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THEORY OF RESONANT RELATIVISTIC OSCILLATOR WITH NONUNIFORM FOCUSING FIELD

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Abstract.

The two-dimensional model of millimeter wave resonant O-type oscillator (as orotron, ledatron, resonant BWO, etc.) with a relativistic electron beam is analyzed. The selfconsistent nonlinear simultaneous equations have been obtained for the arbitrary space distribution of the magnetic guide field. The start generation characteristics are analyzed under small-signal conditions with an analytical solution taken for the case of inclined focusing magnetic field. It is found that the efficiency of electron-wave interaction appreciably depends on the focusing field strength and the relativistic mass factor. The results of numerical optimization of the guide field structure are presented to show possibility of improvement of the start characteristics of the oscillator.

1 Introduction.

It is well known that the application of relativistic voltages for acceleration of electron beams in millimeter wave oscillators allows to obtain high levels of output power of the radiation. The dynamic relativistic variation of electron mass can considerable change the character of the electron-wave energy exchange process. This effect is essential, if the magnetic guide field strength is limited or nonuniform. In this case, the transverse electron-wave interaction and static motion of the electrons in cross direction should be taken into account and for theoretical description of the device a many-dimensional model should be applied.

The goal of this work is to present a theoretical description for the rela-

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tivistic microwave orotron-type generator using a two-dimensional model of the interaction space and to investigate the influence of the relativistic mass factor and of the focusing magnetic field structure (strength and distribution of nonuniformity) on the self-excitation conditions of the device. The investigation of magnetic nonuniformity on the start oscillation characteristics is fulfilled by means of numerical parameter optimization based on the representing of the magnetic displacement vector as a sum of longitudinal component and corresponding values for the group of the local magnetic nonuniformities (LMN) having the gauss form.

2 Model and Governing Equations.

We consider following model of the resonant relativistic generator. A sheet electron beam is passed through the (cavity or open) resonator near to the surface of periodic structure. The electron beam is subjected to the dc longitudinal magnetic field applied in y direction. The focusing field may be nonuniform in a general case. The RF field is assumed to have fixed space structure and to change weakly in the scale of the electron transit time through the resonator. It is possible if the oscillatory system has sufficiently big value of the quality factor Q.

At self-consistent description of the electron-wave interaction process we shall come from the Maxwell-Lorentz equations which can be written accounting the ordinary for resonant devices approximations [1]-[3]

$$-\frac{dC_s}{dt} + i\left(\omega - \omega_s\right) C_s = \frac{1}{N_s \pi} \int\limits_V \int\limits_0^{2\pi} \vec{J}\vec{E}_s^* e^{i\omega t} d(\omega t)\, dV; \tag{1}$$

$$\frac{d\vec{v}}{dt} = -\frac{e}{m_o \gamma}\left\{\vec{E} + \vec{v}\times\vec{B} + \frac{\vec{v}}{c^2}\left(\vec{v}\cdot\vec{E}\right)\right\}, \tag{2}$$

where C_s is the complex amplitude of RF oscillations of the sth resonator mode. The components of the electric field strength vector $\vec{E} = (0, E_y, E_z)$ are given by

$$\begin{aligned} E_y &= C_s f(y)\psi_y(z) e^{i(\Phi\xi - \omega t)}; \\ E_z &= iC_s f(y)\psi_z(z) e^{i(\Phi\xi - \omega t)}. \end{aligned} \tag{3}$$

The functions f and ψ_y, ψ_z define the longitudinal (coordinate y) and transverse (coordinate z) space distribution of the sth mode of the resonator field, respectively; $N_s = \varepsilon_o \int\limits_V \left|\vec{E}_s\right|^2 dV$ is the norm of oscillation; $\omega_s = \omega_s' + i\omega_s'/2Q_s$ is the fundamental complex frequency of the sth mode of the resonator;

Q_s is the resonator quality factor at the sth mode; ω is the generation frequency; $\vec{J}$ is the convection current vector of the beam; e, m_o are the electron charge and rest mass, respectively; $\vec{v}$ is the drift velocity vector of the electron beam; c is the speed of light; $\gamma = (1 - v^2/c^2)^{-1/2}$ is the instantaneous relativistic mass factor; $\Phi = (1 - v_o/v_\phi)\Phi_o$ is the relative transit angle of electrons, describing the difference between the initial electron velocity v_o and the phase speed of slowed down wave v_ϕ; $\Phi_o = \omega L/v_o$ is the unperturbed transit angle of electrons; $\vec{B}$ is the magnetic displacement vector.

The system of equations (1), (2) can be transformed to the form to be convenient for numerical analysis by means of introduction of the following dimensionless variables: $\xi = y/L$; $X = x/H$; $Z = z/H$; $\theta = \omega t - \Phi_o\xi - \varphi_o$ is the electron phase with respect to the hypothetical wave moving with the phase speed which is equal to v_o; $\varphi_o = \omega t_o$ is the initial phase of the electrons; L is the length of the interaction space; $H = 2L/\Phi_o$. The magnetic displacement vector $\vec{B}$ is assumed to have two static (B_y, B_z) and one RF $\tilde{B}_x$ components (H_x polarized wave is excited in the resonator)

$$\vec{B} = \vec{B}_o + \vec{\tilde{B}};\quad \vec{B}_o = (0, B_y, B_z);\quad \vec{\tilde{B}} = (\tilde{B}_x, 0, 0),$$

$$B_y = B_o + \hat{B}_y;\quad B_z \equiv \hat{B}_z;$$

where $\tilde{B}_x$ and E_z are connected by the ratio $\tilde{B}_x = (v_\phi/c^2)E_z$.

The functions ψ_y and ψ_z determining the transverse distribution of field amplitude in the resonator with the comb periodic structure was found to be given by [5]

$$\psi_y(z) = -\mathrm{sh}(\Gamma(z-a))/\mathrm{sh}(\Gamma a);$$
$$\psi_z(z) = \mathrm{ch}(\Gamma(z-a))/\mathrm{sh}(\Gamma a), \tag{4}$$

where $\Gamma = k\sqrt{c^2/v_\phi^2 - 1}$ is the transverse wavenumber; $k = \omega/c$; a is the distance between the resonator mirrors. If the resonator mirrors are considerably removed from each other ($\Gamma a \gg 1$) the expressions (4) become simply

$$\psi_y(z) = \psi_z(z) \equiv \psi(z) = \exp(-\Gamma z). \tag{5}$$

After transition to the dimensionless amplitude F and phase Ψ of the field: $C_s/E_o = F\exp(-i\Psi)$ $(E_o = m_o v_o^2/2eL)$ the equation of oscillation excitation transforms to the standard form for the given class of the resonant generators [6]

$$\frac{dF}{d\tau} + [1 - GS_1(F)]F = 0, \tag{6}$$

$$\frac{d\Psi}{d\tau} + \frac{\omega - \omega_s'}{\omega_s'} 2Q_s + GS_2(F) = 0, \tag{7}$$

where $\tau = \omega t/2Q_s$ is the dimensionless time; $G = 2Q_s|I_o|L^2/N_s U_o \omega_s'$ is the parameter of interaction efficiency; $U_o = m_o c^2(\gamma_o - 1)/e$ is the electron beam accelerating voltage; $\gamma_o = (1 - v_o^2/c^2)^{-1/2}$ is the relativistic mass factor at the initial electron velocity v_o; $S(F) = S_1(F) + iS_2(F)$ is the complex steepness of the oscillatory characteristic of a microwave generator [7]; $\delta\omega = \frac{d\Psi}{d\tau} + \frac{\omega - \omega_s'}{\omega_s'} 2Q_s$ is the electron frequency displacement. The complex steepness of the oscillatory characteristic $S(F)$ can be expressed taking into account the introduced designations as

$$\begin{aligned} S(F) &= \frac{H}{2\pi F \Delta} \int_0^1 f(\xi) \int_{Z^-}^{Z^+} \psi(z) \int_0^{2\pi} \left(1 - i(1 - v_\phi^2/c^2)^{-1/2} \frac{H}{L}\frac{dZ}{d\xi}\right) \times \\ &\quad \times \exp(i(\Theta + \Phi\xi + \varphi)) d\varphi \, dZ_o \, d\xi, \end{aligned} \tag{8}$$

where $Z^{\pm}$ is the integration limits on transverse coordinate Z in the cross section $\xi = 0$ (the plus and minus superscripts refer to upper and lower bound of the electron beam, respectively); Δ is the beam thickness; $\varphi = \varphi_o + \Psi$.

In the chosen designations the equation of motion can be rewritten as

$$\begin{aligned} \frac{d^2\Theta}{d\xi^2} &= \frac{\Phi_o}{2\gamma}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^3 \left\{ \left(1 - \frac{v_o^2}{c^2}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-2}\right)\frac{E_y}{E_o} - \right. \\ &\quad - \frac{v_o^2}{c^2}\frac{H}{L}\frac{dZ}{d\xi}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-2}\frac{E_{pz}}{E_o} + \frac{v_\phi v_o}{c^2}\frac{H}{L}\frac{dZ}{d\xi} \times \\ &\quad \left. \times \left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-1}\left(1 - \frac{v_o}{v_\phi}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-1}\right)\frac{E_z}{E_o}\right\} - \\ &\quad - \Phi_o\Phi_c \frac{H}{L}\frac{dX}{d\xi}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^2 \frac{B_z}{B_o}; \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{d^2X}{d\xi^2} &= \frac{1}{\Phi_o}\frac{dX}{d\xi}\frac{d^2\Theta}{d\xi^2}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-1} - \\ &\quad - \Phi_c \frac{L}{H}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)\left(\frac{B_z}{B_o} - \frac{H}{L}\frac{dZ}{d\xi}\frac{B_y}{B_o}\right) + \\ &\quad + \frac{1}{2\gamma}\frac{v_o^2}{c^2}\frac{dX}{d\xi}\left(\frac{E_y}{E_o} + \frac{H}{L}\frac{dZ}{d\xi}\left(\frac{E_z}{E_o} + \frac{E_{pz}}{E_o}\right)\right); \end{aligned} \tag{10}$$

$$\frac{d^2Z}{d\xi^2} = \frac{1}{\Phi_o}\frac{dZ}{d\xi}\frac{d^2\Theta}{d\xi^2}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-1} + \frac{1}{2\gamma}\frac{v_o^2}{c^2}\frac{dZ}{d\xi}\frac{E_y}{E_o} -$$

$$- \Phi_c \frac{dX}{d\xi}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)\frac{B_y}{B_o} -$$
$$- \frac{1}{2\gamma}\frac{L}{H}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^2 \left\{\left[1 - \frac{v_\phi v_o}{c^2}\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-1} -\right.\right.$$
$$\left. - \left(\frac{v_o}{c}\frac{H}{L}\frac{dZ}{d\xi}\right)^2\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-2}\right]\frac{E_z}{E_o} +$$
$$\left. + \left[1 - (\frac{v_o}{c}\frac{H}{L}\frac{dZ}{d\xi})^2\left(1 + \frac{1}{\Phi_o}\frac{d\Theta}{d\xi}\right)^{-2}\right]\frac{E_{pz}}{E_o}\right\}. \qquad (11)$$

Here $\vec{E}_p$ is the strength vector of static space charge field; $q = \Phi_o(\omega_p/\omega)$ is the space charge parameter; $\omega_p = (e\rho_o/m_o\gamma\varepsilon_o)^{1/2}$ is the relativistic plasma frequency; $\Phi_c = (\omega_c/\omega)\Phi_o$ is the cyclotron transit angle; $\omega_c = eB_o/m_o\gamma_o$ is the relativistic cyclotron frequency.

The written out simultaneous equations (the equations of oscillations excitation (6), (7) and equations of electrons motion (9)–(11)), combined with initial conditions on the resonator input ($\xi = 0$), allow to determine the amplitude and frequency of oscillations on different stages of energy exchange process between the relativistic electron flow and the field of electrodynamic system.

In the work we restrict our consideration on the initial stage of the autooscillations excitation and shall determine the start oscillation conditions of the resonant generator. Relativistic active oscillators usually operate in impulse regime, so starting characteristics (as the increment of oscillation amplitude, the start generate current, the electron displacement of frequency) are of great importance, because a main parameters of generation impulse can be defined in terms of these characteristics.

3 The Small-Signal Conditions.

The amplitude F is small on the stage of self-excitation of oscillations. From the equations of oscillation excitation (6) and (7) under the condition $F \ll 1$ one can obtain the following expressions

$$\alpha = GS_1(0) - 1; \quad G_{st} = 1/S_1(0); \quad \delta\omega = -S_2(0)/S_1(0). \qquad (12)$$

where α is the increment of oscillation amplitude, G_{st} is the starting value of the parameter of interaction efficiency (it's convenient to use G_{st} instead of the start generate current I_{st}, since $G_{st} \sim I_{st}$)).

As we can see from expression (8), the conditions of oscillations beginning depend upon interaction of the electrons both with static fields (the electrical space charge field, the nonuniform magnetic guide field) and with electromagnetic ones. Determination of the contribution of these factors to the process of electron-wave energy exchange is possible on the basis of analysis and decision of the equations of electron motion (9)–(11).

To find out an explicit form of X, Z, Θ, entering in expression for $S(F)$, we express them as a power series in the oscillation amplitude F and then restrict by the linear approximation on F, as follows

$$\begin{aligned} Z(\xi,\varphi_o,Z_o) &\cong Z_o + T_z(\xi,Z_o) + FZ_1(\xi,\varphi_o); \\ X(\xi,\varphi_o,X_o) &\cong X_o + T_x(\xi,Z_o) + FX_1(\xi,\varphi_o); \\ \Theta(\xi,\varphi_o) &\cong F\Theta_1(\xi,\varphi_o). \end{aligned} \tag{13}$$

The values T_x, T_z, X_o, Z_o describe the electron trajectories in static fields only, while X_1 and Z_1 define the dynamic corrections to this trajectories, caused by action of the electromagnetic field.

Besides, we shall consider the case of weak nonhomogeneity of static magnetic field $B_o \gg \hat{B}_y, B_z$. In this case the relativistic change of electron mass (caused by their static and dynamic motion in cross direction) may not be taken into account, so $\gamma \cong \gamma_o$. Then we receive the simplified linear differential equations for values entering in (13)

$$\frac{d^2\Theta_1}{d\xi^2} = \frac{\Phi_o}{2}\gamma_o^{-3} f(\xi)\psi_y(Z)\cos(\Phi\xi+\varphi); \tag{14}$$

$$\frac{d^2T_x}{d\xi^2} + F\frac{d^2X_1}{d\xi^2} = \Phi_c\left(\frac{L}{H}\frac{B_z}{B_o} - \frac{dT_z}{d\xi} - F\frac{dZ_1}{d\xi}\right); \tag{15}$$

$$\begin{aligned} \frac{d^2T_z}{d\xi^2} + F\frac{d^2Z_1}{d\xi^2} &= -\frac{L}{2H}\gamma_o^{-1}\left\{\left(1-\frac{v_o v_\phi}{c^2}\right)Ff(\xi)\psi_y(Z)\times\right. \\ &\left.\times\sin(\Phi\xi+\varphi) + \frac{E_{pz}}{E_o}\right\} - \Phi_c\frac{dT_x}{d\xi} - \Phi_c F\frac{dX_1}{d\xi}. \end{aligned} \tag{16}$$

The initial conditions for the values entering in equations (14)–(16) are specified for nonmodulated beam with account of the scattering action of the anode aperture of electron gun [8]

$$\begin{aligned} T_x(0) &= T_z(0) = \left.\frac{dT_x}{d\xi}\right|_{\xi=0} = 0; \\ \left.\frac{dT_z}{d\xi}\right|_{\xi=0} &= \frac{L}{H}\mathrm{tg}(\alpha_z) \approx \frac{\mathrm{L}}{\mathrm{H}}\alpha_z,\ \alpha_z \to 0; \\ X_1(0) &= Z_1(0) = \left.\frac{dX_1}{d\xi}\right|_{\xi=0} = \left.\frac{dZ_1}{d\xi}\right|_{\xi=0} = 0; \\ \Theta_1(0) &= \left.\frac{d\Theta_1}{d\xi}\right|_{\xi=0} = 0. \end{aligned} \tag{17}$$

The value of the angle of electron injection to the interaction space α_z can be evaluated via the cathode-anode distance d and the coordinate of center of the anode aperture Z_{oc} [8]

$$\alpha_z \cong 2/3(Z_o - Z_{oc})H/d. \tag{18}$$

Only the static part of the space charge field is taken into account

$$E_{pz} = -E_o \frac{H}{L}\frac{2}{\gamma_o} q^2 (Z_o - Z_{oc} + FZ_1). \tag{19}$$

Here we have considered the contracting action of the proper magnetic field of the moving electrons in the flow, which results in reduction of the repulsion force between electrons in comparison with nonrelativistic case γ_o^2 times [9].

The solution of equations (14)–(16) with regard for the initial conditions (17) is given by

$$\begin{aligned}
T_z'(\xi) &= \frac{L}{H}\frac{\Phi_c}{B_o}\int_0^{\xi} \sin(\Phi_{pc}(\xi - \xi'))\int_0^{\xi'} B_z(x')dx'\, d\xi'; \\
T_z''(\xi, Z_o) &= (q/\Phi_c)^2 (Z_o - Z_{oc})(1 - \cos(\Phi_c\xi)) + \\
&+ \frac{1}{\Phi_c}\frac{L}{H}\mathrm{tg}(\alpha_z)\sin(\Phi_c\xi); \\
Z_1(\xi) &= -\frac{L}{2H\Phi_{pc}}\gamma_o^{-1} e^{-\Gamma H Z_o}\frac{1 - v_o v_\phi/c^2}{\sqrt{1 - v_\phi^2/c^2}}\int_0^{\xi} f_1(\xi')\times \\
&\times \sin(\Phi\xi' + \varphi)\sin(\Phi_{pc}(\xi - \xi'))d\xi',
\end{aligned} \tag{20}$$

where $\Phi_{pc}^2 = \Phi_c^2 - q^2$; $T_z = T_z'(\xi) + T_z''(\xi, Z_o)$;

$$f_1(\xi) = f(\xi)\exp(-\Gamma H T_z'(\xi)). \tag{21}$$

The electrons trajectories can be written in analytical form, provided that, a two-component inclined magnetic field is used for focusing of the beam ($B_y = B_o \cos\chi, B_z = B_o \sin\chi$) [10]

$$\begin{aligned}
T_z(\xi, Z_o) &= \frac{L}{H}\xi\sin\chi + \frac{1}{\Phi_c}\frac{L}{H}(\mathrm{tg}\alpha_z - \sin\chi)\sin(\Phi_c\xi) + \\
&+ \left(\frac{q}{\Phi_c}\right)^2 (Z_o - Z_{oc})(1 - \cos(\Phi_c\xi)).
\end{aligned} \tag{22}$$

Here the first summand is connected purely with inclination of magnetic field, the second one takes into account a deviation of the injection angle α_z from the direction of magnetic force lines, the third one connected with the action of static space charge. If magnetic field has more complicated form the equations of motion can be effectively solved only numerically.

Using the found solutions for X_1, Z_1, T_z, we finally obtain the expression for the complex steepness of oscillatory characteristic $S(0)$, determining the starting characteristics of the resonant relativistic generator

$$\begin{aligned} S(0) &= \frac{\hat{H}}{8}\int\limits_0^1 f_1(\xi)e^{i\Phi\xi}\int\limits_0^{\xi} f_1(\xi')e^{-i\Phi\xi'}P(\xi,\xi')\Big\{i\Phi_o\gamma_o^{-3}(\xi-\xi')+ \\ &+ i\Phi_o\frac{\Gamma H}{2}\gamma_o^{-1}\sin(\Phi_{pc}(\xi-\xi'))/\Phi_{pc}- \\ &-\gamma_o^{-1}\frac{1-v_o v_\phi/c^2}{1-v_\phi^2/c^2}\cos(\Phi_{pc}(\xi-\xi'))\Big\}d\xi d\xi', \end{aligned} \tag{23}$$

where

$$P(\xi,\xi') = \int\limits_{Z^-(\xi)}^{Z^+(\xi)} \exp\left(-\Gamma H(2Z_o + T_z''(\xi,Z_o) + T_z''(\xi',Z_o))\right) dZ_o. \tag{24}$$

The expressions (23) for the steepness of oscillatory characteristic has been compared with the analogous expression, received in one-dimensional approximation [7]. It follows that the accounting of transverse motion of the electrons and their interaction with the transverse components of electromagnetic field results in appearance in $S(F)$ additional summands and multipliers, corresponding to the contribution of static and dynamic transverse effects in the electron-wave energy exchange process.

The availability of a magnetic nonhomogeneity results in the distortion of electrons trajectories that can be treat as the change of envelope of the RF field amplitude $f(\xi)$ on a straight electron trajectory. This fact is reflected in appearance of the function $f_1(\xi)$ depending on the form of static electron trajectories (21). The effects, connected with the influence of the space charge field, with the interception of electrons by the slowing down system, and with the lens effect of the anode aperture of the electron gun, are determined by a form of the function $P(\xi,\xi')$.

In the linear theory, the effects connected with longitudinal and transverse interaction are separated. The first term in the curly brackets (23) describes longitudinal interaction for the case of an infinite strength of the magnetic guide field; the second one characterizes the efficiency change of longitudinal interaction, caused by the action of transverse components of the RF field on the electron beam; the third one describes the energy exchange between the transverse components of the RF field and the electron flow. One can see that relativistic mass factor γ_o entered in each term of (23) in negative power so it on the whole reduces $S(0)$.

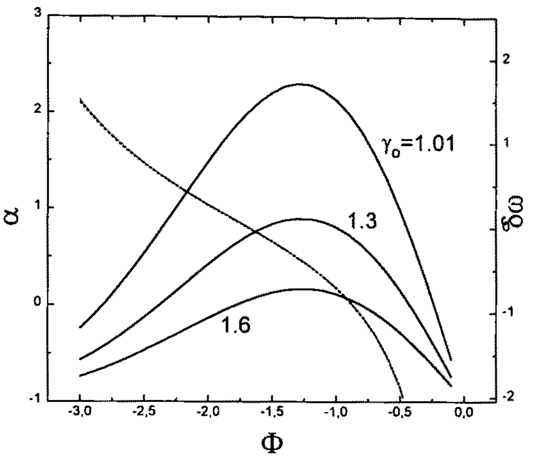

Figure 1: The increment of oscillation amplitude α (solid lines) and the electron displacement of frequency $\delta\omega$ (dashed lines) as a function of the relative transit angle Φ.

4 Discussion.

4.1 Uniform focusing field.

The starting characteristics of the relativistic generator have been calculated by the expressions (12), (23). From the point of view of economies of machine time, the static electron trajectories T_z, T_x is preferable to find out not by analytical formulas (20), but by means of numerical solution of the differential equations (15), (16). Furthermore, if we know the shape of the static electron trajectories (and, therefore, the character of beam interception by the surface of the slowing-down system), the integration limits on transverse coordinates $Z^{\pm}(\xi)$ can be easily determined at any coordinate ξ.

The parameters used for the numerical investigation are

$$w = 0.25, \quad q = 0.01, \quad A_m = 0, \quad \chi = 0, \quad L/d = 5, \quad \delta/H = 0.6, \quad Z_{oc} = 0.3.$$

Thus, the generator with uniform focusing field and weak space charge is considered. The electron beam is injected close to the surface of the slowing down system $\Delta/H = 2Z_{oc}$ (the value of the gap between beam and the grating $Z_{oc} - \Delta/2H$ affects only quantitatively).

The plots of the increment of oscillation amplitude α as a function of the relative transit angle Φ for several values of γ_o are showed by solid curves in Fig. 1. The magnetic displacement of focusing field B_o is assumed to be sufficiently strong ($\omega_c/\omega \cong 0.2$). One can see from Fig. 1, that the effect of γ_o increase is to reduce α over all range of varying of parameter

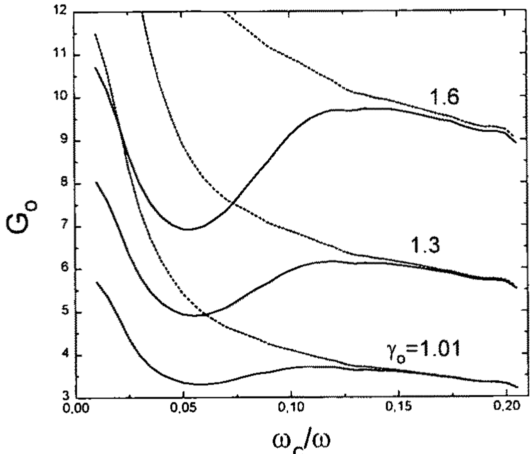

Figure 2: The minimum value of the parameter of interaction efficiency in the generation zone G_{st}^{min} as function of the normalized cyclotron frequency ω_c/ω.

Φ. In addition, the increase of γ_o is accompanied by narrowing of the generation zone on accelerating voltage $U_o \cong \Phi$. This result is analogous to the one obtained in the one-dimensional theory [7], and testifies, that the longitudinal interaction mainly contributes to the electron-wave energy exchange process at chosen conditions.

The dependencies of the electron displacement of frequency $\delta\omega$ on parameter Φ, plotted by dashed lines in Fig. 1, show that a change of relativistic factor almost does not affect on the quality of generator spectrum.

Fig. 2 shows plots of the minimum value of the parameter of interaction efficiency in the generation zone G_{st}^{min} (see Fig. 1) as function of the normalized cyclotron frequency ω_c/ω for various values of γ_o. The solid lines were calculated with accounting of all summands in the expression for linearized steepness of generator oscillatory characteristic (23). The dashed lines in Fig. 2 respect to the case of taking account only first term in (23), that corresponds to consideration of longitudinal electron-wave interaction only. One can see, that for sufficiently strong focusing of the beam ($\omega_c/\omega > 0.12$) the dashed and solid curves practically coincide, thus, the coupling between electrons and transverse components of RF field is possible to be neglected at these conditions. Actually, this result corresponds to passage to the limit of the one-dimensional theory.

As seen from comparison of dashed and solid curves in Fig. 2 a multivariation of electron-wave interaction should be taken into account at weak magnetic fields ($\omega_c/\omega < 0.12$). In this case one-dimensional approximation gives a considerable error. It is in this range of ω_c/ω that the minimum of G_{st}^{min}, corresponding to the optimum of start generate conditions, occurs. It can be explained by presence of the two factors, one of which makes conditions for improvement of start generation conditions, if the value of magnetic displacement $\omega_c/\omega \sim B_o$ is reduced, while other makes it worse. So, coupling between electrons and transverse components of RF field E_z, B_x results in some kind of sorting of the electrons relatively to the phase of E_y component if amplitude of E_y depends on transverse coordinate z. As seen from (5), the amplitude of E_y evanesces away from the grating surface. As a result, general efficiency of electron-wave interaction is increased, that is appeared in reduction of start generate current on stage of oscillation occurrence.

Note that the role of transverse effects in the process of electron-wave interaction increases with growth of the relativistic factor γ_o (see Fig. 2). For explanation of given result we shall analyze the influence of some parameters on the summands of expression (23) defining the steepness of oscillatory characteristic $S(0)$.

Fig. 3 shows the plots of the real part of $S(0)$ as a function of the relative transit angle Φ for several values of γ_o. Here, the case of rather weak focusing field ($\omega_c/\omega = 0.05$) is considered. The curves shown in Fig. 3 (a), (b) and (c) was calculated with accounting of only the first $S^{(1)}$, second $S^{(2)}$ and third $S^{(3)}$ term in curl brackets (23).

One can see from Fig. 3 that $S^{(1)}$ and $S^{(2)}$ are positive while $S^{(3)}$ is negative over all range of Φ. Hence, the processes, described by $S^{(1)}$ and $S^{(2)}$, result in the transmission of energy from the electron beam to the RF field and the interaction mechanism, corresponding to the third term $S^{(3)}$, stipulates the inverse transmission. However, as far as the absolute value of the $S^{(3)}$ is more than on an order less than $S^{(1)}$ and $S^{(2)}$ (see Fig. 3), the analysis of start-oscillation conditions can be fulfilled with taking account of $S^{(1)}$ and $S^{(2)}$ solely.

Besides, it is seen that the growth of the relativistic mass factor γ_o results in reduction of the $S^{(1)}$ and $S^{(2)}$ components of the steepness. This effect becomes apparent especially in Fig. 3(a). Comparing the characteristics between Fig. 3(a) and Fig. 3(b) we see that the values of steepness corresponding to the first and second summand in (23) are close on value for $\gamma_o = 1.6$. Moreover, at further increase of γ_o the second summand appears to be more than first, i.e. the longitudinal bunching is not determining factor of electron-wave interaction under chosen conditions.

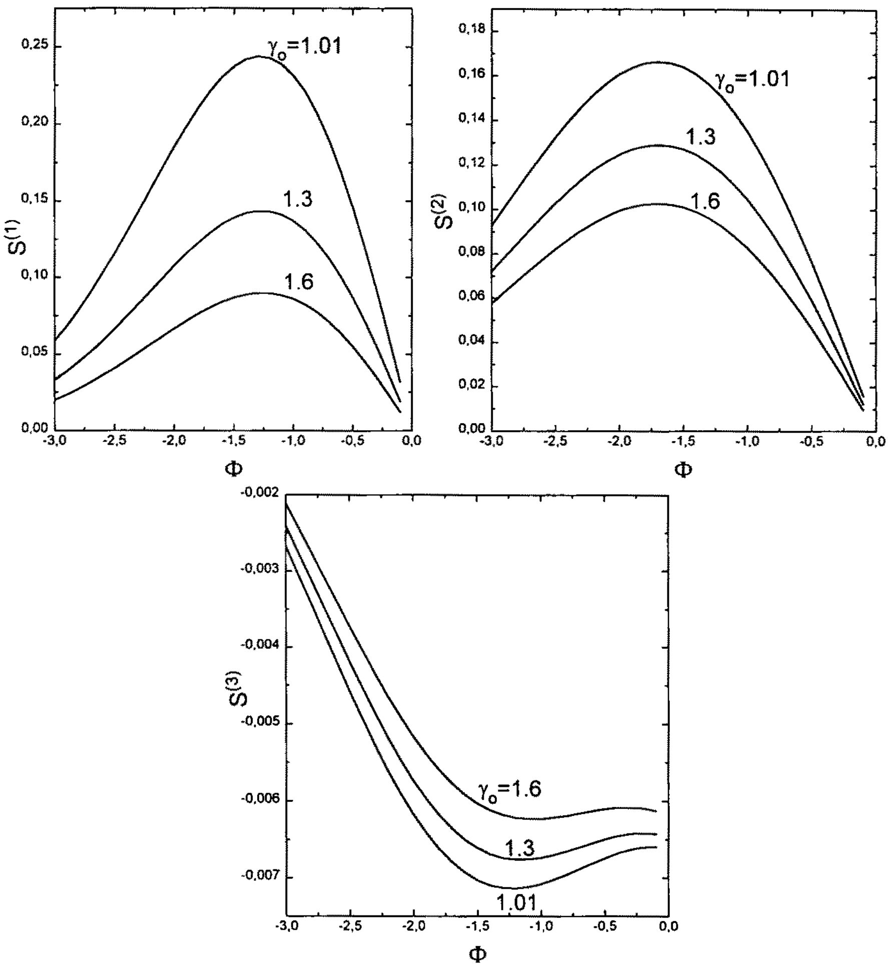

Figure 3: The real part of $S(0)$ as a function of the relative transit angle Φ, calculated with accounting of only the first $S^{(1)}$ (a), second $S^{(2)}$ (b) and third $S^{(3)}$ (c) term in curl brackets (23).

4.2 Optimized nonuniformity of focusing field.

The space distribution structure of the focusing field is assumed to be formed by the group of local magnetic nonuniformitiesis (LMN) [11] and given by

$$B_y = B_o\left[1+\sum_{i=1}^{N} a_i \exp\left(-((\xi-\xi_i)/w_i)^2\right)\right] \tag{25}$$

$$B_z = B_o\frac{2H}{L}\sum_{i=1}^{N}\frac{a_i}{w_i^2}(\xi-\xi_i)(Z-Z_i)\exp\left(-((\xi-\xi_i)/w_i)^2\right) \tag{26}$$

Here a_i is the amplitude of the ith LMN normalized on B_o; ξ_i and w_i are the center coordinate and half-width of the ith LMN, respectively; Z_i is the constant describing decrease of the z component of the magnetic displacement vector at increase of Z. Real value of Z_m should be more then the thickness of electron beam so the strength of the magnetic field changes weakly enough in that scale. The positive values of amplitudes a_i correspond to local increase while negative — to local decrease of magnetic field strength.

The minimum value of the parameter of interaction efficiency in the generation zone G_{st}^{min} is chosen as a quality criterion in the optimization task. Thus, we should find such values of LMN parameters that G_{st}^{min} achieve its minimum for fixed other system parameters. The algorithm of the parameter optimization is based on the successive change of the LMN amplitudes with fixed pitch Δa determining the calculation accuracy.

The parameters of the magnetic nonuniformity are supposed to be

$$w_i = 0.1;\quad \xi_i = -0.1+0.2i,\ i=1,...,5;\quad \Delta a = 0,01.$$

So we restricted our attention on the case of $N = 5$ and fixed the center coordinates and half-widths of each LMN. As numerical results show such an approach allows to find solution of optimization task for any real independent parameter set of the system. It can be explained by the fact that for rather big amplitudes of LMN start oscillation current will be on increase either as a result of considerable electrons interception ($a_i < 0$) or as a result of local electrons moving off from the surface of slowing down system ($a_i > 0$). Consequently, the dependence of G_{st}^{min} on the amplitude of LMN a_i has single minimum, and its position depend on the values of the rest LMN $a_j, j \neq i$.

Fig. 4 shows plots of the minimum value of the parameter of interaction efficiency in the generation zone G_{st}^{min} as function of the normalized cyclotron frequency ω_c/ω. The solid lines were calculated for the optimum distributions of magnetic nonuniformity corresponding to the different values of Φ_c, the dashed line — for uniform focusing field. It is seen from these

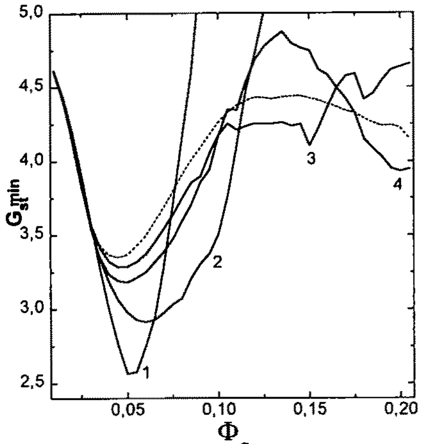

Figure 4: The minimum value of the parameter of interaction efficiency in the generation zone G_{st}^{min}: 1 – optimized focusing field distribution for $\Phi_c = 0.05$, 2 – $\Phi_c = 0.1$, 3 – $\Phi_c = 0.15$, 4 – $\Phi_c = 0.2$.

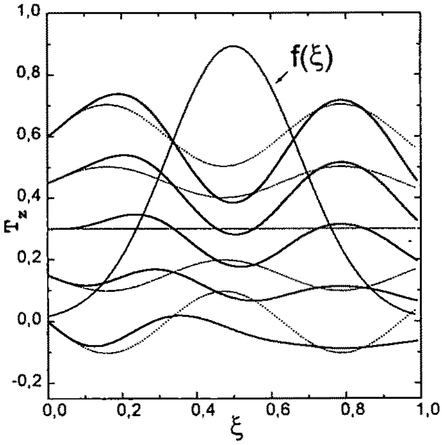

Figure 5: Static electron trajectories for $\Phi_c = 0.1$: the solid lines correspond to optimized nonuniformity, dashed — to the case $A_i = 0, i = 1,...N$.

plots that the start oscillation current is reduced as a result of the action of focusing field nonuniformity only in a rather narrow range of the focusing field strength because of change of the character of current interception. As numerical results show, the effectiveness of the optimization is appreciable just for sufficiently weak focusing field strength if the beam is injected close to the surface of the slowing down system.

The change of the electron beam static structure under action of optimized nonuniformity of the magnetic field is illustrated by the plots in Fig. 5: solid curves represent static electron trajectories $T_z(\xi)$ when we use optimized nonuniformity, dashed curves — when focusing field is uniform. From comparison of the dashed and solid lines one can see that best conditions for oscillation start are realized when static structure of the beam is changed so that lowest beam layer moves most close to the surface of the slowing down system without interception inward the resonator RF field spot $2w$. Also at the end of interaction space (without the RF field spot) active current interception is performed. It can be explained by the changing of the action envelope of the RF field amplitude $f_1(\xi)$ with respect to its optimum form [7].

5 Conclusions.

In this paper we have derived a selfconsistent equation set for a relativistic resonant O-type generator, having applied a two-dimensional model of the interaction space. This equation set is given by (6)–(11) and can be used for investigation of the device in rather general case. Physical reasons of the rise of electron-wave interaction effectiveness at the stage of oscillation beginning have been cleared up in the scope of small-signal approximation. At the small-signal conditions the electrical and magnetic transverse components of RF field improve conditions of coupling between electrons and longitudinal component of RF electrical field to be nonuniform on cross coordinate. This effect is particularly remarkable at weak magnetic guide field. The influence of relativistic mass factor at weak and strong focusing magnetic fields is mainly reduced to growth of start generate current. It was shown that start oscillation current can be appreciably reduced by means of optimization of the space distribution structure of the nonuniform focusing field.

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