Visualization of the Monochromatic Plane Wave Scattering by Multilayer Lens

Yana Sashkova, Yevhen Odarenko

Abstract - **The scattering of electromagnetic wave by the N-layer dielectric cylinder is considered in this work. A universal simulation project is developed for determine of the characteristics of the field scattered by a multilayered cylinder and visualization of the electric field spatial distribution. Amplitude distributions of the electric field for different values of the inner cylinders diameters, material parameters and incident radiation wavelength are obtained.**

Keywords - **Cylinder structure, Monochromatic wave, Bessel functions, Field spatial distribution, GRINs.**

I. INTRODUCTION

Problems associated with the scattering of electromagnetic radiation on cylindrical structures for a long time attracted the attention of researchers. These objects have the basic nature for various branches of radioelectronics and optics. Diffraction of waves on cylinders of different configurations has been studied intensively in various branches of science and technology [1, 2]. In particular, multilayer cylinders are used for the simulation of optical fibers and various devices of gradient optics: the Luneburg lens, the Eaton-Lippmann lens and the "fish-eye" Maxwell lenses [3-8]. In addition, providing a periodic variation of the material parameters of the layers, it can be created effective microwave and optical waveguide system based on multilayer dielectric structures – Bragg waveguides [9].

In this paper the important problem of the electromagnetic field spatial distribution visualization for the wave scattered by a multilayer dielectric cylinder is considered. The solution of this problem allows investigating of the focusing properties of such structures and to optimize their parameters and dimensions.

II. FORMULATION OF THE PROBLEM

An infinite dielectric multilayer cylinder (number of layers is N) with arbitrary material parameters was considered. The generatrix of the cylinder – coordinate axis Oz. The refractive index of the core, the refractive indexes of the layers are generally different (n_j , where j is the layer number).

Along the positive direction of the axis *Ox* monochromatic electromagnetic wave with a parallel polarization (electric field vector is parallel to the cylinder) incident to the structure. Problem scheme is shown in the Fig. 1.

III. SOLVING OF THE PROBLEM

Helmholtz equation for the proposed scheme can be written to the components of the electric field (the case of parallel polarization):

$$\Delta E_z + k^2 E_z = 0 \qquad (1)$$

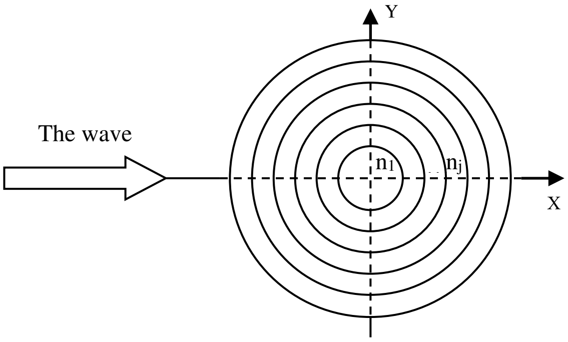

Fig. 1 – The scheme of the multilayer cylinder structure.

This equation is usually solved by the method of separation of variables. Its solution can be represented by Bessel functions of the first and the second kind:

$$E_z = \left(C_1 J_m(kr) + C_2 N_m(kr)\right) e^{im\varphi} \qquad (2)$$

Using the well-known theory of cylindrical functions, which allows to introduce a plane monochromatic wave by an infinite sum of cylindrical waves

$$e^{-ikr\cos\varphi} = \sum_{m=-\infty}^{\infty} (-i)^m J_m(kr) e^{im\varphi} \qquad (3)$$

Inside the cylinder, it takes into account the finite amplitude of the field for *r = 0*, so we consider only first-order Bessel functions.

Field scattered by cylinder, usually presented in the form of cylindrical Hankel functions of the first or second kind. It depends on the sign of the complex exponential factor $e^{\pm i\omega t}$.

The incident wave:

$$E_z^0 = E_0 \sum_{m=-\infty}^{\infty} (-i)^m J_m(k_0 r) e^{im\varphi} \qquad (4)$$

The scattering wave:

$$E_z^a = E_0 \sum_{m=-\infty}^{\infty} A_m H_m^{(1)}(k_0 r) e^{im\varphi} \qquad (5)$$

The field in the core of structure:

$$E_z^i = E_0 \sum_{m=-\infty}^{\infty} B_m J_m(k_1 r) e^{im\varphi} \qquad (6)$$

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The field in the *j*th layer:

$$E^b_{zj} = E_0 \sum_{m=-\infty}^{\infty} (C_{mj}J_m(k_j r) + D_{mj}Y_m(k_j r))e^{im\varphi} \tag{7}$$

The total field outside the cylinder can be represented as a sum of the primary field and the scattered field.

To determine the unknown coefficients of the amplitude the matrix approach has been used:

$$[R] = [M]^{-1}[B] \tag{8}$$

$$[R] = \begin{pmatrix} A_n \\ B_n \\ C_n \\ D_n \end{pmatrix},$$

$$[M] = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix},$$

$$[B] = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}.$$

This form of the equations solution simplifies the calculations for modeling the scattering of electromagnetic waves on the cylindrical structures.

For example, for cylinder with *N* layers we obtain the following relations:

$$[M] = \begin{pmatrix} H_m^{(1)}(kR_N) & -J_m(k_N R_N) & \cdots \cdots & 0 & 0 \\ \frac{k}{\mu_0}H_m^{(1)\prime}(kR_N) & \frac{-k}{\mu_N}J_m{}'(k_N R_N) & \cdots \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots \cdots & Y_m(k_2 R_2) & -J_m(k_1 R_1) \\ 0 & 0 & \cdots \cdots & \frac{k_2}{\mu_2}Y_m{}'(k_2 R_2) & \frac{-k_1}{\mu_1}J_m{}'(k_1 R_1) \end{pmatrix}$$

$$[B] = \begin{pmatrix} -J_m(kR_N) \\ -\frac{k}{\mu}J_m{}'(kR_N) \\ \vdots \\ \vdots \\ 0 \\ 0 \end{pmatrix}.$$

IV. RESULTS AND DISCUSSION

Using the obtained analytical solution for interaction the electromagnetic waves with a multilayered cylindrical structure simulation project was developed. Project was designed for the visualization of the spatial distribution of the amplitude of the electromagnetic field inside and outside the structure. With this project it is possible to calculate the estimated parameters (refractive index, the geometrical dimensions and the focal region) of the multilayer lens with arbitrary input data. Characteristics of the scattered fields for different values of the material parameters of the structure and normalized geometrical dimensions were considered using the developed calculation project.

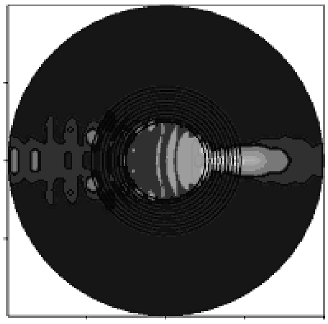
The project allows the study of the structure with an arbitrary number of layers, their dimensions and material parameters. Moreover we can study the structures containing metamaterials.

By analyzing the spatial distribution of the amplitude of the scattered field can be judged about the number and thickness of the layers and the nature of changes in the law of distribution of the refractive index. By changing the source data for visualization, can be clearly achieve the characteristics of the lens we need.

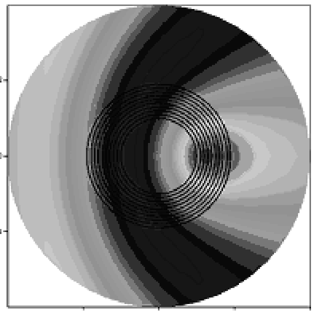
For example, a Luneburg lens has the refractive index:

$$n(r) = \sqrt{2 - \left(\frac{r}{a}\right)^2} \tag{9}$$

Fig. 2 shows the spatial distribution of the electric field amplitude for the multilayer model of the Luneburg lens.



(a)



(b)

Fig. 2. Electric field spatial distribution

Fig. 2(a) corresponds to radiation wavelength λ = 1 micron and core radius $R = 1$ micron. All layers have thickness ρ = 0.1 micron.

The wavelength increase (up to λ = 10 micron) at constant geometric dimensions leads to a change in the spatial distribution of the electric field amplitude (Fig. 2(b)). The focus position in this case does not change, but the focusing spot size increases.

If the wavelength is the constant and the thickness of each layer ρ = 0.3 micron focus point size decreases and will be located between the fifth and seventh layers.

The case when two maximums of the refractive index are realized in the multilayer structure.Here wavelength of incident light λ = 10 micron, the core radius R = 1 micron and the thickness of each layer ρ = 0.1 micron. Field focusing occurs mainly on the right side of the seventh layer (Fig. 3).

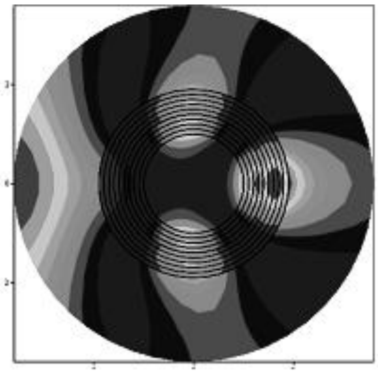


Fig. 3. Electric field spatial distribution
(ρ = 0.1 micron)

If the thickness of the layers varies periodically (0.1 and 0.2 microns), the focus area remains in the seventh layer, and there will be one on the left side of the second layer (Fig. 4). Hence in this case the reflection of wave energy from core is occurred.

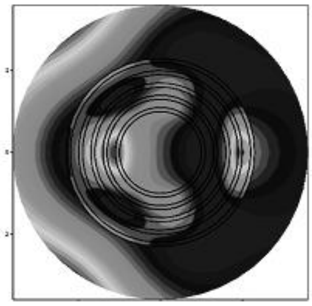


Fig. 4. Electric field spatial distribution
for case of periodic cover

V. CONCLUSION

Therefore, by adjusting the independent variables of the problem it is possible to form a different number of focusing areas a radiation on a multilayer cylinder. In addition, their spatial position is determined by the initial parameters of the problem. It turns out that the focus area can be located outside the structure and inside it.

Developed simulation project is a versatile tool for study of the multi-layer lens characteristics and select their parameters for special applications.

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