Adaptation of the least squares method for determination of oscillating type measuring devices parameters with using gain-frequency characteristic

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***Abstract* — In this article there is suggested a method, which determines both a time constant and a gain-frequency characteristic damp constant for a measuring device, which is simulated by oscillating type dynamic element. The described method is based on a method of least squares, which increases an identification accuracy because of obtaining more information about a measuring device performance during a measurement process. The standard uncertainties of both a time constant and a damp constant are analyzed. The recommendations for measuring process, which was mentioned above, optimization are given.**

Keywords* — *measuring device; oscillating type dynamic element; dynamic characteristic; gain-frequency characteristic; standard uncertainty

I. Introduction

The requirements to exactness and fast-acting of measuring instruments rise today for all human vital activities. It related to aspiring to the unification and standardization of both requirements to the products and requirements to the methods and test facilities for these products. Also that question is actual for a test equipment which contains sensors and measuring facilities with second order transmission functions. Such measuring facilities can be presented as a dynamic oscillating type element, for example, accelerometers [1, 2].

One of the basic metrology descriptions for such sensors and measuring devices are dynamic characteristics [2, 3] that reflect their inertia properties (including speed of response). So therefore identification and correction of dynamic characteristics are important tasks. Their decision will allow to increase exactness and certainty of the got measuring results. On the other hand, the decision of these tasks, especially tasks of identification, is complicated through the substantially nonlinear relations between the dynamic characteristics and the measuring devices parameters, such as a time constant, a damp constant, a damp factor and an angular frequency.

The methods of determination of the parameters of the measuring devices designed by the oscillating type dynamic element by means of their dynamic characteristics considered in [4-7]. But majority from them is operated by limit information content, not using all potential of data that can be got during a measuring experiment. So, graphic and grapho-analitical methods considered in [3], mostly use extremums (minimums and maximums) transient, impulse or gain-frequency characteristic. The methods considered in [6, 7] use two values of gain-frequency characteristic. It can result in the receipt of results with the large extended uncertainty with a high probability in the real terms at presence of hindrances and noises in the entrance and output signals of measuring device. The advantage of the gain-frequency characteristic over other dynamic characteristic is foremost in that measuring of gain-frequency characteristic come true in the set mode. It augments exactness of authentication due to reduction of random measurement error.

The purpose of this work is adaptation of least-squares method, whose efficiency at treatment of measuring results does not cause doubts, to the task of determination of parameters of measuring devices, designed by an oscillating type dynamic element, using gain-frequency characteristics.

II. Theoretical relationships

The gain-frequency characteristic of measuring device, simulated by the oscillating type dynamic element, described by the formula

$$A(\omega)=\frac{k}{\sqrt{(1-\omega^2T^2)^2+(2\omega\xi T)^2}}, \qquad (1)$$

where ω is an angular frequency, k is a static conversion factor, T is a time constant and ξ is a damp constant of the measuring device.

This expression can be converted to the form

$$(1-\omega^2T^2)^2+(2\omega\xi T)^2=\frac{k^2}{A^2(\omega)} \qquad (2)$$

where ω and k are known, and values $A(\omega)$ can be measured.

Unknown parameters can be expressed from formula (2) as follows

$$T=\frac{1}{\omega}\sqrt{1+2\xi^2\pm\sqrt{4\xi^2(\xi^2-1)+\frac{k^2}{A^2(\omega)}}}; \qquad (3)$$

$$\xi=\frac{1}{2\omega T}\sqrt{\frac{k^2}{A^2(\omega)}-(1-\omega^2T^2)^2}. \qquad (4)$$

If two observations $A(\omega_1)$ and $A(\omega_2)$ of the gain-frequency characteristic are experimentally obtained at different frequencies, you can get a system of equations

$$\begin{cases} T = \dfrac{1}{\omega_1}\sqrt{1+2\xi^2 \pm \sqrt{4\xi^2(\xi^2-1)+\dfrac{k^2}{A^2(\omega_1)}}}\,; \\ \xi = \dfrac{1}{2\omega_2 T}\sqrt{\dfrac{k^2}{A^2(\omega_2)}-(1-\omega_2^2T^2)^2}\,. \end{cases} \quad (5)$$

Solving the system (5) by substituting one equation into another, we find the time constant

$$T = \sqrt[4]{\frac{\omega_2^2-\omega_1^2+A_2^2\omega_1^2-A_1^2\omega_2^2}{\omega_1^2\omega_2^2(\omega_2^2-\omega_1^2)}}\,. \quad (6)$$

The damp constant can be obtained by substituting T into the second equation of system (5).

If measurements are carried out at frequencies ω_1 and $\omega_2 = n\omega_1$, the formulas for calculating the time constant and attenuation coefficient will be

$$T = \frac{1}{\omega_1}\sqrt[4]{\frac{\left(\dfrac{k^2}{A^2(\omega_2)}-1\right)-n^2\left(\dfrac{k^2}{A^2(\omega_1)}-1\right)}{n^2(n^2-1)}}\,; \quad (7)$$

$$\xi = \frac{1}{2n\omega_1 T}\sqrt{\frac{k^2}{A^2(\omega_2)}-(1-n^2\omega_1^2T^2)^2}\,. \quad (8)$$

If to substitute formula (7) into formula (8), a damp constant can be obtained as

$$\xi = \sqrt{\frac{n^4\left(\dfrac{k^2}{A^2(\omega_1)}-1\right)-\left(\dfrac{k^2}{A^2(\omega_2)}-1\right)}{4n\sqrt{(n^2-1)\left[\left(\dfrac{k^2}{A^2(\omega_2)}-1\right)-n^2\left(\dfrac{k^2}{A^2(\omega_1)}-1\right)\right]}}+\frac{1}{2}}\,. \quad (9)$$

Expression (2) can also be converted to

$$1-2\omega^2T^2+\omega^4T^4+4\omega^2T^2\xi^2 = \frac{k^2}{A^2(\omega)}\,. \quad (10)$$

This implies

$$\omega^2T^4+2T^2(2\xi^2-1) = \frac{1}{\omega^2}\left(\frac{k^2}{A^2(\omega)}-1\right). \quad (11)$$

If to enter the notation

$$\beta(\omega) = \frac{1}{\omega^2}\left(\frac{k^2}{A^2(\omega)}-1\right);$$

$$T^4 = a\,;\ 2T^2(2\xi^2-1) = b\,, \quad (12)$$

can get a system of linear equations

$$a\omega_i^2 + b = \beta(\omega_i)\,, \quad (13)$$

where $i = 1\ldots N$, N – the number of frequencies at which the gain-frequency characteristic was measured.

The system (13) in the simplest case can be solved in the presence of the gain-frequency characteristic values $A(\omega_1)$ and $A(\omega_2)$ obtained at two frequencies ω_1 and ω_2

$$\begin{cases} a\omega_1^2 + b = \beta(\omega_1); \\ a\omega_2^2 + b = \beta(\omega_2), \end{cases}. \quad (14)$$

where $\beta(\omega_1)$ and $\beta(\omega_2)$ are determined by substitution of values ω_1, ω_2 and $A(\omega_1)$, $A(\omega_2)$ into the expression (12).

The solution of the system (14) is

$$a = \frac{\begin{vmatrix} \beta(\omega_1) & 1 \\ \beta(\omega_2) & 1 \end{vmatrix}}{\begin{vmatrix} \omega_1^2 & 1 \\ \omega_2^2 & 1 \end{vmatrix}} = \frac{\beta(\omega_1)-\beta(\omega_2)}{\omega_1^2-\omega_2^2}\,; \quad (15)$$

$$b = \frac{\begin{vmatrix} \omega_1^2 & \beta(\omega_1) \\ \omega_2^2 & \beta(\omega_2) \end{vmatrix}}{\begin{vmatrix} \omega_1^2 & 1 \\ \omega_2^2 & 1 \end{vmatrix}} = \frac{\omega_1^2\beta(\omega_2)-\omega_2^2\beta(\omega_1)}{\omega_1^2-\omega_2^2}\,. \quad (16)$$

The time constant T and damp constant ξ are determined in accordance with expressions (12) using the formulas

$$T = \sqrt[4]{a}\,; \quad (17)$$

$$\xi = \sqrt{\frac{1}{2}\left(\frac{b}{2\sqrt{a}}+1\right)}\,. \quad (18)$$

To improve the accuracy of parameter T and ξ identification under conditions of a measuring experiment (in the presence of interference of various origins), it is proposed to apply the method of least squares, which was theoretically well developed for a system of linear equations [8].

The sum of the residuals squares

$$Q = \sum_{i=1}^{N}\delta_i^2 = \sum_{i=1}^{N}(a\omega_i^2+b-\beta(\omega_i))^2$$

reaches a minimum when all its partial derivatives are zero, i.e.

$$\begin{cases} \dfrac{\partial Q}{\partial a} = 2\sum\limits_{i=1}^{N} \omega_i^2 (a\omega_i^2 + b - \beta(\omega_i)) = 0; \\ \dfrac{\partial Q}{\partial b} = 2\sum\limits_{i=1}^{N} (a\omega_i^2 + b - \beta(\omega_i)) = 0. \end{cases} \tag{19}$$

Entering notations $[\omega^2] = \sum\limits_{i=1}^{N} \omega_i^2$; $[\omega^4] = \sum\limits_{i=1}^{N} \omega_i^4$; $[\beta(\omega)] = \sum\limits_{i=1}^{N} \beta(\omega_i)$; $[\omega^2\beta(\omega)] = \sum\limits_{i=1}^{N} \omega_i^2 \beta(\omega_i)$, can get a system of normal equations

$$\begin{cases} a \cdot [\omega^4] + b \cdot [\omega^2] = [\omega^2 \beta(\omega)]; \\ a \cdot [\omega^2] + N \cdot b = [\beta(\omega)]. \end{cases} \tag{20}$$

The solution of this system is

$$a = \frac{N[\omega^2\beta(\omega)] - [\omega^2][\beta(\omega)]}{N[\omega^4] - [\omega^2]^2}; \tag{21}$$

$$b = \frac{[\omega^4][\beta(\omega)] - [\omega^2][\omega^2\beta(\omega)]}{N[\omega^4] - [\omega^2]^2}. \tag{22}$$

The parameters T and ξ are determined by the formulas (17) and (18).

The standard uncertainties of the coefficients a and b are estimated by the formulas

$$u(a) = \sqrt{\frac{N}{N[\omega^4] - [\omega^2]^2}} u(\delta); \tag{23}$$

$$u(b) = \sqrt{\frac{[\omega^4]}{N[\omega^4] - [\omega^2]^2}} u(\delta), \tag{24}$$

where $u(\delta) = \sqrt{\dfrac{\sum\limits_{i=1}^{N} \delta_i^2}{N-2}}$ – standard uncertainty of the residuals δ_i, which are calculated by substitution of the coefficient estimates a and b into the system of equations

$$\delta_i = a\omega_i^2 + b - \beta(\omega_i). \tag{25}$$

The standard uncertainty of measuring the time constant T in accordance with the measurement equation (17) will be

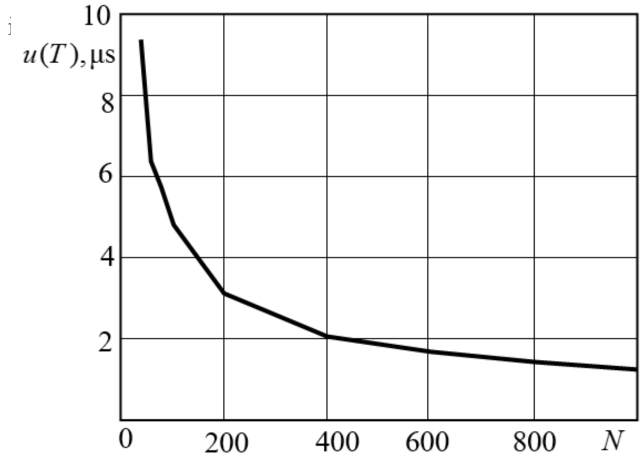
$$u(T) = \frac{\partial T}{\partial a} u(a) = \frac{1}{4T^3} u(a). \tag{26}$$

The standard uncertainty of measuring the damp constant ξ in accordance with the measurement equation (18) will be

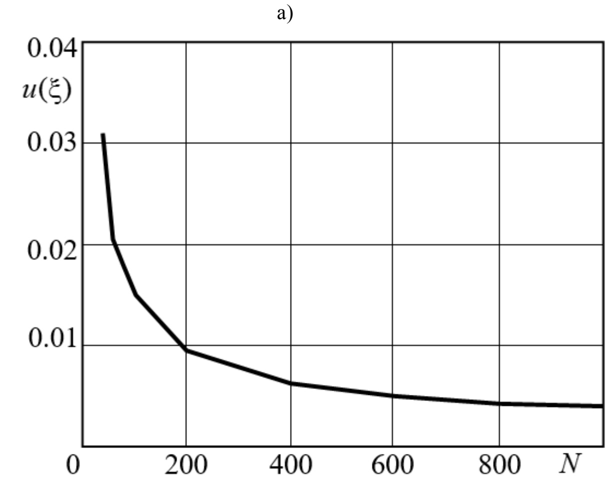
$$\begin{aligned} u(\xi) &= \sqrt{\left(\frac{\partial \xi}{\partial a}\right)^2 u^2(a) + \left(\frac{\partial \xi}{\partial b}\right)^2 u^2(b)} = \\ &= \frac{1}{4T^2\xi} \sqrt{\frac{1}{T^4}\left(\xi^2 - \frac{1}{2}\right)^2 u^2(a) + \frac{1}{4} u^2(b).} \end{aligned} \tag{27}$$

III. RESULTS AND THEIR DISCUSSION

The dependencies of standard uncertainties $u(T)$ and $u(\xi)$ on the number of frequencies N at which the gain-frequency characteristic was measured for an accelerometer with the static conversion $k = 1g = 9.8\,\mathrm{m/s^2}$, the time constant $T = 8$ ms and the damp constant $\xi = 0.6$ are shown



a)



b)

Fig.1 The dependencies of standard uncertainties $u(T)$ (a) and $u(\xi)$ (b) on the number of frequencies N

The research was performed by mathematical modeling with averaging 20 observations at each point. Wherein the standard deviation for $u(T)$ ranged from 3 μs wits $N = 40$ up to 0.06 μs with $N = 1000$; the standard deviation for $u(\xi)$ ranged from 0.013 wits $N = 40$ up to 0.0002 with $N = 1000$.

Analyzing the dependencies of Fig. 1 it is obvious that the standard uncertainties decrease with increasing number of observations.

The dependencies of standard uncertainties $u(T)$ and $u(\xi)$ on $u[A(\omega_i)]$ with $N = 100$ are shown in Fig. 2.

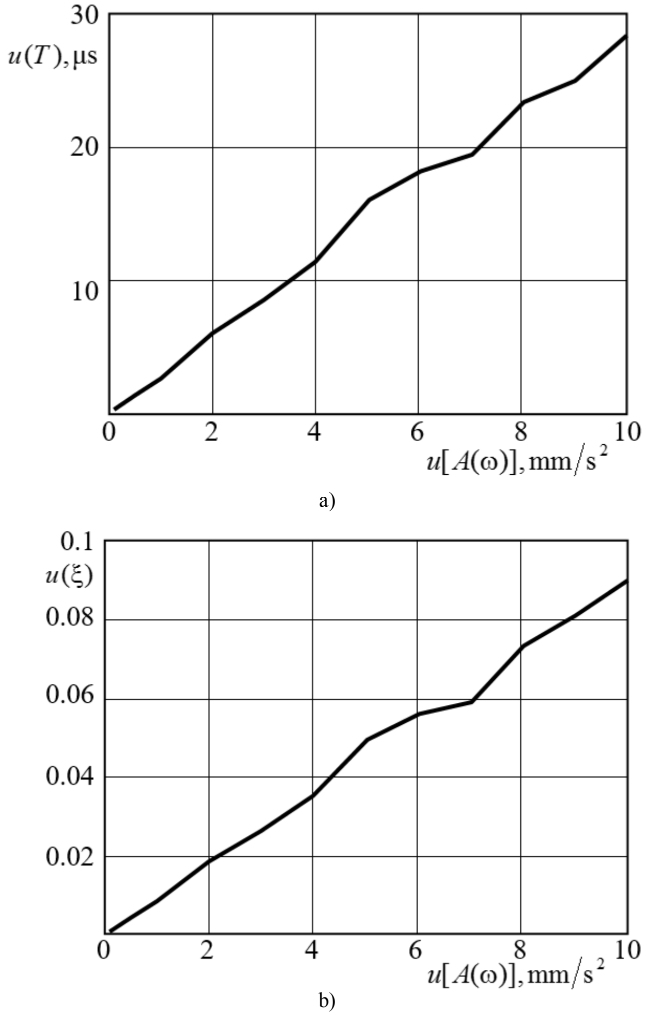


Fig. 2 The dependencies of standard uncertainties $u(T)$ and $u(\xi)$ on $u[A(\omega_i)]$ with $N = 100$

The standard deviation for $u(T)$ ranged from 0.04 μs wits $u[A(\omega)] = 10^{-4}\ \frac{\mathrm{m}}{\mathrm{s}^2}$ up to 5 μs with $u[A(\omega)] = 10^{-2}\ \frac{\mathrm{m}}{\mathrm{s}^2}$; the standard deviation for $u(\xi)$ ranged from 0.0001 with $u[A(\omega)] = 10^{-4}\ \frac{\mathrm{m}}{\mathrm{s}^2}$ up to 0.02 with $u[A(\omega)] = 10^{-2}\ \frac{\mathrm{m}}{\mathrm{s}^2}$.

The dependencies in Fig. 2 show that standard uncertainties $u(T)$ and $u(\xi)$ increase in proportion to the uncertainty $u[A(\omega)]$. Therefore, in measuring experiment, in order to reduce, arising mainly due to the presence of noise in measuring the gain-frequency characteristic, it is advisable to make additional repeated observations at each frequency under study ω_i, followed by averaging the values of the gain-frequency characteristic.

CONCLUSIONS

The method based on the least squares method for determining the time constant and the damp constant of the measuring devices described by the oscillating type dynamic element has been developed. This allows to improve the accuracy of determining the parameters of the measuring devices by the gain-frequency characteristic due to obtaining more information about the behavior of the measuring devices in terms of the measuring experiment.

The standard uncertainties of measuring the time constant and the damp constant are investigated. Compared to the data obtained in [6], the standard uncertainty measuring the time constant can be reduced by more than 10 times, the standard uncertainty of measuring the damp constant – by more than 4 times.

Recommendations for optimizing the experimental conditions for determining the parameters of measuring devices modeled by the oscillating type dynamic element with using the gain-frequency characteristic are given.

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