Modeling the Electromagnetic Wave Diffraction on Conical Gratings Consisting of Zero-Thickness PEC Strips

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***Abstract* — The periodic gratings are effectively used for producing modern electronics and radio engineering equipment and complexes. The electromagnetic wave diffraction problem for cone gratings is considered. Impedance boundary conditions at cone structure are proposed. The problem of electromagnetic wave diffraction on cone gratings with imperfectly conducting strips is studied. Diagrams of electromagnetic field distribution are given.**

Keywords — cone gratings, impedance, boundary conditions, spectrum, diagrams.

I. INTRODUCTION

Periodical structures are widely used in modern radio engineering and electronics devices, equipments and complexes [1-3]. Various model metamaterials can be produced by employing periodical gratings [4]. Changing gratings geometry (from one hand) can lead to complicating solution of the corresponding diffraction problem, but (from the other hand) essentially widen the area of practical application of the gratings. It's well known that structures with angular parameters (cones. angular sectors, wedges, bicones etc.) have wideband and super wideband properties [5]. Diffraction harmonic wave boundary problems for radially and athimully conducting cone structures and cones with athimutal slots are considered in [6-8] .

The task of this article is to study a problem of electromagnetic harmonic wave diffraction on cone gratings with longitudinal slots.

II. PROBLEM STATEMENT

The simple periodic perfectly conducting and thin cone gratings Σ with N strips is in the field of an electric ($\chi = 1$) or magnetic ($\chi = 2$) radial harmonic ($e^{ia\omega t}$, $a = \pm 1$) dipole. The dipole is placed at the point B_0 , $\vec{p}_r$ is a moment of the dipole (Fig.1). Let us introduce a spherical coordinate system r , θ , φ with the origin at the cone tip ($r = 0$). The open cone angle is 2γ and the cone is defined by an equation $\theta = \gamma$. The cone period $l = 2\pi / N$ and the slot width d are values of dihedral angles. These angles are formed by planes those pass through the cone axis and slot edges. The problem is to find a total electromagnetic field $\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + \vec{E}_1(\vec{r})$ and $\vec{H}(\vec{r}) = \vec{H}_0(\vec{r}) + \vec{H}_1(\vec{r})$ in the presence of the conical gratings, here $\vec{E}_1(\vec{r})$, $\vec{H}_1(\vec{r})$ are diffracted fields. The total fields satisfy the Maxwell's equations, the boundary condition at the cone strips, the infinity condition and the condition of the limited energy. The solution of this problem is unique. By introducing the Debay's potentials $\upsilon^{(\chi)}(\vec{r})$ the given boundary electromagnetic problem is reduced to solving the 1-st ($\chi = 1$) or the 2-nd problem of the mathematical physics boundary problem for the Helmholtz equation with respect to $\upsilon^{(\chi)}(\vec{r})$ that is sought in the form $\upsilon^{(\chi)}(\vec{r}) = \upsilon_0^{(\chi)}(\vec{r}) + \upsilon_1^{(\chi)}(\vec{r})$, here $\upsilon_0^{(\chi)}(\vec{r})$ is a dipole potential, $\upsilon_1^{(\chi)}(\vec{r})$ is a potential that is caused by gratings presence,

$$\upsilon_1^{(\chi)} = \frac{2}{\pi^2 r_0} \sum_{m=-\infty}^{\infty} (-1)^m e^{-im\varphi_0} \times$$

$$\times \int_0^{\infty} \tau sh\pi\tau \frac{\Gamma(1/2 - m + i\tau)}{\Gamma(1/2 + m + i\tau)} a_{0\tau}^{(\chi)} b_{m\tau}^{(\chi)} \frac{K_{i\tau}(qr)}{\sqrt{r}} \hat{U}_{m,i\tau}^{(\chi)}(\theta,\varphi) d\tau,$$

$$\hat{U}_{m,i\tau}^{(\chi)}(\theta,\varphi) = \sum_{n=-\infty}^{+\infty} x_{m,n+m_0}^{(\chi)} \frac{P_{-1/2+i\tau}^{m+nN}(\pm\cos\theta)}{\frac{d^{\chi-1}}{d\gamma^{\chi-1}} P_{-1/2+i\tau}^{m+nN}(\pm\cos\gamma)} e^{i(m+nN)\varphi},$$

$a_{0\tau}^{(\chi)} = \frac{\hat{p}_\chi}{4ch\pi\tau} \frac{K_{i\tau}(qr_0)}{\sqrt{r_0}}$, $\hat{p}_\chi$, $b_{m\tau}^{(\chi)}$ are known coefficients, $x_{m,s}^{(\chi)}$ are unknown coefficients, $K_{i\tau}(qr)$ - is a Mackdonald's function. $\Gamma(z)$ is a gamma-function, $P_{-1/2+i\tau}^{m+nN}(\pm\cos\theta)$ is a Legendre's function, $q = iak$, k is a wave number, $\mathrm{Im}\, ak \le 0$, $\frac{m}{N} = m_0 + \nu$, m_0 is the nearest integer to $\frac{m}{\tilde{N}}$, $-1/2 \le \nu < 1/2$.

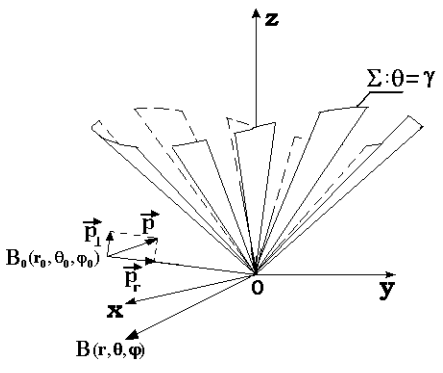

Fig. 1. Conical gratings.

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III. RESULTS

A. Perfectly conducting conical gretings

If the gratings strips are perfectly conducting then $\vec{E}\times\vec{n}\big|_{\Sigma}=0$, $\vec{n}$ is a normal to strips. By using the boundary field condition at the strips and the field continuity condition in slots one can obtain the following systems of equations for finding coefficients $x_{m,n}^{(\chi)}$:

$$\sum_{n=-\infty}^{+\infty} x_{m,n}^{(\chi)} e^{inN\varphi} = e^{im_0 N\varphi}, \qquad \text{strips } \Sigma, \qquad (1)$$

$$\sum_{n=-\infty}^{+\infty} \left[N(n+\nu)\right]^{\breve{\rho}(\chi)} \frac{|n|}{n}(1-\hat{\varepsilon}_n^{(\chi)}) x_{m,n}^{(\chi)} e^{inN\varphi} = 0, \text{ slots } \Sigma, \qquad (2)$$

$$\left[N(n+\nu)\right]^{\breve{\rho}(\chi)} \frac{|n|}{n}(1-\hat{\varepsilon}_n^{(\chi)}) =$$

$$= \frac{(-1)^{(n+\nu)/N+\chi-1} ch\pi\tau}{\pi(\sin\gamma_2)^{1-\breve{\rho}(\chi)}} \frac{\Gamma(1/2+i\tau+(n+\nu)N)}{\Gamma(1/2+i\tau-(n+\nu)N)} \times$$

$$\times \frac{1}{\frac{d^{\chi-1}}{d\gamma_2^{\chi-1}} P_{-1/2+i\tau}^{(n+\nu)N}(\cos\gamma_2) \frac{d^{\chi-1}}{d\gamma_2^{\chi-1}} P_{-1/2+i\tau}^{(n+\nu)N}(-\cos\gamma_2)},$$

$$\breve{\rho}(\chi) = (-1)^{\chi-1}.$$

Let us consider a special case of the cone gratings provided that the limit exists

$$W_\chi = \lim_{\substack{N\to+\infty \\ d/l\to 2-\chi}} \left[-\frac{1}{N} \ln\sin\left(\frac{\pi}{2}(\chi - d/l)\right)\right] > 0. \qquad (3)$$

The cone gratings is supposed to be transformed into a conical net structure that consists of lots number of strings ($\chi=1$) or a thick slot structure ($\chi=2$). In this case of the cone gratings (a semi- transparent cone) the analytical solution is obtained. Electromagnetic fields components at the semi- transparent cone surface satisfy average boundary conditions like these ones:

$$\chi=1, E_r^+ = E_r^-, \ \frac{-q}{wW_1\sin\gamma} E_r\Big|_{\Sigma} = \left(\frac{\partial^2}{\partial r^2} - q^2\right)(rj_r),$$

$$G^{\pm} = G\big|_{\theta=\gamma\pm 0}, \qquad (4)$$

here j_r is a radial component of a surface current density that is induced at the surface of the semi- transparent cone Σ;

$$\chi=2, \ E_\varphi^+ = E_\varphi^-, \ \left(\frac{\partial^2}{\partial r^2} - q^2\right)(rE_\varphi)\Big|_{\Sigma} = \frac{-wq}{4W\sin\gamma} j_\varphi \qquad (5)$$

here j_φ is an azimuthal component of a surface current density that is induced at the surface of the semi- transparent cone Σ.

B. Conical gratings consisting of imperfectly conducting strips

Let strips of the cone gratings be semi-transparent at which the average boundary conditions (4), (5) are fulfilled:

$$\vec{n}\times\vec{E}^+ = \vec{n}\times\vec{E}^-,$$

$$\vec{n}\times\left\{\vec{n}\times\left[\vec{E}^{(\chi)+} + \vec{E}^{(\chi)-}\right]\right\} = 2R^{(\chi)}\vec{n}\times L^{(1)}\left[\vec{H}^{(\chi)+} - \vec{H}^{(\chi)-}\right],$$

here $R^{(\chi)} = \frac{w}{q} W_\chi \sin\gamma$, w is a media wave resistance, W_χ is a transparency parameter (3), $\vec{n}$ is an outer normal surface vector, $L^{(1)}$ is a special differential operator of the 2-nd order.

We have the following system for $x_{m,s}^{(\chi)}$:

$$\sum_{n=-\infty}^{+\infty} \left\{1 + 2^{3-2\chi}\left(W_\chi\right)^{\breve{\rho}(\chi)} [N(n+\nu)]^{\breve{\rho}(\chi)} \frac{|n|}{n}(1-\varepsilon_n^{(\chi)})\right\} \times$$

$$\times x_n^{(\chi)} e^{inN\varphi} = 1, \qquad \text{strips;} \qquad (6)$$

$$\sum_{n=-\infty}^{+\infty} [N(n+\nu)]^{\breve{\rho}(\chi)} \frac{|n|}{n}(1-\varepsilon_n^{(\chi)}) x_n^{(\chi)} e^{inN\varphi} = 0, \text{ slots.} \qquad (7)$$

The systems (6), (7) turn into (1), (2) for $W_1=0$ or $W_2\to+\infty$ respectively. For the strips of high transparency parameter ($W^{(1)}\gg 1$) the Debay's potential has the following form ($\chi=1$):

$$\upsilon_1^{(1)} = -\frac{2}{\pi^2}\frac{\beta^*}{2W^{(1)}} \int_0^{+\infty} \tilde{K}\cdot f(\tau) \frac{P_{-1/2+i\tau}(\pm\cos\theta)}{P_{-1/2+i\tau}(\pm\cos\gamma)} d\tau +$$

$$+ \frac{2}{\pi^3}\frac{\beta^*}{W^{(1)}} \sum_{n=1}^{+\infty} \cos Nn\varphi \times \qquad (8)$$

$$\times \int_0^{+\infty} f(\tau) \frac{\tilde{K}\sin n\delta}{n} \frac{P_{-1/2+i\tau}^{-nN}(\pm\cos\theta)}{P_{-1/2+i\tau}^{-nN}(\pm\cos\gamma)} d\tau + O\left(\left(W^{(1)}\right)^{-2}\right),$$

$$D_{i\tau}^{(1)} = \frac{\pi}{ch\pi\tau} P_{-1/2+i\tau}(\cos\gamma) P_{-1/2+i\tau}(-\cos\gamma), \ \beta = 1 - \frac{\delta}{\pi},$$

$$\delta = \pi d/l, \ f(\tau) = \frac{\tau sh\pi\tau a_{0\tau}^{(1)} b_{0\tau}^{(1)}}{1/D_{i\tau}^{(1)} + \beta^*/2W^{(1)}}, \ \tilde{K} = \frac{K_{i\tau}(qr)}{\sqrt{r}}.$$

In this case the boundary problem spectrum is defined by roots of the following equation:

$$\frac{\cos\pi\varsigma}{\pi} \frac{1}{P_{-1/2+\varsigma}(\cos\gamma) P_{-1/2+\varsigma}(-\cos\gamma)} + (1-\delta/\pi)/2W = 0, \qquad (9)$$

$$(-1)^{pN} \frac{\cos\pi\varsigma}{\pi} \frac{\Gamma(1/2+\varsigma-pN)}{\Gamma(1/2+\varsigma+pN)} \frac{1}{P_{-1/2+\varsigma}^{-pN}(\cos\gamma) P_{-1/2+\varsigma}^{-pN}(-\cos\gamma)} +$$

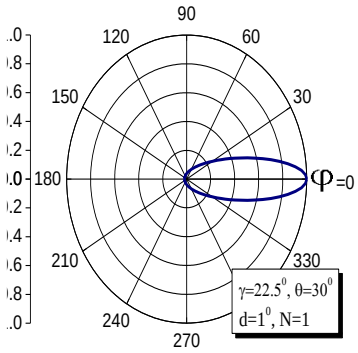
$$+ (1-\delta/\pi)/2W = 0, \ p = 1,2... \qquad (10)$$

For $W_1\gg 1$ roots of (9) are near roots of the equation $\cos\pi\varsigma = 0$:

$$\tilde{\varsigma}_s^{(1)*} = \frac{1}{2} + s + \frac{1-\delta/\pi}{2W}\left[P_s(\cos\gamma)\right]^2 + O\left(W^{-2}\right), \qquad (11)$$

$s = 0,1,2...$, and roots of (10) are near roots of $\Gamma(1/2+\varsigma-pN)\cos\pi\varsigma = 0$.

The least spectrum value ($W_1\gg 1$) is $\tilde{\varsigma}_0^{(1)*}$ that defines field behavior at the tip of the unclosed semi-transparent cone. The set of eigenvalues of (11) for the semi-transparent cone with a slot ($W_1\gg 1, N=1$) is an exited spectrum of the isotropic semi-transparent cone $\{\varsigma_s\}$



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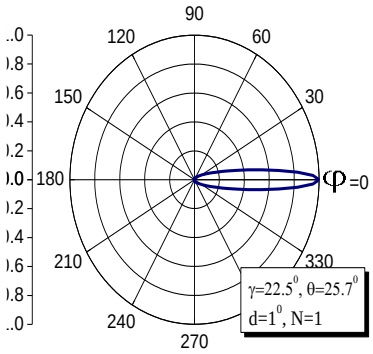

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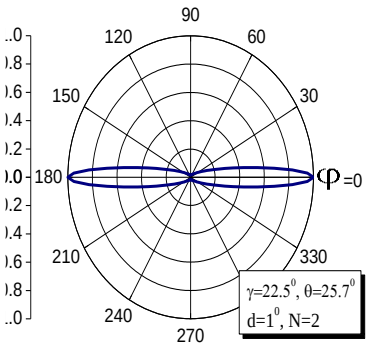

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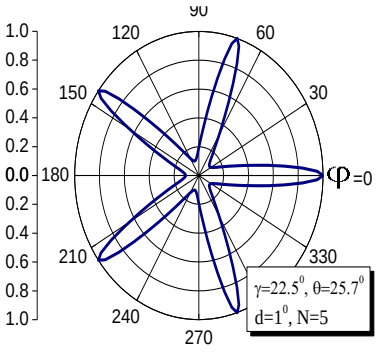

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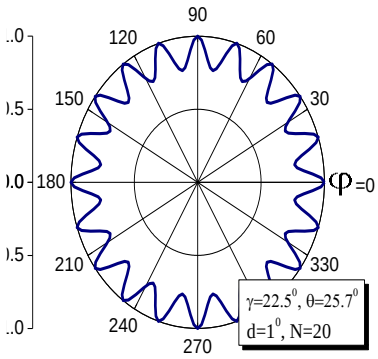

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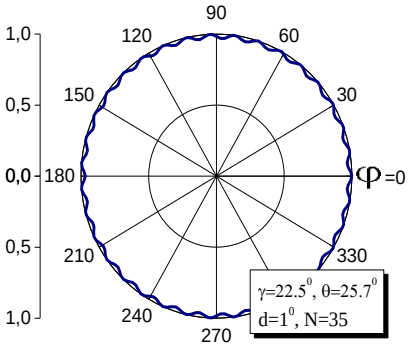

g)

Fig.2 Normal space field distribution.

$$\varsigma_s = \frac{1}{2} + s + \frac{1}{2W}\left[P_s\,(\cos\gamma)\right]^2 + O\left(W_1^{-2}\right),\ s = 0,1,2,\ldots,$$

and

$$\tilde{\varsigma}_s^{(1)*} = \varsigma_s - \frac{\delta/\pi}{2W_1}\left[P_s\,(\cos\gamma)\right]^2,\ \tilde{\varsigma}_0^{(1)*} = \varsigma_0 - \frac{\delta/\pi}{2W_1},\ \tilde{\varsigma}_s^{(1)*} \le \varsigma_s .$$

Electromagnetic field behavior in the vicinity of the cone tip ($qr << 1, W_1 >> 1$) can be estimated as

$$\left|\vec{E}\right| \sim \left|qr\right|^{-1+1/2W_1-\delta/2\pi W_1},\ \left|\vec{H}\right| \sim \left|qr\right|^{1/2W-\delta/2\pi W}.$$

We can conclude that the slot presence leads to increasing an electric field singularity at the cone tip.

For the semi-transparent cone with a narrow slot ($N = 1, \delta << 1$) the Debay's potential can be represented in the form (far from the slot)

$$\upsilon_1^{(1)} = -\frac{2}{\pi^2}\int_0^{+\infty}\frac{\tilde{K}\tau sh\pi\tau a_{0\tau}b_{0\tau}}{\left(2W/D_{i\tau}+1\right)-\delta/\pi}\frac{P_{-1/2+i\tau}(\pm\cos\theta)}{P_{-1/2+i\tau}(\pm\cos\gamma)}d\tau +$$

$$+\frac{2\delta}{\pi^3}\sum_{n=-\infty}^{+\infty}e^{in\varphi}\int_0^{+\infty}\frac{\tilde{K}\tau sh\pi\tau a_{0\tau}b_{0\tau}}{\left(2W/D_{i\tau}^{(-p)}+1\right)-\delta/\pi}\frac{P_{-1/2+i\tau}^{-n}(\pm\cos\theta)}{P_{-1/2+i\tau}^{-n}(\pm\cos\gamma)}d\tau + \quad (12)$$

$$+O\left(\frac{1}{W}\delta^2\ln\frac{1}{\delta}\right),$$

$$D_{i\tau}^{(M)} = (-1)^M \frac{\pi}{ch\pi\tau} \frac{\Gamma(1/2+i\tau-M)}{\Gamma(1/2+i\tau+M)} P_{-1/2+i\tau}^{M}(\cos\gamma) P_{-1/2+i\tau}^{M}(-\cos\gamma),$$

$$\frac{1}{W}\delta^2 \ln\frac{1}{\delta} \ll 1.$$

Representation of $\upsilon_1^{(1)}$ (12) can be written as the series of integrand residues. Terms of the series are waves caused by the unclosed semi-transparent cone. These waves are defined by boundary problem spectrum values satisfying the equation

$$B_{\varsigma}^{-p}(\gamma, W_1) - \frac{\delta}{\pi} = 0, \qquad (13)$$

$$\hat{B}_{\varsigma}^{-p}(\gamma, W) = (-1)^p \frac{\pi}{\cos\pi\varsigma} \frac{\Gamma(1/2+\varsigma+p)}{\Gamma(1/2+\varsigma-p)} \times$$

$$\times P_{-1/2+\varsigma}^{-p}(\cos\gamma) P_{-1/2+\varsigma}^{-p}(-\cos\gamma) + 2W, \ p = 0,1,2\ldots .$$

If $\delta \ll 1$ the roots of (13) $\hat{\varsigma}_s^{-p} = \hat{\varsigma}_s^{-p}(\gamma, W_1, \delta)$ are near the roots $\hat{\varsigma}_s^{-p} = \hat{\varsigma}_s^{-p}(\gamma, W_1, \delta)$, $\hat{B}_{\varsigma}^{-p}(\gamma, W_1) = 0$ and

$$\hat{\varsigma}_s^{-p} = \hat{\varsigma}_s^{-p} + \frac{\delta}{\pi}\left(\frac{d}{d\varsigma}\hat{B}_{\varsigma}^{-p}\Big|_{\varsigma=\hat{\varsigma}_s^{-p}}\right)^{-1} + O\left(\delta^2\right), \ s = 0,1,2\ldots .$$

The least spectrum value is $\hat{\varsigma}_0^0 =$

$$= \varsigma_0^0 + \frac{\delta}{\pi^2}\left(\frac{d}{d\varsigma}\left\{\frac{1}{\cos\pi\varsigma} P_{-1/2+\varsigma}(\cos\gamma) P_{-1/2+\varsigma}(-\cos\gamma)\right\}\Big|_{\varsigma=\hat{\varsigma}_0}\right)^{-1} +$$

$$+ O\left(\delta^2\right).$$

If $W_1 \gg 1$ and $\delta \ll 1$ then

$$\hat{\hat{\varsigma}}_0 = \frac{1}{2} + \frac{1}{2W_1} + \frac{\delta}{2\pi\left[\ln\left(\frac{1}{2}\sin\gamma\right) - W_1\right]} + O\left(\tilde{\tilde{\eta}}\right),$$

$$\delta \ln\left(\frac{1}{2}\sin\gamma\right) \ll 1,$$

or

$$\hat{\hat{\varsigma}}_0 = \frac{1}{2} + \frac{1}{2W_1}\left(1 - \frac{\delta}{\pi}\right) + O\left(\tilde{\tilde{\eta}}\right), \ \tilde{\tilde{\eta}} = \max\left(\delta^2, \delta/W, 1/W^2\right).$$

Electromagnetic field behavior ($qr \ll 1$) is estimated as

$$\left|\vec{E}\right| \sim \left|qr\right|^{-3/2+\hat{\hat{\varsigma}}_0}, \ \left|\vec{H}\right| \sim \left|qr\right|^{-1/2+\hat{\hat{\varsigma}}_0}.$$

The representation (8) is true for any slot width and any number of slots.

Let the dipole be at the cone axis ($\theta_0 = \pi$, $\chi = 2$) and near the cone tip ($kr_0 \ll 1$).

The analytical solutions are obtained and space field distributions are studied for the cone gratings with narrow slots ($d/l << 1$). It is showed that perpendicular field components have squared singularities at the slot edge whereas parallel ones are bounded. In Fig. 2 diagrams space field distribution in the azimuth plane are represented.

The field distribution analysis shows that the fields are concentrated near slots. Increasing number of slots changes the field distribution diagram form.

IV. Conclusions

On the basis of the foregoing material of the article one can draw the following conclusions:

1. The boundary conditions at the imperfectly conducting cone strips are proposed. These impedance boundary conditions contain the open angle of the cone.

2. In the particular case of the impedance conical strips the rigorous analytical solution of the diffraction problem is obtained.

3. The field distribution diagrams are given for one mode field approximation (the electromagnetic field source is placed in the closed neighborhood of the cone tip).

4. The results given in the paper can be used for creating modern matemeterial of the cones surfaces and producing complex radioengineering and electronics equipment.

References

[1] L. Deng, T. Hening, P.J. Klar, S. Feng, Z. Xie, S. Wang, J.Ye. Sun, P. Han and Y. Zhang, "Optimization of the Rayleigh anomaly of metallic gratings for terahertz sensor applications," IOP J. Opt., vol. 16, no 9, pp. 094015-7, 2014.

[2] S. Edward, A. Antoncecchi, H. Zhang, H. Sielcken, S. Witte, and P. C. M. Planken, "Detection of periodic structures through opaque metal layers by optical measurements of ultrafast electron dynamics," Optics Express, vol. 26, no 18, pp. 23380-23396, 2018.

[3] Y. Han, X.-Q. Luo, T. Li, W. Zhang, S.-P. Wang, J.S. Tsai, F. Nori and J. Q. You, "Time-domain grating with a periodically driven qutrit," Phys. Rev., vol. 11, pp. 014053-5, 2019.

[4] I.V. Shadrivov, M. Lapine and Yu.S. Kivshar, "Nonlinear, tunable and active metamaterials," Springer Intern. Publishing, 324 p., 2015.

[5] C. A. Balanis, "Antenna theory: analysis and design," 4th Edition. Wiley, 1096 p., 2016.

[6] G. G. Goshin, G. G. Kravchenko and V. G. Myshkin, "Scattering of planar electromagnetic waves by a radially conducting bicone reflector," Radiotekhnika and Elektronika, vol. 25, pp. 232-240, 1980.

[7] G.G. Goshin and A. R. Chesnokov, "Diffraction of electromagnetic waves at a plane with azimuthal conductivity," Radiophysics and Quantum Electron., vol. 4, pp. 5-9, 1985.

[8] D. B. Kuryliak and O. B. Trishchuk, "TM- waves radiation from the azimuthal slot in the sphere-conical resonator," Radio-physics and Radio astronomy, vol. 18, no 1, pp. 87-95, 2013.