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Modelling of Electromagnetic Scattering by a Slot Cone Reflector

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Abstract

The problem of electromagnetic plane wave diffraction on an infinite perfectly conducting circular cone with periodical longitudinal slots is considered. This cone structure is a model of a slot cone antenna and reflector. The plane wave propagates along the cone axis. The method for solving electrodynamics boundary problem is based on using the Kontorovich-Lebedev integral transforms and the Rieman-Hilbert problem method. Analytical solutions for a partly transmitted cone and a narrow cone sector (strip) are derived. Boundary problem spectra, field structure and its behavior near boundary singularities are investigated.

1. INTRODUCTION

It's well known that the investigation of open screens scattering problems is of interest theory and practice both [1-3]. The one of most serious problems of modern technologies is the wide band and super wide band electromagnetic waves radiation. Cones and bicones structures have application in radiolocation, surveillance, communications, telemetry because of them wide bands and ultra wide band behaves [4,5,7]. There have been several theoretical investigations on the scattering by open cones and bicones [5-8]. Circumferential slots on cones with elliptic cross section have been studied by Blume and Grafmuller [6]. As for circumferential slots on circular cone some results are presented in [8]. Work [7] is devoted to diffraction problems for radially conducting circular cones and bicones (number of conductors and their characteristic dimensions were ignored). The electromagnetic field of radially slotted perfectly conducting elliptic cone has been tackled by Vafiadis and Sahalos in [8]. However the slot was finite and the slot width was assumed to be much smaller than its length. The structure under consideration is an infinite perfectly conducting circular cone with slots cut periodically along generatricies. This structure can be regarded as a suitable model of controlled slot antennas and reflectors. There are no restrictions for the slot width of the cone. The radial dipole field scattered by the cone with periodical longitudinal slots has been given by the present author [9,10].

The purpose of this study is to solve the problem of electromagnetic plane wave diffraction on the cone with longitudinal slots.

2. FORMULATION OF THE PROBLEM

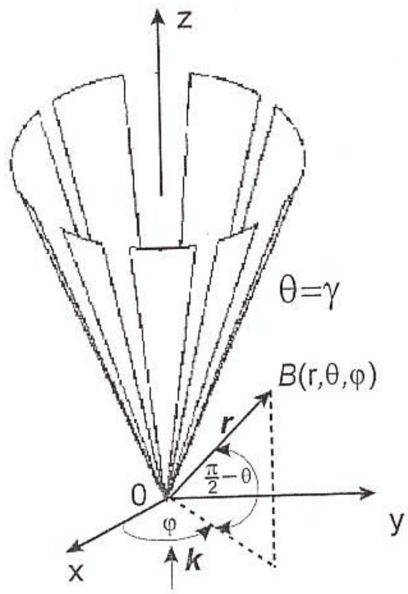

Figure1

In spherical coordinate system r, ϑ, φ the cone structure with N periodical slots cut along generatricies is defined by an equation $\vartheta = \gamma$ (Fig.1). The period of the structure $l = 2\pi / N$ and the slot width d are angular values. The slot width is a value of dihedral angle formed by planes that pass through the cone axis and cone strip edges. Let the incident linear polarized electromagnetic plane wave $\{\vec{E}^i, \vec{H}^i\}$ propagates along the cone axis (with time dependence exp(-*iwt*) suppressed throughout). The total electromagnetic field consists of the incident and diffracted field:

$$\vec{E} = \vec{E}^i + \vec{E}^d \, , \; \vec{H} = \vec{H}^i + \vec{H}^d . \tag{1}$$

The field components can be expressed via electric and magnetic Debay potentials U_1, U_2, as follows [11]

$$\vec{E} = \nabla \times \nabla \times (\vec{r} U_1) + ik \nabla \times (\vec{r} U_2) , \tag{2}$$

$$\vec{H} = -ik \nabla \times (\vec{r} U_1) + \nabla \times \nabla \times (\vec{r} U_2) . \tag{3}$$

The potentials $U_s s = 1,2$ satisfy the Helmholtz equation, boundary conditions at the cone, infinity condition, singularity condition (the cone tip, cone strip edges). Let the incident field be

$$\vec{E}^i = (E^i_x, 0, 0), \vec{H}^i = (0, H^y_i, 0) , \tag{4}$$

where $E^i_x = H^i_y = \exp(ikz)$ with k=w/c being the free-space wavenumber.

One of effective analytical methods for solving diffraction problems with conic boundaries is the method of the Kontorovich-Lebedev integral transform [12]. The pair of the transforms is presented in the form

$$F(\tau)=\int_0^{\infty} f(r)e^{-\frac{\pi\tau}{2}}\frac{H_{i\tau}^{(1)}(kr)}{\sqrt{r}}d\tau \ , \tag{5}$$

$$f(r)=-\frac{1}{2}\int_0^{+\infty}\tau sh\pi\tau e^{-\frac{\pi\tau}{2}}F(\tau)\frac{H_{i\tau}^{(1)}(kr)}{\sqrt{r}}d\tau \ , \tag{6}$$

here $H_{i\tau}^{(1)}(kr)$ is the Hankel function. The solution for U_s can be written as the sum of a free-space field potential U_s^i and a diffracted field potential U_s^d, due to the presence of the slotted cone:

$$U_s=U_s^i+U_s^d, s=1,2, \tag{7}$$

where

$$U_1^i(r,\vartheta,\varphi)=-\frac{\cos\varphi}{k^2 r\sin\vartheta}\left(\cos kr+i\cos\vartheta\sin kr-e^{ikr\cos\vartheta}\right), \tag{8}$$

$$U_2^i(r,\vartheta,\varphi)=U_1^i(r,\vartheta,\pi/2-\varphi) \ , \tag{9}$$

$$U_s^d=\overline{U}_s^d+\hat{U}_s^d, \tag{10}$$

$$\overline{U}_s^d=-\frac{i}{2k^2}\frac{\sin kr}{r}\frac{d^{s-1}}{d\gamma^{s-1}}\left[tg\frac{\gamma}{2}\sum_{m=-1,1}(-i\frac{|m|}{m})^{s-1}\overline{V}_m^{(s)}\right], \tag{11}$$

$$\overline{V}_m^{(s)}=\sum_{n=-\infty}^{+\infty}\xi_{m+nN}^{(s)}\frac{P_0^{-|m+nN|}(\pm\cos\vartheta)}{\frac{d^{s-1}}{d\gamma^{s-1}}P_0^{-|m+nN|}(\pm\cos\gamma)}e^{i(m+nN)\varphi} \ , \tag{12}$$

$\hat{U}_s^d$ is represented in terms of the Kontorovich-Lebedev transform (4)

$$\hat{U}_s^d=-\frac{1}{2}\int_0^{+\infty}\tau sh\pi\tau e^{-\frac{\pi\tau}{2}}\hat{u}_s^d\frac{H_{i\tau}^{(1)}(kr)}{\sqrt{r}}d\tau \ , \tag{13}$$

here

$$\hat{u}_s^d=\sum_{m=-1;1}(-i\frac{|m|}{m})^{s-1}\frac{d^{s-1}}{d\gamma^{s-1}}P_{-\frac{1}{2}+i\tau}^{m}(\cos\gamma)b_{m\tau}G_{m\tau}^{(s)} \ , \tag{14}$$

$$b_{m\tau}=\frac{1}{k}\sqrt{\frac{\pi}{2k}e^{i\frac{\pi}{4}}}\frac{\tau}{ch\pi\tau}\left(\tau^2+\tfrac{1}{4}\right)^{\alpha(m)}, \alpha(m)=-\frac{1}{2}\left(\frac{|m|}{m}+1\right), \tag{15}$$

$$G_{m\tau}^{(s)}(\tau) = \sum_{n=-\infty}^{+\infty} x_{m+nN}^{(s)} \frac{P_{-\frac{1}{2}+i\tau}^{m+nN}(\pm\cos\vartheta)}{\frac{d^{s-1}}{d\gamma^{s-1}} P_{-\frac{1}{2}+i\tau}(\pm\cos\gamma)} e^{i(m+nN)\varphi}, \tag{16}$$

$P_\mu^q(\pm\cos\vartheta)$ are the associated Legendre functions, the upper sign corresponds to $0<\vartheta<\gamma$ and the lower one $\gamma<\vartheta<\pi$; $\xi_p^{(s)}, x_p^{(s)} (s=1,2)$ are unknown coefficients,

$$\xi_p^{(s)} = \lim_{i\tau\to 1/2} x_p^{(s)}. \tag{17}$$

2. SUMMARY EQUATIONS

Taking into account the boundary condition and the field continuity condition at the slots we obtain the following function equations

$$\sum_{n=-\infty}^{+\infty} x_{m,n}^{(s)} e^{inN\varphi} = e^{im_0N\varphi}, \qquad \frac{\pi d}{l} < |N\varphi| \le \pi \tag{18}$$

$$\sum_{n=-\infty}^{+\infty} x_{m,n}^{(s)} [N(n+\nu]^{\beta(s)} \frac{|n|}{n}(1-\varepsilon_{m,n}^{(s)}) e^{inN\varphi} = 0, \qquad |N\varphi| < \frac{\pi d}{l} \tag{19}$$

where

$$[N(n+\nu)]^{\beta(s)} \frac{|n|}{n}(1-\varepsilon_{m,n}^{(s)})$$
$$= \frac{(-1)^{(n+\nu)N+s-1}}{\pi(\sin\gamma)^{1-\beta(s)}} \frac{\Gamma(1/2+i\tau+(n+\nu))}{\Gamma(1/2+i\tau-(n+\nu))} \frac{ch\pi\tau}{\frac{d^{s-1}}{d\gamma^{s-1}} P_{-\frac{1}{2}+i\tau}^{(n+\nu)N}(\cos\gamma) \frac{d^{s-1}}{d\gamma^{s-1}} P_{-\frac{1}{2}+i\tau}^{(n+\nu)N}(-\cos\gamma)}, \tag{20}$$

$\Gamma(z)$ - Gamma-function, $\frac{m}{N} = m_0 + \nu, -1/2 \le \nu < 1/2$, m_0 is the nearest integer number to $\frac{m}{N}$, $\beta(s) = (-1)^{s+1}$. By using the Rieman-Hilbert method [1] function equations (18), (19) can be brought to the pair of infinite systems of linear algebraic equations for coefficients $x_{m,n}^{(s)}, s=1,2$. The is an infinite system of linear algebraic equations for $x_{m,n}^{(2)}$ [10]

$\Gamma(z)$

$$x_{m,n}^{(2)} - \delta_n^{m_0} = -\frac{|m_0|}{m_0}(1-\varepsilon_{m,m_0}^{(2)}) V_{n-1}^{m_0-1}(u) + \sum_{p\ne 0} (x_{m,n}^{(2)} - \delta_p^{m_0}) \frac{|p|}{p} \varepsilon_{m,n}^{(2)} V_{n-1}^{p-1}(u)$$
$$+ (x_{m,0}^{(2)} - \delta_0^{m_0}) [P_n(u) + \varepsilon_{m,0}^{(2)} V_{n-1}^{-1}(u)], n \ne 0$$

(21)

$$\frac{2P_{\nu-1}(u)}{P_\nu(-u)+P_{\nu-1}(-u)}(x_{m,0}^{(2)}-\delta_0^{m_0}) = -\nu\frac{|m_0|}{m_0}(1-\varepsilon_{m,m_0}^{(2)})V^{m_0}(u)$$
$$+\nu\sum_{p=-\infty}^{+\infty}(x_{m,p}^{(2)}-\delta_p^{m_0})\frac{|p|}{p}\varepsilon_{m,p}^{(2)}V^p(u), \tag{22}$$

here

$$u=\cos(\frac{l-d}{l}\pi),\ \delta_n^{m_0}=0, n\neq m_0; \delta_n^{m_0}=1, n=m_0;$$

$$V_{n-1}^{m-1}(u)=\frac{n}{2(n-m)}\left[P_{n-1}(u)P_m(u)-P_n(u)P_{m-1}(u)\right], \tag{23}$$

$$V^p(u)=\frac{1}{p+\nu}\left\{P_p(u)-\frac{P_{\nu-1}(u)}{P_\nu(-u)+P_{\nu-1}(-u)}\left[P_p(u)-P_{p-1}(u)\right]\right\}. \tag{24}$$

The matrix operators of infinite systems are compact and coefficients $x_{m,n}^{(s)}$ not depend on wave number k. It's convenient for analyzing far field $(kr>>1)$ and field near the cone tip $(kr<<1)$. For any problem parameters the system solutions can be obtained with the reduction method and for special cases of the cone (partly transmitted cone, narrow cone strips) with the iteration one.

4. RESULTS

Let's consider the cone configuration when the limit

$$\lim_{\substack{N\to+\infty \\ d/l\to 1}}\left[-\frac{1}{N}\ln\left(1-d/l\right)\right]=Q>0 \tag{25}$$

exits. This conic surface is a partly transmitted cone. By using the iteration method for solving the infinite systems one can derive analytical solution of the diffraction problem for the partly transmitted cone:

$$U_1^d=-\frac{1}{k}\sqrt{\frac{\pi}{2k}}e^{i\frac{\pi}{4}}\cos\varphi\int_0^{+\infty}\frac{H_{i\tau}^{(1)}(kr)}{\sqrt{r}}\frac{\tau(\tau^2+1/4)}{\sigma_{i\tau}}e^{-\frac{\pi\tau}{2}}th\pi\tau\left[P_{-\frac{1}{2}+i\tau}^{-1}(\cos\gamma)\right]^2$$
$$\times P_{-\frac{1}{2}+i\tau}^{-1}(-\cos\vartheta)d\tau-\frac{i}{k^2}\frac{\cos\varphi}{1+2Q}tg^2\frac{\gamma}{2}ctg\frac{\vartheta}{2}\frac{\sin kr}{r}, \gamma<\vartheta<\pi, \tag{26}$$

$$\sigma_{i\tau}=(\tau^2+1/4)P_{-\frac{1}{2}+i\tau}^{-1}(\cos\gamma)+2Q\frac{ch\pi\tau}{\pi}, \tag{27}$$

$$U_2^d=0. \tag{28}$$

The expression in (26) yields the tip-diffracted field for the incident cone axis plane wave and observation points provided, however, that $\vartheta > 2\gamma$. The potential U_1^d in the region $0 < \vartheta < \gamma$ has the analogous form. The diffracted field due to the presence of this partly transmitted cone is determined by potential U_1^d only. That's why the diffracted field is the TM one. The field components satisfy the boundary-averaged conditions at the partly transmitted cone:

$$E_r^+ = E_r^- \ , \tag{29}$$

$$\frac{ik}{Q\sin\gamma} E_r = \left(\frac{\partial^2}{\partial r^2} + k^2\right)(r\widetilde{H}_\varphi) \ , \tag{30}$$

where $\widetilde{H} = H^+ - H^-, H^+ = H|_{\vartheta=\gamma+0}, H^- = H|_{\vartheta=\gamma-0}$.

The potential U_1 (7) can be presented as residue series around positive poles of the integrand in (26)

$$U_1 = -\frac{\pi i}{k}\sqrt{\frac{2\pi}{kr}} e^{i\frac{\pi}{4}} \cos\varphi \sum_{q=1}^{+\infty} \frac{\widetilde{\nu}_q \left(\widetilde{\nu}_q^{\,2} - 1/4\right) I_{\widetilde{\nu}_q}(kr)}{\cos\pi\widetilde{\nu}_q \frac{d}{d\mu}\sigma_\mu |_{\mu=\widetilde{\nu}_q}} \left[P^{-1}_{-\frac{1}{2}+\widetilde{\nu}_q}(\cos\gamma)\right]^2 P^{-1}_{-\frac{1}{2}+\widetilde{\nu}_q}(-\cos\vartheta), \tag{31}$$

$$\gamma < \vartheta < \pi$$

$$\sigma_{\widetilde{\nu}_q} = 0 . \tag{32}$$

The infinite series in (31) is useful for computation only in those parameter ranges where it converges rapidly: in the vicinity of the cone tip (*kr<<1*). To investigate the far field behavior (*kr>>1*) one should make use of the "quasi-optic" integral representation for the Debay potential in (26). The boundary problem spectrum for the partly transmitted cone is determined by the positive roots $\widetilde{\nu}_q$ of the equation (32). Solving this equation asymptotically we find

1) for any γ and *Q<<1*

$$\widetilde{\nu}_q^{\,\pm} = \alpha_q^{1\pm} + \frac{2Q\cos\pi\alpha_q^{1\pm}}{\pi\left[\left(\alpha_q^{1\pm}\right)^2 - 1/4\right]\frac{d}{d\mu}[P^{-1}_{-\frac{1}{2}+\mu}(\cos\gamma)P^{-1}_{-\frac{1}{2}+\mu}(-\cos\gamma)]|_{\mu=\alpha_q^{1\pm}}} + O(Q^2) \ , \tag{33}$$

$P^{-1}_{-\frac{1}{2}+\alpha_q^{1+}}(\cos\gamma) = 0$, $P^{-1}_{-\frac{1}{2}+\alpha_q^{1-}}(-\cos\gamma) = 0$;

2) for any γ and *Q>>1*

$$\tilde{\nu}_q = \frac{1}{2} + q + \frac{1}{2Q} q(q+1)\left[P_q^{-1}(\cos\gamma)\right]^2 + O(Q^{-2}) \,, \tag{34}$$

3) for any Q and $\gamma << 1$

$$\tilde{\nu}_q = \frac{1}{2} + q + \frac{q(q+1)}{4(1+2Q)}\gamma^2 + O\left(\gamma^4 \ln {}^2\!/_{\gamma}\right), \tag{35}$$

$q=1,2,3\ldots.$

Near the partly transmitted cone tip $(kr << 1)$ the total electric field components are equivalent to $(kr)^{-3/2+\tilde{\nu}_1}$, where $\tilde{\nu}_1$ is the smallest positive root of the equation (32). The total electric field behavior at the isotropic perfectly conducting cone tip $(\gamma << 1)$ is of the type [13,14] $|\bar{E}| \sim |kr|^{\gamma^2/2}$. For small cone angles $(\gamma << 1)$

$$\tilde{\nu}_1 \approx \frac{3}{2} + \frac{\gamma^2}{2(1+2Q)}. \tag{36}$$

Thus the total electric field scattered by the partly transmitted small angle cone decreases fast (for $kr \to 0$) as one for the isotropic perfectly conducting cone $(\gamma << 1)$. The partly transmitted cone is a model of cone surface formed by many narrow conductors (a wire cone antenna or reflector). That's why the surface current density has a radial component only

$$j_r = -\sqrt{\frac{2}{\pi kr}} \frac{e^{-i\pi/4}}{\sin\gamma} \cos\varphi \int_0^{+\infty} \frac{\tau}{\sigma_{i\tau}} sh\pi\tau e^{-\frac{\pi\tau}{2}} H_{i\tau}^{(1)}(kr) P_{-\frac{1}{2}+i\tau}(\cos\gamma) d\tau + \frac{1}{1+2Q} \frac{\cos\varphi}{\cos^2(\gamma/2)} \frac{\sin kr}{kr} \tag{37}$$

The diffracted field is TM type field of elliptic polarization, as follows (26), (31).

The analytical solution for a narrow cone sector ($N=1$ and $\beta = 2\pi - d << 1$), is determined mainly by the electric potential $V_d^{(1)}$ that may be written in the form

$$V_d^{(1)} = -\frac{1}{2\ln(\beta/4)} \frac{1}{k} \sqrt{\frac{\pi}{2k}} e^{i\frac{\pi}{4}} \sum_{n=-\infty}^{+\infty} (-1)^n e^{in\varphi} \times$$

$$\int_0^{+\infty} \tau th\pi\tau e^{-\frac{\pi\tau}{2}} P^{-1}_{-\frac{1}{2}+i\tau}(\cos\gamma) \frac{\frac{1}{|n|}\left(1-\delta_p^{(1)}\right)}{A_{i\tau}\left[1-\frac{1}{A_{i\tau}}\sum_{p\neq 0}\frac{1}{|p|}\delta_p^{(1)}\right]M_{i\tau}} \frac{P^n_{-\frac{1}{2}+i\tau}(\pm\cos\vartheta)}{P^n_{-\frac{1}{2}+i\tau}(\pm\cos\gamma)} d\tau \;+$$

$$\frac{i}{k}\frac{\sin kr}{kr} + O\left(\frac{\beta^2}{\ln\beta}\right), \tag{38}$$

here

$$A_{i\tau} = \frac{\pi}{ch\pi\tau} P_{-\frac{1}{2}+i\tau}(\cos\gamma) P_{-\frac{1}{2}+i\tau}(-\cos\gamma),\ 1-\delta_n^{(1)} = \frac{1}{1-\varepsilon_n^{(1)}},$$

$$M_{i\tau} = \frac{\frac{1}{A_{i\tau}}}{1-\frac{1}{A_{i\tau}}\sum_{p\neq 0}\frac{1}{|p|}\delta_p^{(1)}} - \frac{1}{2\ln(\beta/4)},$$ β is a cone sector width.

The expression (38) is true for observation points, which are far from the cone sector edges. The smallest value of the boundary problem spectrum for the narrow cone sector is

$$\zeta_0 = \frac{1}{2} - \frac{1}{2\ln(\beta/4)} + O\left(\frac{1}{\ln^2\beta}\right). \tag{39}$$

It corresponds to the wave that propagates along the narrow sector (a sector wave) and defines the field singularity at the sector tip $(kr \ll 1)$. Thus E_ϑ field component behavior in the vicinity of the tip is reduced to the following one

$$E_\vartheta \sim -\frac{1}{2\ln(\beta/4)}(kr)^{-3/2+\zeta_0} D(\vartheta,\varphi), \tag{40}$$

$$D(\theta,\varphi) = A\left\{ctg(\vartheta/2) + \frac{1}{\sin\vartheta}\left[-1+\mathrm{Re}\left(\frac{1-Ce^{2i\varphi}}{\sqrt{C^2e^{2i\varphi}+2Ce^{i\varphi}\cos(\beta/2)+1}}\right)\right]\right\}, \gamma<\vartheta<\pi,$$

$C = ctg(\vartheta/2)/ctg(\gamma/2)$.

For region $0<\vartheta<\gamma$ one should change ϑ to $\pi-\vartheta$ in (40).

5. CONCLUSION

An efficient algorithm for solving electrodynamics boundary problems with slot cone geometry is described. By using this algorithm one can derive analytical and numerical solution of plane electromagnetic wave diffraction on cones with longitudinal slots (three-dimensional gratings). Analytical solutions are obtained for the partly transmitted cone and the cone sector. The boundary problem spectrum is investigated. Spectrum analysis shows eigenvalue independence on wave number. Field behavior at the tip is defined by the smallest eigenvalue.

The algorithm described may be extended over transient electrodynamics problems for inhomogeneous cone structures.

REFERENCES

[1] V.P. Shestopalov, "Spectral theory and excitation of open structures", Kiev: Naukova Dumka Publ., 1987 (in Russian).

[2] Z.T. Nazarchuk, "Numerical investigation of wave diffraction on cylindrical structures", Kiev: Naukova Dumka Publ., 1989 (in Russian).

[3] A.I. Nosich, "Scattering of the surface wave of an open impedance waveguide by an unclosed cylindrical screen", *Soviet Physics-Tech. Physics (Engl. Transl.)*, vol.31, No.8, 1986.

[4] S.N. Samaddar, E.L. Mokole, "Biconical antennas with unequal cone angles", *IEEE Trans. on Antennas and Propagat*, vol.46, No.2, 1998.

[5] B.I. Kolodii, D.B. Kuriliak, "Axisymmetrical problems of electromagnetic wave diffraction on the conical surfaces", Kiev: Naukova Dumka Publ., 1995 (in Ukraine).

[6] S. Blume, B. Grafmuller, "Biconical antennas and conical horns with elliptic cross section", *IEEE Trans. on Antennas and Propagat.*, vol.36, 1988.

[7] G.G. Goshin, "Boundary problems of electrodynamics in conic regions", Tomsk: Tomsk State University Publ., 1987 (in Russian).

[8] E. Vafiadis, J.N. Sahalos, "The electromagnetic field of a slotted elliptic cone", *IEEE Trans. on Antennas and Propagat.*, vol.38, No.11, 1988.

[9] V.A. Doroshenko, V.G. Sologub, "Excitation of an unclosed cone by an electric radial dipole", *Radiotechnika i elektronika (Engl. Transl.)*, vol.35, No.12, 1990.

[10] V.A. Doroshenko, "Excitation of a cone with longitudinal slots by a magnetic radial dipole", *Radioteknika (Engl. Transl.)*, Kharkov: Osnova Public., No.97, 1992.

[11] L.A. Vainstein, "Electromagnetic waves", Moscow: "Radio i sviaz Publ., 1988 (in Russian).

[12] N.N. Lebedev, M.I. Kontorovich, "On application of the converse formula to solving some electrodynamics problems", *Zhurnal Experim. i Theoret. Fiziki (Engl. Transl.)*, vol.9, No.6, 1939.

[13] A.S. Goriainov, "Diffraction of an electromagnetic plane wave propagating along a cone axis", *Radiotekhnika i electronika (Engl. Transl.)*, vol.6, No.1, 1966.

[14] L.B. Felsen, N. Marcuvitz, "Radiation and Scattering of waves", Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1973.