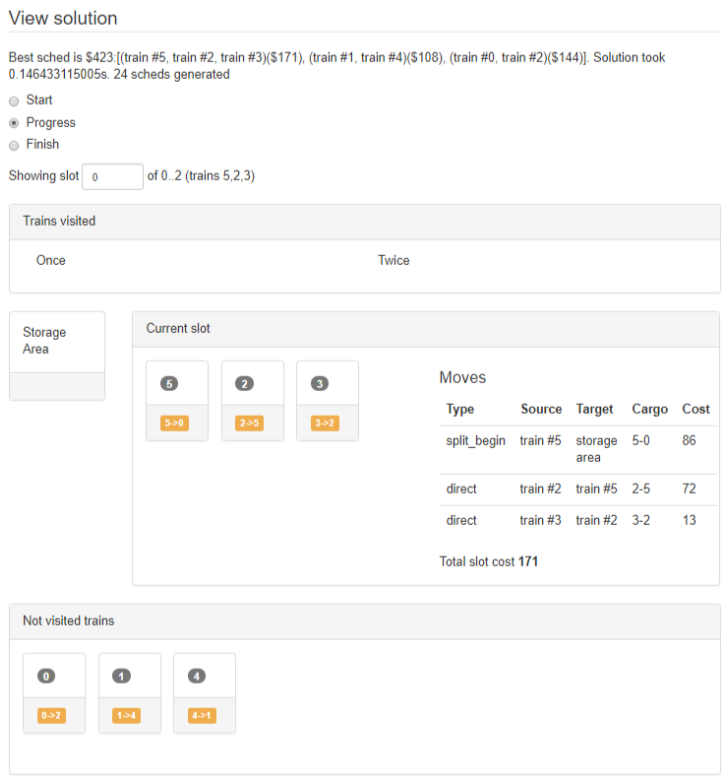


Fig. 8. Screenshot of developed application (solution, first slot)

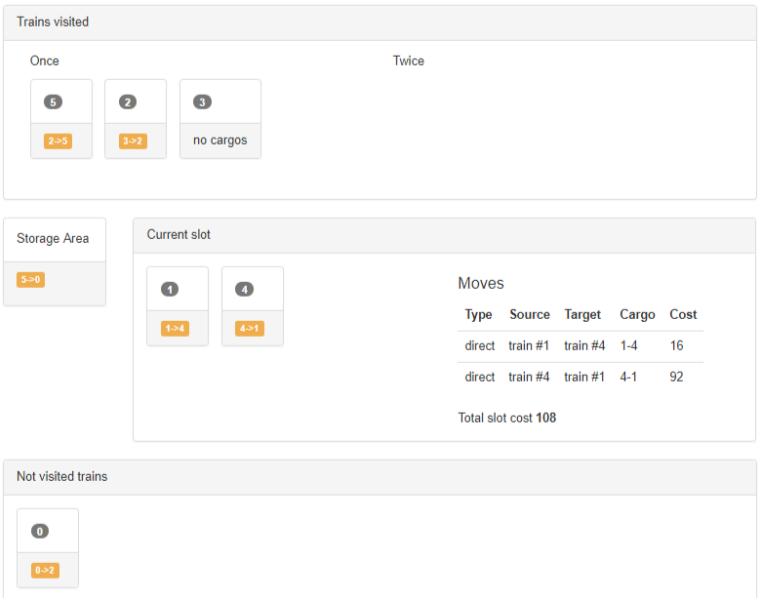


Fig. 9. Screenshot of developed application (solution, second slot)

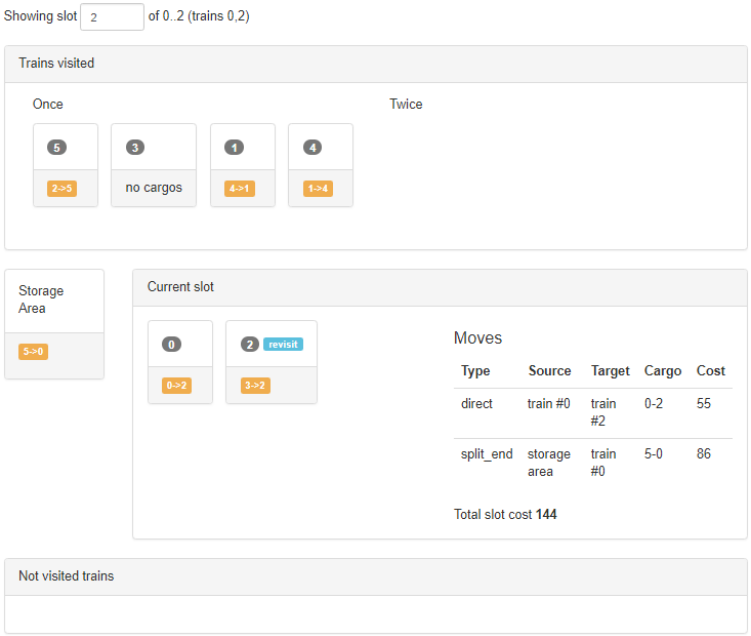


Fig. 10. Screenshot of developed application (solution, third slot)

Table 1. Comparison of exact and heuristic solutions for problem of optimization of linear function on cyclic permutations set

n	Exact solution			Heuristic solution			Relative difference between objective function value
	Value of objective function	Vertex count	Solution time	Value of objective function	Vertex count	Solution time	
10	169.2	12290	6 s	172	1782	16 ms	1.65%
15	309.3	33990	12 s	319.3	4642	47 ms	3.23%
20	525.2	523760	102 s	571.3	3817	62 ms	8.78%
30	804.5	1600350	244 s	915.8	21540	485 ms	13.83%
40	1621.1	6105360	1074 s	1889.2	6662	203 ms	16.54%

7. Conclusion

In this article, software for solving various combinatorial generation and optimization problems were given.

Algorithms for generating classical combinatorial sets k-compositions of combinatorial sets were implemented in Delphi 7. Also, algorithm for generation of permutations with partially fixed order of elements which are generalization of well-known permutations with given ups and downs was implemented as a module for software for generating k-sets.

Programmatic implementations of algorithms of solving three various combinatorial optimization problems were also considered. For the pickup and delivery problems with 3D

loading constraints and for the problem of scheduling freight trains in rail-rail transshipment yards with train arrangement, web applications which are available online were developed.

Finally, software solving the problem of optimization of linear function on cyclic permutations set were considered. Comparison of results obtained with exact algorithm (which uses branch and bound method) and with heuristic algorithm was given.

Developed software has friendly interface and allows to set input data combinations in a quite flexible way.

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