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ELECTRODYNAMICS AND WAVE PROPAGATION

Scattering of the Field of an Electric Dipole by a Conic Structure with Longitudinal Slots

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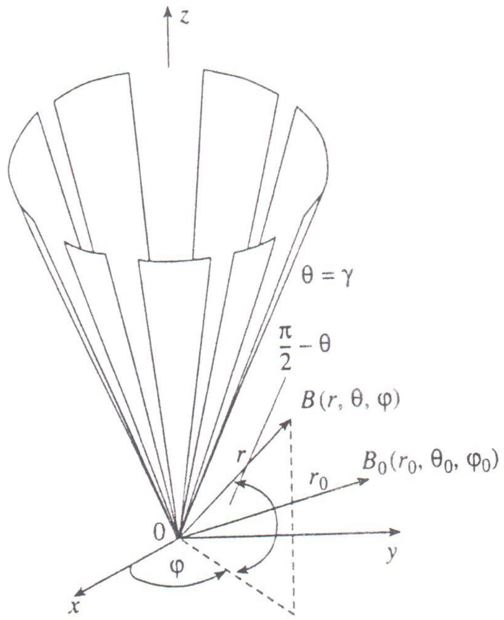
Abstract—The excitation of a semi-infinite perfectly conducting infinitely thin circular cone with semi-infinite slots by a radial electric dipole is investigated. The slots are cut along the cone generatrices and equally spaced. The problem is solved based on the Kontorovich–Lebedev integral transform and the Riemann–Hilbert problem method. Analytic solutions are obtained for a semitransparent cone and a cone with narrow slots. The effect of slots on the field structure, polarization, and behavior in the proximity of the cone vertex is studied.

INTRODUCTION

The generation of super-wide-band radiation is an important problem in modern information technology. Conic structures are omnidirectional and super-wide-band in radiation pattern and matching [1]. They are widely applied in radar, communications, and telemetry [2–5]. The presence of inhomogeneities, in particular, of slots on the screen surface, affects the structure and polarization of the scattered field [2–5], while, by varying their number and size, one can control scattering characteristics and create directive radiation. Electrodynamic problems for inhomogeneous cones and bicones are investigated in [2–6]. Conic structures with transverse (azimuthal) slots are studied in [2, 4]; spirally or radially conducting surfaces are studied in [5]. Scattering from a semi-infinite perfectly conducting elliptic cone with a longitudinal slot whose width is much smaller than its length is investigated in [6]. We consider a semi-infinite perfectly conducting infinitely thin circular cone with semi-infinite slots. The slots are cut along the generatrices of the cone and equally spaced. This structure is a model of a conical antenna with a controlled radiation pattern and polarization. Our purpose is to investigate the excitation of the cone with equally spaced longitudinal slots by a radial electric dipole. Particular cases of this structure (a cone with a longitudinal slot; a planar angular sector, e.g., a quadrant; a cone with two symmetric slots, which is a model of the *V*-shaped antenna, etc.) are of interest in themselves in both theory and applications. The method proposed for solving the problem is based on the Kontorovich–Lebedev integral transform and the method of the Riemann–Hilbert problem. In the cases of a semitransparent cone and a cone with narrow slots, an analytical solution to the problem is derived and the structure and polarization of the field are analyzed as well as the field behavior in the proximity of the boundary inhomogeneities (the tip of the cone and edges of the conic strips).

FORMULATION OF THE PROBLEM

In the spherical coordinate system (r, ϑ, φ), a semi-infinite perfectly conducting infinitely thin circular cone Σ with N equally spaced slots cut along the generatrices is specified by the equation $\vartheta = \gamma$. We denote the angular width of the slots and the spacing of the conic structure by d and $l = 2\pi/N$, respectively (d and l are dihedral angles formed by the planes that pass through the axis of the cone and the edges of two neighboring cone strips). The cone is excited by a radial electric dipole that is located at point $B_0(r_0, \vartheta_0, \varphi_0)$ and has electric moment $\vec{e}$. The dipole produces time-harmonic



Problem geometry.

fields $\vec{E}^{(i)}$ and $\vec{H}^{(i)}$. Fields $\vec{E}^{(s)}$ and $\vec{H}^{(s)}$ are scattered by the cone. Total fields $\vec{E}$ and $\vec{H}$

$$\vec{E} = \vec{E}^{(i)} + \vec{E}^{(s)}, \quad \vec{H} = \vec{H}^{(i)} + \vec{H}^{(s)} \tag{1}$$

satisfy the Maxwell equations in a medium with constant permeability μ and permittivity ε, the boundary condition on the strips

$$E_t|_\Sigma = 0,$$

the radiation condition, and the energy finiteness condition. In this formulation, the electrodynamic problem has a unique solution. In order to find it, it is convenient to use Debye potential U [7], which satisfies the homogeneous Helmholtz equation outside the cone and the source, the Dirichlet boundary condition on the strips, the principle of ultimate absorption, and the edge condition in the proximity of inhomogeneities of the boundary. In accordance with the structure of the total field, we represent U in the form

$$U = U^{(i)} + U^{(s)},$$

where

$$U^i = \frac{|\vec{e}|}{r_0}\frac{\exp(-ikR)}{R}, \quad k = \frac{\omega}{c}\sqrt{\varepsilon\mu}, \tag{2}$$
$$\operatorname{Im} k \le 0, \text{ and } R = |\vec{r} - \vec{r}_0|.$$

One of the efficient methods for investigating boundary-value problems with the conic geometry is the Kontorovich–Lebedev integral transform [8]

$$G(\tau) = \int_0^\infty g(r)\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}dr, \tag{3}$$

$$g(r) = -\frac{1}{2}\int_0^{+\infty} \tau\sinh\pi\tau\exp(\pi\tau)G(r)\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}d\tau. \tag{4}$$

Since $U^{(i)}$ can be represented as [5]

$$U^{(i)} = -\frac{1}{2}\int_0^{+\infty} \tau\sinh\pi\tau\exp(\pi\tau) \times \sum_{m=-\infty}^{+\infty} a_m(\tau, k)\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}U_{m\tau}^{(i)}(\vartheta, \vartheta_0)d\tau,$$

$$a_m(\tau, k) = \frac{\pi|\vec{e}|}{r_0}\frac{(-1)^m\exp(-im\varphi_0)}{\cosh\pi\tau}\frac{H_{i\tau}^{(2)}(kr_0)}{\sqrt{r_0}} \times \frac{\Gamma\left(\frac{1}{2} - m + i\tau\right)}{\Gamma\left(\frac{1}{2} + m + i\tau\right)},$$

$$U_{i\tau}^{(i)}(\vartheta, \vartheta_0) = \begin{cases} P^m_{-1/2+i\tau}(\cos\vartheta)P^m_{-1/2+i\tau}(-\cos\vartheta_0), \\ 0 < \vartheta < \vartheta_0 \\ P^m_{-1/2+i\tau}(-\cos\vartheta)P^m_{-1/2+i\tau}(\cos\vartheta_0), \\ \vartheta_0 < \vartheta < \pi, \end{cases}$$

we look for the solution to the boundary-value problem in the form of the Kontorovich–Lebedev integral (4)

$$U^{(s)} = -\frac{1}{2}\int_0^{+\infty} \tau\sinh\pi\tau \sum_{m=-\infty}^{+\infty} b_{m\tau}^{(s)}U_{m\tau}^{(s)}(\vartheta, \varphi)\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}d\tau, \tag{5}$$

where

$$b_{m\tau}(\tau, k) = -a_m(\tau, k)P^m_{-1/2+i\tau}(-\cos\vartheta_0)P^m_{-1/2+i\infty}(\cos\gamma),$$

$$U_{m\tau}^{(s)} = \sum_{n=-\infty}^{+\infty} x_{m, n+m_0}(\tau)\frac{P^{m+nN}_{-1/2+i\tau}(\pm\cos\vartheta)}{P^{m+nN}_{-1/2+i\tau}(\pm\cos\gamma)} \times \exp(i(m+nN)\varphi), \tag{6}$$

$H_{i\tau}^{(2)}(kr)$ is the Hankel function, $\Gamma(z)$ is the gamma function, $P_\zeta^m(\cos\vartheta)$ is the associated Legendre function, $x_{m,n}$ are unknown coefficients, m_0 is the integer closest to m/N, $\nu = m/N - m_0$, and $-1/2 \le \nu < 1/2$. In (6), the upper and lower signs correspond to the regions $0 < \vartheta < \gamma$ and $\gamma < \vartheta < \pi$, respectively. The condition that the energy must be finite imposes restrictions on coefficients $x_{m,n}$, requiring that they belong to the Hilbert space l^2 with the norm

$$\|\xi\|^2 = \sum_{n=-\infty}^{+\infty}(1 + |n|)|\xi_n|^2.$$

PAIRED SUMMATION EQUATIONS: THE INFINITE SYSTEM OF LINEAR ALGEBRAIC EQUATIONS OF THE SECOND KIND

The boundary condition imposed on the conical strips and the field continuity condition on the slots yield a system of paired functional summation equations of the first kind whose kernels have the form of trigonometric functions

$$\sum_{n=-\infty}^{+\infty} x_{m,n}\exp(inN\varphi) = \exp(im_0N\varphi), \quad \pi d/l < |N\varphi| \le \pi, \tag{7}$$

$$\sum_{n=-\infty}^{+\infty} N(n+\nu)\frac{|n|}{n}(1 - \varepsilon_{m,n})x_{m,n}\exp(inN\varphi) = 0, \quad |N\varphi| < \pi d/l, \tag{8}$$

where

$$N(n+\nu)\frac{|n|}{n}(1-\varepsilon_{m,n})$$
$$= \frac{(-1)^{(n+\nu)N}}{\pi}\cosh\pi\tau\frac{\Gamma(1/2+i\tau+(n+\nu)N)}{\Gamma(1/2+i\tau-(n+\nu)N)} \quad (9)$$
$$\times\frac{1}{P^{(n+\nu)N}_{-1/2+i\tau}(\cos\gamma)P^{(n+\nu)N}_{-1/2+i\tau}(-\cos\gamma)}.$$

Using the asymptotics of $P^m_\nu(\cos\gamma)$ for $m \gg 1$ [9], one can show that the following estimate for $\varepsilon_{m,n}$ is valid:

$$\varepsilon_{m,n} = O\left(\frac{1}{N^2(n+\nu)^2}\right). \quad (10)$$

The considered excitation induces only the radial component j_r of the surface current density, which can be expressed in terms of $x_{m,n}$ as

$$j_r = \frac{2ik}{\sin\gamma}\sum_{m,n=-\infty}^{+\infty}\exp(i(n+\nu)N\varphi) \quad (11)$$
$$\times\int_0^{+\infty}\frac{H^{(2)}_{i\tau}(kr)}{\sqrt{r}}b_m(\tau,k)N(n+\nu)\frac{|n|}{n}(1-\varepsilon_{m,n})x_{m,n}(\tau)d\tau.$$

The operator of functional equations (7) and (8) is unbounded and non-self-adjoint. Therefore, these equations cannot be solved using the apparatus of operator equations in Hilbert spaces [10, 11]. We have managed to construct the left regularizer for (7) and (8) and reduce this system to the Fredholm infinite system of linear algebraic equations (ISLAE) of the second kind [11, 12]. We modify the system of paired equations into a form that is suitable for regularization. Introduce coefficients $y_{m,n}$ coupled with $x_{m,n}$ by

$$y_{m,n} = (-1)^{n-m_0}\frac{n+\nu}{m_0+\nu}\frac{|n|}{n}(1-\varepsilon_{m,n})x_{m,n}. \quad (12)$$

Differentiating (7) with respect to φ yields the following system of functional equations:

$$\sum_{n=-\infty}^{+\infty}\frac{|n|}{n}(1-\delta_{m,n})y_{m,n}\exp(in\psi) = \exp(im_0\psi), \quad (13)$$
$$|\psi| < (l-d)\pi/l$$

$$\sum_{n=-\infty}^{+\infty}y_{m,n}\exp(in\psi) = 0, \quad (l-d)\pi/l < |\psi| \le \pi, \quad (14)$$

with the additional condition

$$\sum_{n=-\infty}^{+\infty}\frac{1}{n+\nu}\frac{|n|}{n}(1-\delta_{m,n})y_{m,n} = \frac{1}{m_0+\nu}, \quad (15)$$

where

$$1-\delta_{m,n} = \frac{1}{1-\varepsilon_{m,n}}, \quad \psi = N\varphi - \frac{|\varphi|}{\varphi}\pi. \quad (16)$$

Splitting the operator corresponding to the left-hand side of the system into leading and compact parts and inverting the leading part by the Riemann–Hilbert problem method [13], we obtain the Fredholm ISLAE of the second kind for $y_{m,n}$

$$M_\nu(u)y_{m,0} = V^{m_0}(u) + \sum_{s=-\infty}^{+\infty}\frac{|s|}{s}\delta_{m,s}V^s(u)y_{m,s}, \quad (17)$$

$$M_\nu(u) = \frac{1}{\nu}\frac{P_\nu(-u)-P_\nu(-u)}{P_{\nu-1}(-u)+P_\nu(-u)},$$

$$y_{m,q} = V^{m_0}_{q-1}(u)$$
$$+\sum_{s=-\infty}^{+\infty}\frac{|s|}{s}\delta_{m,s}y_{m,s}V^{s-1}_{q-1}(u) + y_{q,0}P_q(u), \quad (18)$$

where $u = \cos(\pi(l-d)/l)$, $P_\nu(u)$ is the Legendre function, and functions $V^{m-1}_{n-1}(u)$ and $V^m(u)$ are defined and calculated as in [14]. As a result of regularizing paired functional equations (7) and (8), system (17) and (18) is equivalent to them. Thus, the initial electrodynamic problem is reduced to solving an infinite system of equations for Fourier coefficients of the Debye potential. Unknown $y_{m,n}$ are independent of the wave-number, which is convenient for determining the field both near the vertex ($kr \ll 1$) and far from it ($kr \gg 1$). A solution to the ISLAE exists, is unique, and can be found by the truncation (reduction) method. The higher the order of truncation, the more exact is the approximate solution obtained by the truncation method [11]. For a semitransparent cone with a large number of slots whose width is comparable to the structure spacing and for a cone with narrow slots, the norm of the matrix operator is less than unity. Hence, we can solve system (17) and (18) by the method of successive approximations (iterations). Taking into account only the first iteration, we obtain the following expressions for $y_{m,n}$:

$$y_{m,0} = \frac{V^{m_0}(u) + \sum_{s\neq 0}A_{ms}[V^{m_0-1}_{s-1}(u)+\Phi^{m_0}_s(u)]}{M_\nu(u)+\frac{1}{\nu}(1-\delta_{m,0}) - \sum_{s\neq 0}A_{ms}W^{m_0}_s(u)}, \quad (19)$$

$$y_{m,s} = \frac{1}{1-\frac{|s|}{s}\delta_{m,s}V^{s-1}_{s-1}(u)} \quad (20)$$
$$\times[V^{m_0-1}_{s-1}(u)+\Phi^{m_0}_s(u)+y_{m,0}W^m_p(u)], \quad s\neq 0,$$

where

$$W_s^m(u) = P_s(u) + \delta_{m,0}V_{s-1}^{-1}(u) + \Phi_s(u),$$

$$A_{ms} = \frac{1}{1 - \frac{|s|}{s}\delta_{m,s}V_{s-1}^{s-1}(u)}\frac{|s|}{s}\delta_{m,s}V^s(u) \qquad (21)$$

and the estimates for $\Phi_s^{m_0}(u)$ and $\Phi_s(u)$ are

$$|\Phi_s^{m_0}(u)| < \frac{p}{1-p}[1 - u + \sqrt{1-u}\ln(1 + |m_0|)], \quad m_0 \neq 0,$$

$$|\Phi_s(u)| < \frac{p}{1-p}(1 + |\delta_{m,0}|\sqrt{1-u}), \qquad (22)$$

$$p < \text{const}\frac{\sqrt{1-u}^2}{N^2}\sin^2\gamma.$$

A SEMITRANSPARENT CONE

Passing to the limit in (19) and (20) as $N \longrightarrow +\infty$ and $d/l \longrightarrow 1$, we find $y_{m,n}$ and $x_{m,n}$ for a semitransparent cone, which is the limit case of the cone with longitudinal slots when the following limit exists:

$$\lim_{\substack{N \to +\infty \\ d/l \to 1}}\left[-\frac{1}{N}\ln\cos\frac{\pi d}{2l}\right] = Q > 0. \qquad (23)$$

Substituting the limit values into (6) yields the expression for the Debye potential ($\gamma < \vartheta_0$)

$$U^{(s)} = -\frac{1}{2}\sum_{m=-\infty}^{+\infty}\exp(im\varphi)\int_0^{+\infty}\frac{\tau\sinh\pi\tau\exp(\pi\tau)a_m(\tau, k)}{1 + 2m(1-\varepsilon_{m,0})Q} \qquad (24)$$
$$\times \frac{H_{i\tau}^{(2)}(kr)}{\sqrt{kr}}\frac{P_{-1/2+i\tau}^m(-\cos\vartheta)}{P_{-1/2+i\tau}^m(-\cos\gamma)}d\tau, \quad \gamma < \vartheta < \pi.$$

This expression is valid for the source and observation points coupled by the condition $\vartheta + \vartheta_0 > \pi + 2\gamma$, which follows from the condition that the integral must converge and which corresponds to the region where the field scattered by the vertex exists. A similar representation can also be obtained for $U^{(s)}$ when $0 < \vartheta < \gamma$. On the surface of the semitransparent cone, components of the electromagnetic field satisfy the averaged boundary conditions of the form

$$E_r^+ = E_r^-, \qquad (25)$$

$$-\frac{ik}{wQ\sin\gamma}E_r = \left(\frac{\partial^2}{\partial r^2} + k^2\right)(r\tilde{H}_\varphi), \qquad (26)$$

where $\tilde{H} = H^+ - H^-$, while f^+ and f^- stand for the limit values of f at $\vartheta = \gamma \pm 0$. Consider the axisymmetric excitation of the cone ($\varphi_0 = 0$, $\vartheta_0 = \pi$). Passing to the integration over the imaginary axis ($\mu = i\tau$), we modify the expression for $U^{(s)}$ to the form

$$U^{(s)} = \frac{\pi^2|\vec{e}|}{2r_0\sqrt{rr_0}}\int_{-i\infty}^{+\infty}\frac{\mu T_\mu(r, r_0)}{\Delta_\mu\cos\pi\mu}[P_{-1/2+\mu}(\cos\gamma)]^2 \qquad (27)$$
$$\times P_{-1/2+\mu}(-\cos\vartheta)d\mu, \quad \gamma < \vartheta < \pi,$$

where

$$\Delta_\mu = \pi P_{-1/2+\mu}(\cos\gamma)P_{-1/2+\mu}(-\cos\gamma) + 2Q\cos\pi\mu, \qquad (28)$$

$$T_\mu(r, r_0) = \begin{cases} I_\mu(kr)H_\mu^{(2)}(kr_0), & r < r_0, \\ H_\mu^{(2)}(kr)I_\mu(kr_0), & r > r_0. \end{cases}$$

Employing the Cauchy residue theorem, we can expand the integral in (27) in residues at the poles of the integrand and obtain the solution to the problem in the form of a series.

Integral representation (27) is useful for analyzing the far field ($kr \gg 1$); the series representation is useful for analyzing the field near the cone vertex and when the source is located near the vertex ($kr_0 \ll 1$). We provide a series representation of one of the components of the total field

$$E_\vartheta = -\frac{\pi^3 i|\vec{e}|}{rr_0\sqrt{r_0}}\sum_{s=0}^{+\infty}\frac{\mu_s(\mu_s^2 - 1/4)\frac{d}{dr}(\sqrt{r}T_{\mu_s}(r, r_0))}{\cos\pi\mu_s\frac{d}{d\mu}\Delta_\mu|_{\mu=\mu_s}} \qquad (29)$$
$$\times [P_{-1/2+\mu_s}(\cos\gamma)]^2 P_{-1/2+\mu_s}(-\cos\vartheta), \quad \gamma < \vartheta < \pi,$$
$$\Delta_{\mu_s} = 0.$$

The spectrum of the boundary-value problem for the semitransparent cone is determined by roots of the equation $\Delta_\mu = 0$. The minimum root characterizes the field behavior near the cone vertex. In the particular case of a semitransparent cone, when $Q \ll 1$, these roots are located near the zeros of function $P_{-1/2+\mu}(\pm\cos\gamma)$

$$\mu_s^\pm = \alpha_s^\pm$$
$$-\frac{2Q\cos\pi\alpha_s^\pm}{\pi\frac{d}{d\mu}[P_{-1/2+\mu}(\cos\gamma)P_{-1/2+\mu}(-\cos\gamma)]|_{\mu=\alpha_s^\pm}} + O(Q^2), \qquad (30)$$
$$P_{-1/2+\alpha_s^+}(\cos\gamma) = 0, \quad P_{-1/2+\alpha_s^-}(-\cos\gamma) = 0.$$

Thus, in this case, the spectrum has the form of a perturbed spectrum of the Dirichlet boundary-value problem for an axisymmetrically excited solid cone [15]. Near the cone vertex, the electric and magnetic fields are proportional to $(kr)^{-3/2+\mu_0^-}$ and $(kr)^{-1/2+\mu_0^-}$, respectively, where

$$\mu_0^- = \alpha_0^- - AQ + O(Q^2),$$

$$A = \frac{2\cos\pi\mu_0}{\pi P_{-1/2+\alpha_0^-}\frac{d}{d\mu}P_{-1/2+\mu}(-\cos\gamma)\big|_{\mu=\alpha_0^-}}. \quad (31)$$

Taking into consideration the behavior of function $P_{-1/2+\mu}(-\cos\gamma)$ in the vicinity of first root α_0^- [16] and assuming that $\gamma \le \pi/2$ $(1/2 < \alpha_0^- \le 3/2)$, we find that $A > 0$. Near the vertex of the solid circular cone, the electric and magnetic fields are proportional to $(kr)^{-3/2+\alpha_0^-}$ and $(kr)^{-1/2+\alpha_0^-}$, respectively [17]. Hence, near the vertex of the semitransparent cone ($Q \ll 1$), the field has a higher-order singularity than near the vertex of the solid cone. For the semitransparent cone, when $Q \gg 1$,

$$\mu_s = \frac{1}{2} + s + \frac{1}{2Q}[P_s(\cos\gamma)]^2 + O(Q^{-2}), \quad s = 0, 1, 2\ldots, \quad (32)$$

and the field near the vertex has a singularity of the order $(kr)^{-1+1/2Q}$.

ASYMPTOTIC SOLUTION IN THE CASE OF RADIATION FROM NARROW SLOTS

Suppose that the dipole is located inside the cone on its axis ($\varphi_0 = 0$, $\vartheta_0 = 0$, and $m = m_0 = 0$). Consider radiation from narrow slots ($d/l \ll 1$). Using estimates and asymptotics of functions $V_{p-1}^{n-1}(u)$ for $(1 + u) \ll 1$ [14] and formulas (21), (22), (12), (5), and (6), we obtain the asymptotic expansion of potential $U^{(s)}$ at a large distance from the slots in terms of parameter $(1 + u)$

$$U^{(s)} = \int_0^{+\infty}\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}L(\tau)P_{-1/2+i\tau}(\pm\cos\vartheta)d\tau$$

$$-\frac{(1+u)}{2N}\sum_{n\neq 0}\exp(in\varphi)$$

$$\times\int_0^{+\infty}\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}\frac{L_n(\tau)}{F_{i\tau}^n}\frac{P_{-1/2+i\tau}^{nN}(\pm\cos\vartheta)}{P_{-1/2+i\tau}^{nN}(\pm\cos\gamma)}d\tau$$

$$+O((1+u)^2\ln(1+u)), \quad (33)$$

$$L(\tau) = -\frac{1}{2}\tanh\pi\tau\exp(\pi\tau)b_0(\tau, k)\frac{P_{-1/2+i\tau}(\mp\cos\gamma)}{D_{i\tau} - \frac{1}{N}\ln\left(\frac{1-u}{2}\right)},$$

$$L_n(\tau) = -\frac{1}{2}\frac{\tau\sinh\pi\tau\exp(\pi\tau)b_0(\tau, k)}{D_{i\tau} + \frac{1-\delta_n}{N|n|\delta_n}}\frac{1}{N|n|}\frac{1-\delta_n}{\delta_n},$$

$$D_{i\tau} = \frac{1}{N|n|}(1-\delta_n)\big|_{n=0},$$

$$F_{i\tau}^{nN} = \frac{D_{i\tau}\frac{1}{N|n|}(1-\delta_n)}{\delta_n D_{i\tau} + \frac{1}{N|n|}(1-\delta_n)} + \frac{1}{2N}(1+u). \quad (34)$$

The upper and lower signs in (33) correspond to the regions $0 < \vartheta < \pi$ and $\gamma < \vartheta < \pi$, respectively. We let the slot width go to zero ($d \longrightarrow 0$ and $u \longrightarrow -1$) in (33) to obtain the known expression for the Debye electric potential for a solid cone axisymmetrically excited by a radial electric dipole [15]. The field scattered by the solid cone is characterized by the same polarization as the source field and has three components (E_r, E_ϑ, and H_φ). In the presence of slots, the field (of the TM type) contains all the components, which indicates that the field polarization has changed. Passing to integration over the imaginary axis ($\mu = i\tau$) in (33), we express potential U of the total field as a series of residues at the poles of the integrand

$$U = \frac{1}{N}(1+u)\frac{\pi i|\vec{e}|}{r_0\sqrt{r_0 r}}\sum_{n=0}^{+\infty}\chi_n\cos nN\varphi$$

$$\times\left\{\sum_{s=0}^{+\infty}\frac{\mu G_\mu(\xi_s^+, r, r_0)}{\frac{d}{d\mu}P_{-1/2+\mu}}\frac{P_{-1/2+\mu}^{-nN}(-\cos\vartheta)}{P_{-1/2+\mu}^{-nN}(-\cos\gamma)}\Bigg|_{\mu=\alpha_s^+}\right.$$

$$\left.+\sum_{q=0}^{+\infty}\frac{\mu G_\mu(\xi_q^{nN-}, r, r_0)}{P_{-1/2+\mu}(\cos\gamma)}\frac{P_{-1/2+\mu}^{-nN}(-\cos\vartheta)}{\frac{d}{d\mu}P_{-1/2+\mu}^{-nN}(-\cos\gamma)}\Bigg|_{\mu=\alpha_q^{nN-}}\right\} \quad (35)$$

$$+O((1+u)^2\ln(1+u)), \quad \gamma < \vartheta < \pi,$$

$$G_\beta(\alpha, r, r_0) = \begin{cases}\left(\frac{kr}{2}\right)^{\beta-\alpha}I_\alpha(kr)H_\alpha^{(2)}(kr_0), & r < r_0,\\ \left(\frac{kr_0}{2}\right)^{\beta-\alpha}H_\alpha^{(2)}(kr)I_\alpha(kr_0), & r > r_0,\end{cases}$$

$$\chi_n = \begin{cases}1/2, & n = 0\\ 1, & n \neq 0\end{cases}, \quad P_{-1/2+\alpha_s^{nN\pm}}^{-nN}(\pm\cos\gamma) = 0, \quad (36)$$

$$F_\mu^{nN}\Big|_{\mu = \xi_s^{nN\pm}} = 0, \qquad (37)$$

$$\xi_s^{nN\pm} = \alpha_s^{nN\pm} - \frac{(1+u)}{2N}\tilde{D}_\mu^{nN}\Big|_{\mu = \alpha_s^{nN\pm}} + O((1+u)^2), \qquad (38)$$

$$\tilde{D}_\mu^{nN} = \frac{(-1)^{nN}\cos\pi\mu}{\pi\frac{\Gamma(1/2+\mu+nN)}{\Gamma(1/2+\mu-nN)}\frac{d}{d\mu}[P_{-1/2+\mu}^{-nN}(\cos\gamma)P_{-1/2+\mu}^{-nN}(-\cos\gamma)]}, \quad \alpha_s^{nN\pm}\Big|_{n=0} = \alpha_s^{\pm},$$

where roots α_q^{nN+} and α_q^{nN-} correspond, respectively, to the upper and lower signs of the argument of the Legendre function. The field scattered by the cone with narrow slots contains components of the field scattered by the solid cone and components of the field due to the presence of slots. The scattered field is represented as an infinite set of elliptically polarized *TM* modes. The spectrum of the boundary-value problem for the cone with narrow slots consists of roots $\xi_s^{nN\pm}$ (38) of equation (37) and has the form of spectrum ($\{\alpha_s^{nN\pm}\}$) of the boundary-value problem for the solid cone [15] perturbed by slots. Near the vertex ($\gamma \le \pi/2$), the field behavior is characterized by the minimum eigenvalue of the spectrum

$$\xi_0^- = \alpha_0^- - \frac{(1+u)}{2N}A + O((1+u)^2); \qquad (39)$$

therefore, the electric field has a singularity of the order $(kr)^{-3/2+\xi_0^-}$ and the magnetic field decreases as $(kr)^{-1/2+\xi_0^-}$ as the observation point approaches the vertex. Thus, the longitudinal narrow slots increase the order of the field singularity near the cone vertex. Near the boundary inhomogeneities (the slot edges and vertex), field component E_ϑ perpendicular to the edges is determined by the expression

$$C_N(kr)^{-3/2+\xi_0^-}\left\{-1 + \mathrm{Re}\left[\frac{1-\rho}{\sqrt{\rho^2+2u\rho+1}}\right]\right\}, \qquad (40)$$

$$r < r_0, \quad 0 < \vartheta < \gamma, \quad \rho = \left(\tan\frac{\vartheta}{2}\cot\frac{\gamma}{2}\exp(-i\varphi)\right)^N.$$

The behavior of the E_r component parallel to the edge is characterized by the term

$$\tilde{C}_N(kr)^{-3/2+\xi_0^-}\ln\left|1-\rho\sqrt{\rho^2+2u\rho+1}\right|, \qquad (41)$$
$$r < r_0, \quad 0 < \vartheta < \gamma,$$

where C_N and $\tilde{C}_N$ are known coefficients.

When the cone has one narrow slot ($N = 1$), expressions (40) and (41) can be reduced to simpler forms, which implies that, near the slot edge, the field components perpendicular to the edge depend on ϑ as $|\gamma - \vartheta|^{-1/2}$. The azimuthal component of the electric field in the slot is represented by

$$E_\varphi^{(s)} = -\frac{1}{\sin\gamma}\frac{\sqrt{2}\pi i|\vec{e}|}{rr_0\sqrt{r_0}}\frac{\sin(\varphi/2)}{\sqrt{\cos\varphi - \cos(d/2)}}$$

$$\times\sum_{s=0}^{+\infty}\frac{\mu(kr_0/2)^{\xi_s^+-\mu}I_\mu(kr_0)}{\frac{d}{d\mu}P_{-1/2+\mu}(\cos\gamma)}\Bigg|_{\mu=\alpha_s^+} \times\frac{d}{dr}[\sqrt{r}H_{\alpha_s^+}^{(2)}(kr)] \qquad (42)$$

$$\times[1+0(1+u)], \quad \vartheta = \gamma, \quad r > v_0, \quad |\varphi| < d/2.$$

Formula (42) implies that, near the slot, the tangential component of the electric field has a singularity of the order of $((d/2)^2 - \varphi^2)^{-1/2}$. This fact is in agreement with the Meixner condition [18].

CONCLUSION

Our investigations show that, using the Kontorovich–Lebedev transform and the Riemann–Hilbert problem method, the electrodynamic problem for a semi-infinite perfectly conducting infinitely thin cone with equally spaced longitudinal slots excited by a radial electric dipole can be reduced to solving a Fredholm ISLAE of the second kind for the Fourier coefficients of the field components. In the particular cases when the cone is semitransparent and when it has narrow slots, we obtained integral and series representations for the Debye electric potential and the electric field components. The spectrum of the boundary-value problem and the structure, polarization, and behavior of the field near the surface inhomogeneities (the cone vertex and slot edges) are studied. It is demonstrated that longitudinal slots affect the field polarization and increase the order of the field singularity near the vertex. The proposed algorithm can be used for solving electrodynamic problems for a more complicated geometry of a scattering surface.

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