Evaluation of Unsteady Non-Harmonic Fields in Microwave Devices. II. Decomposition in the Partial Oscillators

A.V. Gritsunov
Kharkiv National University of Radio Electronics
14 Lenin Ave., Kharkiv, 61166, Ukraine
E-mail: gritsunov@kture.kharkov.ua – Phone/fax: +38 057 7021362

Introduction

Described in the Part I *partial modes* of an electrodynamic line are derived with 1D linear transform of the normal eigenmodes in the line longitudinal direction. The advantage of the partial modes over the normal eigenmodes (i.e., the longitudinal localization of the first) makes tempting generalization of this idea in all spatial dimensions. A nonstationary non-harmonic field in arbitrary electrodynamic system can be expanded in 2D or 3D set of coupled *partial oscillators*.

The Lattice Approximation

Partial oscillators (synonyms: *oscillets*, *partial functions* of an electrodynamic system) are an alternative to the eigenfunctions as a basis for expansion of the fields whose spectrum in the wavenumber $\vec{k}$ domain is finite: $|\vec{k}| \le k_{max} < \infty$. The *generic potential* can be evaluated as a finite series of the partial oscillators in the Hilbert L^2 space (generally, this is not a Fourier series):

$$\mathcal{A}(t,x,y,z) = \mathfrak{A}_p(x,y,z)\mathbf{u}_p(t)$$

The vector of N partial functions $\mathfrak{A}_p(x,y,z)$ is a solution of so-called *intervalues problem* for the differential operator $-\nabla^2$ in an electrodynamic system. This is finding a nontrivial solution of the matrix equation

$$\nabla^2 \mathfrak{A}_p + \left[k_p^2\right]\mathfrak{A}_p = 0$$

with homogeneous boundary conditions, which spatially localizes all items of the sought vector. $\left[k_p^2\right]$ is a unknown $N \times N$ matrix of the line *intervalues* (squared mutual wavenumbers of the oscillets). The intervalues problem is a multivariate conditional optimization problem with a vector criterion. The N must be such that the matrix $\left[k_p^2\right]$ contains all eigenvalues of the system with $k_m^2 \le k_{max}^2$ including multiple ones. In addition to the fixed eigenvalues, restrictions in the optimization are forms of the matrices $\left[k_p^2\right]$ and inverted unit mutual *pseudoenergies* $\left[\tilde{W}_p\right]^{-1}$, which must remain sufficiently localized around the leading diagonal.

On the other hand, the vectors of the partial functions and the eigenfunctions $\mathfrak{A}_r(x,y,z)$ of a system may be related as $\mathfrak{A}_r = [F]\mathfrak{A}_p$; $\mathfrak{A}_p = [F]^{-1}\mathfrak{A}_r$ where $[F]$ is a $N \times N$ *form-matrix* of the eigenfunctions. Like the partial modes, the form-matrix for an electrodynamic system may be arbitrary nonsingular, however, only the matrices spatially localizing all the oscillets are of interest. There is a guess that for $N \ge 8$ (if there are no multiple eigenvalues) such form-matrix exists for any system. The matrices of intervalues and eigenvalues $\left[k_r^2\right]$ are related as

$$\left[k_r^2\right]=\left[F\right]\left[k_p^2\right]\left[F\right]^{-1};$$
$$\left[k_p^2\right]=\left[F\right]^{-1}\left[k_r^2\right]\left[F\right].$$

The relations of other electrodynamic factors are also the same as for the partial modes (see the Part I).

Like the partial mode, the oscillet is a "cloud" of the field that vibrates as a single whole. This allows describing transient phenomena in a set of the coupled partial oscillators in terms of the lumped-element oscillating systems theory. In respect to waves in a homogeneous medium, this set is regular and seems like an oscillating atomic lattice in a crystal; therefore, respective approximation may be referred to as the lattice one. The "lattice" is, however, not a predetermined discretization grid, as in pseudospectral methods [1], but an "aftereffect" of identity of the $m_x m_y m_z$-th normal eigenmode phase shifts $\Delta\varphi_{m_x}$, $\Delta\varphi_{m_y}$, $\Delta\varphi_{m_z}$ between adjacent oscillets in x, y, and z directions, as well as the medium uniformity. The oscillets in an inhomogeneous medium may be different in shape and non-equidistant just as the partial modes in an irregular line.

The oscillets joint the advantages of the D'Alembert and the Bernoulli (Fourier) approaches to solving of the wave equation. They produce descriptive solutions both for first stages of a transient process in an electrodynamic system (when progressive waves move away from a pulse source) and for a steady state of one (when standing waves have arisen). The oscillets also can simulate fields of continuous spectra in an "open" system (i.e., with outer radiation boundary conditions) keeping their own discreteness.

When the $\phi-\vec{A}$ formalism is used in a 3D space, there are four mutually orthogonal subsets of the eigenfunctions: scalar functions ($\vec{A}\equiv 0$); potential vector functions with $\vec{\nabla}\times\vec{A}=0$; and two subsets of solenoidal vector ones with $\vec{\nabla}\cdot\vec{A}=0$, which are called conventionally as the magnetic and the electric (such names are of the physical sense only for some simple boundary conditions). Only solenoidal vector subset exists in a 2D space. The linear transform of above subsets produces four type of oscillets named as O (scalar), O_P (potential), O_{SM} (solenoidal magnetic), and O_{SE} (solenoidal electric). For a 2D problem, these are O, O_P, and O_S oscillets. The O type does not exist if the $\vec{E}-\vec{B}$ formalism is used.

For an uniform medium and periodic with the periods ΔX and ΔY boundary conditions in the directions x and y respectively (a 2D problem is considered), a discrete 2D set of the complex eigenfunctions $\mathfrak{A}_{rm_x m_y}$ may be related to a 2D array of the oscillets $\mathfrak{A}_{pn_x n_y}$ as the 2D discrete Fourier transform:

$$\mathfrak{A}_{rm_x m_y}(x,y)=\sum_{n_x=-N_x}^{N_x}\sum_{n_y=-N_y}^{N_y}\mathfrak{A}_{pn_x n_y}(x,y)\cdot\exp\left(-\mathrm{i}\frac{2\pi n_x m_x}{2N_x+1}\right)\exp\left(-\mathrm{i}\frac{2\pi n_y m_y}{2N_y+1}\right),$$

so:

$$\mathfrak{A}_{pn_x n_y}(x,y)=\frac{1}{N}\sum_{m_x=-N_x}^{N_x}\sum_{m_y=-N_y}^{N_y}\mathfrak{A}_{rm_x m_y}(x,y)\cdot\exp\left(\mathrm{i}\frac{2\pi m_x n_x}{2N_x+1}\right)\exp\left(\mathrm{i}\frac{2\pi m_y n_y}{2N_y+1}\right)$$

where $N=(2N_x+1)(2N_y+1)$ is the number of the oscillets whose fields are taken into account in each point of the space:

$$\mathfrak{A}=\sum_{n_x=-N_x}^{N_x}\sum_{n_y=-N_y}^{N_y}\mathfrak{A}_{pn_x n_y}(x,y)u_{pn_x n_y}(t).$$

$u_{pn_xn_y}(t)$ is an instantaneous value of the n_xn_y-th oscillet field.

A 2D array of intervalues $k^2_{pn_xn_y}$ can be derived from the eigenvalues $k^2_{rm_xm_y}$ as a 2D discrete Fourier transform. An approximate result for the uniform medium can be obtained with a semidiscrete Fourier transform:

$$k^2_{p00} = \frac{\pi^2}{3}\left[\frac{(2N_x+1)^2}{\Delta X^2} + \frac{(2N_y+1)^2}{\Delta Y^2}\right];$$

$$k^2_{pn_x0} = 2\cdot(-1)^{n_x}(2N_x+1)^2/\Delta X^2 n_x^2\ ;$$

$$k^2_{p0n_y} = 2\cdot(-1)^{n_y}(2N_y+1)^2/\Delta Y^2 n_y^2\ ;$$

$$k^2_{pn_xn_y} = 0, \quad \text{where } n_x \neq 0 \text{ and } n_y \neq 0.$$

Arrays of mutual damping factors $\delta_{pn_xn_y}$ and unit mutual pseudoenergies $\tilde{W}_{pn_xn_y}$ of the oscillets can be obtained from the eigenmode damping factors $\delta_{rm_xm_y}$ and unit pseudoenergies $\tilde{W}_{rm_xm_y}$ in the same way. An array of inverted unit mutual pseudoenergies $\tilde{W}^i_{pn_xn_y}$ is a solution of the system of N linear equations:

$$\sum_{n_x=-N_x}^{N_x}\sum_{n_y=-N_y}^{N_y}\tilde{W}_{p\ -n_x+\nu_x\ -n_y+\nu_y}\tilde{W}^i_{pn_xn_y} = \begin{cases}1 & \text{if } \nu_x=0 \text{ and } \nu_y=0\,;\\ 0 & \text{otherwise}\end{cases}$$

where $\nu_x = -N_x \ldots N_x$, $\nu_y = -N_y \ldots N_y$, and the sums $-n_x+\nu_x$, $-n_y+\nu_y$ periodically recur into the ranges $-N_x \ldots N_x$, $-N_y \ldots N_y$ respectively.

In addition to the amplitude and the energy normalizations, the truncated Gaussian normalization of the oscillets, e.g.:

$$\max\left|\mathfrak{A}_{rm_xm_y}\right| = \exp\left(-\psi\frac{m_x^2+m_y^2}{N_x^2+N_y^2}\right)\text{V}\cdot\text{s/m}$$

($\psi > 0$) allows extra localization of ones reducing “truncation” errors (i.e., the Gibbs’ phenomenon) in evaluation of the fields, as N_x and N_y are limited. However, the oscillets become non-orthogonal at that. If ψ increases, the oscillets at first lower and “shrink”, but the array $\tilde{W}^i_{pn_xn_y}$ grows and “spreads”. The latter enlarges the truncation errors in evaluation of the right-hand side of the excitation equation and the PIC model noise. Thus, an optimal value of ψ must exist.

The excitation equation for an n_xn_y-th oscillet time factor is

$$\frac{d^2u_{pn_xn_y}}{dt^2} + 2\sum_{\nu_x=-N_x}^{N_x}\sum_{\nu_y=-N_y}^{N_y}\delta_{p\nu_x\nu_y}\frac{du_{p\ n_x+\nu_x\ n_y+\nu_y}}{dt} + \sum_{\nu_x=-N_x}^{N_x}\sum_{\nu_y=-N_y}^{N_y}\omega^2_{p\nu_x\nu_y}u_{p\ n_x+\nu_x\ n_y+\nu_y}$$
$$= \frac{1}{2}\sum_{\nu_x=-N_x}^{N_x}\sum_{\nu_y=-N_y}^{N_y}\tilde{W}^i_{p\nu_x\nu_y}\int_{\Delta Z}dz\int_{\Delta X\,\Delta Y}dxdy \cdot\mathfrak{A}_{p\ n_x+\nu_x\ n_y+\nu_y}(x,y)\,j(t,x,y)$$

where $\omega^2_{p\nu_x\nu_y} = c^2k^2_{p\nu_x\nu_y}$; ΔZ is the size of the 2D system in the third direction.

The ϕ/c, A_x, and A_y components of the O, O_P, and O_S 2D oscillets with n_x=0 and n_y=0 in the free space are shown in Figs. 1, 2, and 3 respectively. They are derived from the normal eigenmodes of the

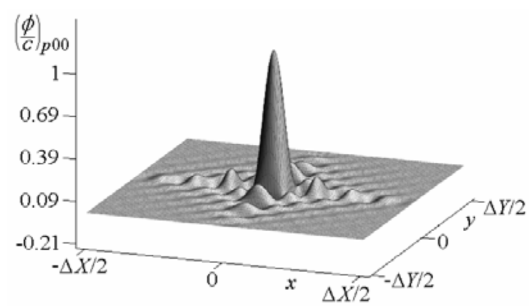

Fig. 1. The O_{00} oscillet

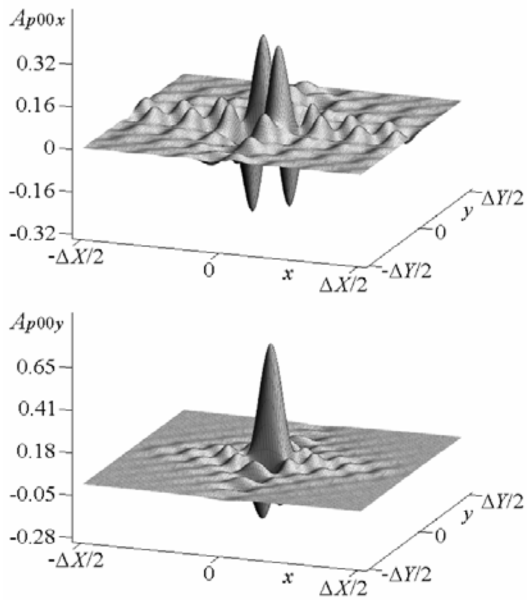

Fig. 2. The O_{P00} oscillet

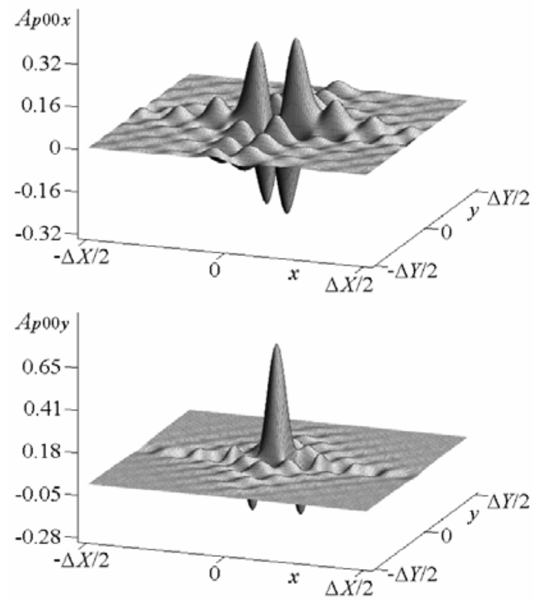

Fig. 3. The O_{S00} oscillet

space using the amplitude normalization. These oscillets are orthogonal (i.e., only $\tilde{W}_{p00}$ and $\tilde{W}^{i}_{p00}$ are not equal zero). For a bounded space, the oscillator shapes are more complicate and differ from one oscillet to another. As an example, Fig. 4 show the longitudinal (A_z) component of two solenoidal oscillets intended for synthesis of the fields in the LE_0 and LE_1 passbands of a 2D (y,z) regular vane delay line.

Conclusion

The decomposition of the fields in a set of the coupled partial oscillators is a "large-scale" alternative to the FDTD and FETD methods, as the oscillet "lattice" spacing is determined by physical reasons (upper bound of considered wavenumbers) rather than computational ones. The lattice approximation is adaptable to anisotropic media, because the model of coupled partial oscillators is close to the physical mechanism causing this phenomenon (i.e., the atomic oscillations). The fields in a nonlinear or nonstationary medium also can be expanded in a set of nonlinearly coupled nonlinear or parametric oscillators.

References

[1] J. P. Boyd, *Chebyshev and Fourier Spectral Methods*. New York: Dover, 2000.

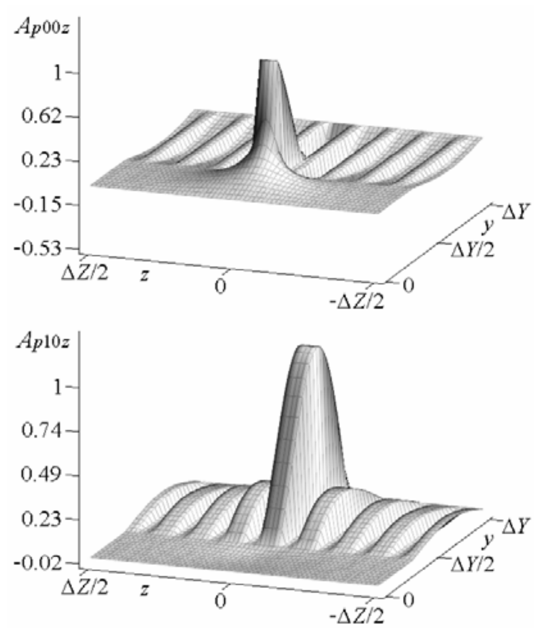

Fig. 4. The O_{SE00} and O_{SE10} oscillets