Numerical study of algorithms to construct optimal trajectories for collaborative robots in Industry 5.0 manufacturing scenarios

Elgun Jabrayilzade[1]

1. CITAR Department, Kharkiv National University of Radio Electronics, UKRAINE, Kharkiv, Nauki Ave. 14., e-mail: vladyslav.yevsieiv@nure.ua

***Abstract*:** This study considers the problem of numerical modeling of algorithms for constructing optimal trajectories for collaborative robots in Industry 5.0 production scenarios, where the key aspect is safe and effective interaction with a dynamic environment and humans. The developed mathematical models are based on potential function methods and multi-criteria optimization, which allows combining obstacle avoidance, energy consumption reduction, and task execution time minimization. Numerical modeling has shown that the actual trajectory of the robot remains convergent to the desired one even in the presence of obstacles, confirming the adaptability of the algorithms to unpredictable changes in the environment. Analysis of the time error showed its short-term increase during maneuvering and subsequent stabilization, which indicates the reliability of the proposed models. The study of the smoothness of movement confirmed the absence of sharp changes in the curvature of the trajectory, which ensures safety and comfort of operation in production processes. The results obtained prove the feasibility of using the developed approaches to create intelligent robotic systems capable of increasing production efficiency, reducing accident risks, and improving the quality of interaction between robots and humans in the Industry 5.0 concept.

***Keywords*:** optimal trajectories, collaborative robots, numerical modeling, obstacle avoidance, adaptive control, Industry 5.0, multi-criteria optimization, trajectory planning, safe interaction.

I. INTRODUCTION

The current stage of robotics development in Industry 5.0 is characterized by the integration of intelligent control systems that ensure safe and effective interaction between humans and robots in a shared working environment. One of the key tasks in this regard is to construct optimal trajectories for collaborative robots, since it is the quality of trajectory planning that determines the level of productivity, the accuracy of production tasks, and the minimization of collision risks. The relevance of the research is determined by the need to adapt planning algorithms to dynamic and uncertain production scenarios, where there are moving objects, variable constraints, and humans as active participants in the process. Traditional methods of trajectory construction do not always guarantee the necessary level of flexibility and stability of the system, while the use of numerical methods and optimization algorithms allows to increase the efficiency of calculations and the accuracy of results. Therefore, the development and research of new approaches to optimal trajectory planning is an important step in ensuring the adaptability and safety of collaborative robots, which determines the scientific and practical significance of this work.

II. MATHEMATICAL SUPPORT FOR NUMERICAL RESEARCH OF ALGORITHMS FOR CONSTRUCTING OPTIMAL TRAJECTORIES FOR COLLABORATIVE ROBOTS

Kinematic model in configuration space. Let $q \in \mathbb{R}^n$ be the configuration vector (joint angles + base position) at time moment t. Then the position of the end effector in the workspace is given by the mapping $p(t) = f(q(t))$, where $f: \mathbb{R}^n \to \mathbb{R}^m$. The Jacobian is defined as $J(q) = \frac{\partial f}{\partial q}$ and links the velocities: $\dot{p} = J(q)\dot{q}$. Let us describe the parameters: $q(t)$ - robot configuration; $p(t)$ - effector position; f - direct kinematic transformer; $J(q)$ – Jacobian; n - number of degrees of freedom; m - size of the workspace (usually 3 or 6). This model is needed for transformation between the joint space and the task space (working space), for formulating constraints on the end effector, and for calculating collisions/distances.

The dynamics of the manipulator (model in joint space) is described as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + D\dot{q} = \tau + J(q)^T F_{ext} \tag{1}$$

Where: $q, \dot{q}, \ddot{q}$ - joint status (position, velocity, acceleration); $p(q)$ - effector position in the workspace; $J(q)$ - Jacobian; $M(q)$ - inertia matrix (positive definite); $C(q,\dot{q})$ - Coriolis/centrifugal forces matrix; $G(q)$ - gravity vector; τ - control moment vector (all joints); F_{ext} - external forces (contacts, reactions); D - viscous loss matrix.

Objective functionality (optimal control). The general problem of optimal trajectory planning is formulated as minimizing the functionality:

$$J[q(\cdot),\tau(\cdot)] = \Phi\big(q(T)\big) + \int_0^T l(q(t),\dot{q}(t),\tau(t),t)dt \tag{2}$$

subject to dynamic equations and constraints. Here Φ is the terminal value (e.g., distance to the target), and l is the integral value, for example:

$$l = w_p\|p(t) - p_d(t)\|^2 + w_u\|\tau(t)\|^2 + w_{\dot{q}}\|\dot{q}(t)\|^2 \quad (3)$$

Where: $p_d(t)$ - desired trajectory/target; $w_p, w_u, w_{\dot{q}}$ - integral weights. The model's functionality combines the criteria of tracking accuracy, drive energy/effort, and smoothness of motion; weight coefficients allow balancing between trajectory quality and costs.

Safety and comfort constraints (continuous inequalities). Collisions and safety are formalized as a set of inequalities:

$$h_k(q(t), t) \geq 0, k = 1, \ldots, K \quad (4)$$

where an example is the distance constraint between critical points of the robot and a person/obstacle:

$$h_{col}(q, t) = d(q, t) - d_{safe} \geq 0 \quad (5)$$

With $d(q, t) = min_{x\in R(q), y\in O(t)}\|x - y\|$

Where: d_{safe} - minimum permissible distance; $R(q)$ - - set of body points of the robot; $O(t)$ - set of obstacles/positions of the human.

Restrictions (5) guarantee a minimum safe distance and take into account the human comfort zones.

Starting potential for numerical implementation of avoidance. To ensure a differentiated form of restriction, the following potential is introduced:

$$U_{rep}(q, t) = \begin{cases} \frac{1}{2}k_r\left(\frac{1}{d(q,t)} - \frac{1}{d_0}\right), d(q, t) < d_0 \\ 0, \quad d(q, t) \geq d_0 \end{cases} \quad (6)$$
$$F_{rep} = -\nabla_q U_{rep}(q, t)$$

Where: $k_r > 0$ - repulsion coefficient; d_0 - potential influence radius; $d(q, t)$ - distance; ∇_q - configuration gradient.

Discretization — direct collocation. We divide the interval $[0, T]$ into N nodes t_i and represent the trajectory as a set of variables $\{q_i, \dot{q}_i, \tau\}_{i=0}^N$. We minimize the total discrete functional:

$$J_d = \Phi(q_N) + \sum_{i=0}^{N-1} l(q_i, \dot{q}_i, \tau_i)\Delta t \quad (7)$$

subject to discrete constraint equations (collocations), for example, using the trapezoidal method:

$$q_{i+1} = q_i + \frac{\Delta t}{2}(\dot{q}_i + \dot{q}_{i+1}) \quad (8)$$

Where: N - number of nodes; $\Delta t = T/N$ - steps; $q_i, \dot{q}_i, \tau_i$ - discrete variables; collocation equations ensure compatibility with dynamics. Direct transcription converts a continuous optimal problem into an NLP (nonlinear program), which is solved numerically by standard optimizers.

Gradient/variational expression for updating (for gradient algorithms and CHOMP/TrajOpt). Gradient of the goal along the trajectory in discrete representation:

$$\nabla_{q_i} J_d = 2w_p(p_i - p_{d,i})^T \frac{\partial p_i}{\partial q_i} + \frac{\partial U_{rep}(q_i)}{\partial q_i} \quad (9)$$

Where: $\frac{\partial p_i}{\partial q_i} = J(q_i)$ - Jacobian at the node; $p_{d,i}$ - desired position of the node; regularizers - terms that penalize high derivatives (velocity, acceleration).

The gradient (9) is used in local optimizers (CHOMP, TrajOpt) for iterative trajectory smoothing, taking into account collision penalties and smoothness.

MPC formulation for online optimization. At each time step t_k the problem of minimizing the functional on the horizon T_p is solved:

$$\min_{\{q_{k+j}, \tau_{k+j}\}_{j=0}^{N_p-1}} \Phi\left(q_{k+N_p}\right) + \sum_{j=0}^{N_p-1} l(\cdot) \quad (10)$$

subject to discrete dynamics and constraints, after which only the first step τ_k (receding horizon) is applied.

Where: N_p - forecast horizon; k - current time index; weight palette is the same as in the functional l. MPC allows you to adapt the trajectory in real time as the environment and human movement change, combining global goals and local safety constraints.

Stochastic/robust formulation (necessary for uncertainties). If the person's position or model parameters are random, we minimize the expected value of costs with guarantees:

$$\min \mathbb{E}\left[\Phi\big(q(T)\big) + \int_0^T l(\cdot)dt\right] \; under \; \Pr\{h_k(q, t) \geq 0\} \geq 1 - \delta \quad (11)$$

or in the form of robust optimization:

$$\min \max_{\xi\in\Xi} J[q(\cdot); \xi] \quad (12)$$

Where: ξ - unknown parameters/noise; δ - acceptable risk of violation of the constraint; Ξ - set of possible scenarios/uncertainties.

Expressions 11-12 allow us to obtain trajectories that are highly likely to be safe and perform tasks even with random variations.

Coordination of multiple robots (cooperative constraints). For a group M of robots, we introduce a state vector $Q = [q^1, \ldots, q^M]$ and set additional mutual avoidance constraints:

$$d_{ij}(q^i, q^j, t) - d_{min} \geq 0, \forall_i \neq j \quad (13)$$

or as a penalty in the objective: $\sum_{i<j} U_{rep}^{ij}(q^i, q^j)$.

Where: M - number of robots; d_{ij} - distance between robots i and j; d_{min} - minimum inter-robot distance.

Model 13 ensures safe interaction between robots and allows searching for optimal distributed trajectories.

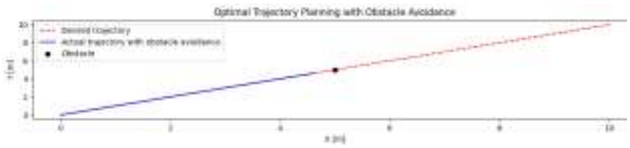
In practice, the choice of discretization step Δt weights $w_p, w_u, \ldots$,, potential coefficients k_r, and radii d_0 determines the trade-off between accuracy, computation speed, and safety; their tuning is usually done experimentally or through separate optimization (meta-optimization). For a large number of DOFs, it is recommended to use direct methods (direct collocation) or iterative local methods with analytical gradients, and for online adaptation — MPC with a limited number of nodes. If there are stochastic components (human movement, sensor noise), it makes sense to include a stochastic or robust component in the formulation.

The developed mathematical models of algorithms for constructing optimal trajectories for collaborative robots in Industry 5.0 production scenarios have a number of significant advantages that determine their effectiveness and practical significance. First, they allow for both kinematic and dynamic characteristics of robots to be taken into account, ensuring the accuracy of trajectory formation in complex production conditions. Second, the models integrate safety constraints and human comfort zones, ensuring the adaptability of robot movements and reducing the risk of accidents in a shared environment. An important advantage is the possibility of multi-criteria optimization, which combines energy cost minimization, trajectory smoothing, and task execution time reduction without loss of accuracy. The use of potential functions and MPC allows modeling the response to dynamic changes in the environment in real time, while maintaining stability and efficiency. In addition, the proposed expressions are easily scalable for multi-robot systems, which facilitates the development of coordination strategies in collaborative scenarios. Importantly, the models take into account uncertainties, thanks to which the system remains resistant to unpredictable human actions or external factors. As a result, the developed approaches form the basis for the creation of a new generation of intelligent robotic systems capable of effectively collaborating with humans and increasing productivity in Industry 5.0 manufacturing processes.

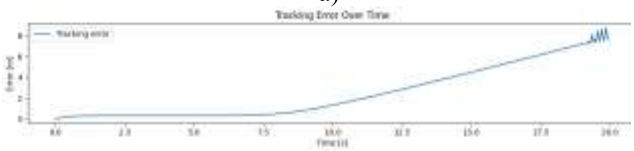
II. NUMERICAL MODELING AND ANALYSIS OF THE RESULTS OBTAINED

The goal of the first experiment, shown in the graph “Optimal Trajectory Planning with Obstacle Avoidance,” was to test the ability of the optimal trajectory planning algorithm to ensure the robot's movement toward the target while avoiding obstacles. The expected result is a deviation of the actual trajectory from the desired one in the obstacle zone, followed by a return to the optimal path, confirming the effectiveness of the route construction model. The second experiment, shown in the “Tracking Error Over Time” graph, aimed to evaluate the dynamics of the error between the desired and actual trajectories over time. The expected result is an initial increase in error at the moment of maneuvering near the obstacle and subsequent stabilization, demonstrating the convergence of the algorithm to the specified trajectory. The third experiment, shown in the “Trajectory Smoothness” graph, is aimed at analyzing the smoothness of movement and the curvature of the trajectory when avoiding obstacles. The expected result is a slight increase in curvature during maneuvering, which quickly stabilizes, confirming the algorithm's ability to form safe and smooth trajectories without sudden changes in motion. Together, these experiments demonstrate the ability to construct efficient, safe, and optimal trajectories for collaborative robots in Industry 5.0 manufacturing scenarios.

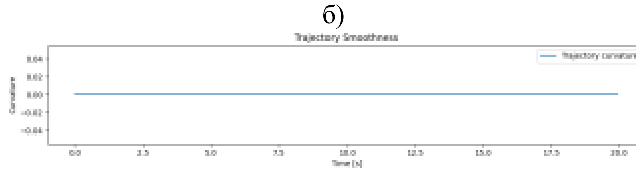
The results of numerical simulations are presented in Figure 1.



a)



б)



в)

a); Optimal Trajectory Planning with Obstacle Avoidance б) Tracking Error Over Time; в) Trajectory Smoothness

Figure 1. – Numerical modelling results

Based on numerical modeling, the effectiveness of algorithms for constructing optimal trajectories in an environment with obstacles was investigated. The first graph (Fig. 1a) shows the desired and actual trajectories of the robot's movement, where it can be seen that thanks to potential functions, the robot successfully avoids obstacles and finds the optimal path to the goal. The second graph (Fig. 1b) shows the change in tracking error over time, which initially increases due to maneuvering around the obstacle but then decreases, indicating convergence to the optimal trajectory. The third graph (Fig. 1c) shows the curvature of the trajectory, which increases in the obstacle avoidance zone but stabilizes over time, ensuring smooth movement. The results confirm the feasibility of using the developed models to optimize the movement of collaborative robots in production scenarios.

III. CONCLUSION

The study confirmed the effectiveness of the developed algorithms for constructing optimal trajectories for collaborative robots in the context of Industry 5.0 production scenarios. The results of numerical modeling showed that the proposed approaches allow forming trajectories that not only take into account the optimality of movement but also ensure safe avoidance of obstacles in a shared working environment. Motion analysis demonstrated that the actual trajectory of the robot coincides with the desired one after a local deviation in the obstacle zone, confirming the stability and reliability of the models. It was found that the tracking error increases during maneuvering but gradually decreases and stabilizes, indicating the algorithm's ability to adaptively correct motion. The study of trajectory smoothness confirmed the absence of sharp changes in curvature, which is critical for safety and ease of operation in production scenarios involving humans. Thus, the proposed algorithms have proven their effectiveness in combining multi-criteria requirements, in particular, minimizing energy consumption, task execution time, and compliance with safety requirements. The results obtained lay the foundation for the practical implementation of collaborative robot systems in new-generation production processes, increasing their flexibility, productivity, and level of interaction with humans...

REFERENCES

[1] Zia, A., & Haleem, M. (2025). Bridging research gaps in industry 5.0: Synergizing federated learning, collaborative robotics, and autonomous systems for enhanced operational efficiency and sustainability. IEEE Access.

[2] Attar, H., Abu-Jassar, A. T., Yevsieiev, V., Lyashenko, V., Nevliudov, I., & Luhach, A. K. (2022). Zoomorphic mobile robot development for vertical movement based on the geometrical family caterpillar. Computational intelligence and neuroscience, 2022(1), 3046116.

[3] Nevliudov, I., Yevsieiev, V., Baker, J. H., Ahmad, M. A., & Lyashenko, V. (2020). Development of a cyber design modeling declarative Language for cyber physical production systems. J. Math. Comput. Sci., 11(1), 520-542.

[4] Nevliudov, I., & et al. (2020). Method of Algorithms for CyberPhysical Production Systems Functioning Synthesis. International Journal of Emerging Trends in Engineering Research, 8(10), 7465-7473.

[5] Lyashenko, V., Abu-Jassar, A. T., Yevsieiev, V., & Maksymova, S. (2023). Automated Monitoring and Visualization System in Production. International Research Journal of Multidisciplinary Technovation, 5(6), 9-18.

[6] Mustafa, S. K., Yevsieiev, V., Nevliudov, I., & Lyashenko, V. (2022). HMI Development Automation with GUI Elements for Object-Oriented Programming Languages Implementation. SSRG International Journal of Engineering Trends and Technology, 70(1), 139-145.

[7] Nevliudov, I., Yevsieiev, V., Lyashenko, V., & Ahmad, M. A. (2021). GUI Elements and Windows Form Formalization Parameters and Events Method to Automate the Process of Additive Cyber-Design CPPS Development. Advances in Dynamical Systems and Applications, 16(2), 441-455.

[8] Kragic, D., Gustafson, J., Karaoguz, H., Jensfelt, P., & Krug, R. (2018, July). Interactive, Collaborative Robots: Challenges and Opportunities. In IJCAI (pp. 18-25).

[9] Nevliudov, I., Yevsieiev, V., Maksymova, S., Gopejenko, V., & Kosenko, V. (2025). Development of mathematical support for adaptive control for the intelligent gripper of the collaborative robot manipulator. Advanced Information Systems, 9(3), 57-65.

[10] Maksymova, S., Yevsieiev, V., Chala, O., & Ababneh, J. (2025). DECISION-MAKING MODEL FOR CONTROLLING A COLLABORATIVE ROBOT-MANIPULATOR BASED ON THE SENSOR FUSION METHOD AND THE RULES OF RULE-BASED SYSTEMS. Multidisciplinary Journal of Science and Technology, 5(6), 526-538.

[11] Невлюдов, І. Ш., Євсєєв, В. В., & Гурін, Д. В. (2025). Model development of dynamic representation a model description parameter for the environment of a collaborative robot manipulator within the industry 5.0 framework. Системи управління, навігації та зв'язку. Збірник наукових праць, 1(79), 42-48.

[12] Yevsieiev, V., Abu-Jassar, A., Maksymova, S., & Demska, N. (2025). Development of a model for recognizing various objects and tools in a collaborative robot workspace. ACUMEN: International journal of multidisciplinary research, 2(1), 224-239.

[13] Yevsieiev V. Mobile Robots and Autonomous Vehicles in the Mobility as a Service (MAAS) Concept / V. Yevsieiev // Sustainable smart cities and communities: business and innovation solutions 2025: Thesises of Reports of I st I International Conference, April 21, 2025. - Kharkiv, 2025. - P.7-8.

[14] Yevsieiev V. Using Multi-Agent Systems in the Management of Collaborative Robots / V. Yevsieiev // Computer-integrated technologies, automation and robotics 2025: Thesises of Reports of II st All-Ukrainian Conference, May 16-17, 2025. - Kharkiv, 2025. - P. 13-17

[15] Yevsieiev, V., Maksymova, S., Gurin, D., & Alkhalaileh, A. (2024). HR data visualization of the distance to the object in the collaborative robot workspace based on hc-sr04 sensor. ACUMEN: International journal of multidisciplinary research, 1(4), 388-401.

[16] Yevsieiev, V., Maksymova, S., Abu-Jassar, A., & Ababneh, J. (2025). MATHEMATICAL MODEL OF LOCAL DECISION-MAKING FOR COLLABORATIVE ROBOTS USING EDGE COMPUTING. Multidisciplinary Journal of Science and Technology, 5(6), 34-46.

[17] Igor Nevliudov, Vladyslav Yevsieiev, Svitlana Maksymova, Nataliia Demska, Nikolaj Starodubcev, Oleksandr Klymenko, "Monitoring System Development for Equipment Upgrade for IIoT", 2023 IEEE 5th International Conference on Modern Electrical and Energy System (MEES), pp.1-5, 2023

[18] Nevliudov, I., Yevsieiev, V., Maksymova, S., & Filippenko, I. (2020). Development of an architecturallogical model to automate the management of the process of creating complex cyberphysical industrial systems.Eastern-European Journal of Enterprise Technologies, 2020, 4(3-106), 44-52.

[19] Невлюдов, І., Клименко, О., Євсєєв, В., & Максимова, С. (2025). IMPROVEMENT OF THE ENCODING INFORMATION METHOD FOR PHARMACEUTICAL PRODUCTS QR-CODES DURING SORTING ON A ROBOTIC CONVEYOR LINE. Вісник Національного технічного університету «ХПІ». Серія: Технології в машинобудуванні, (1 (11)), 128-134.

[20] Yevsieiev, V., Ababneh, J., Maksymova, S., & Abu-Jassar, A. (2025). DEVELOPMENT OF A MATHEMATICAL MODEL FOR SIMULATING A DECENTRALIZED CONTROL SYSTEM FOR COLLABORATIVE ROBOT NETWORKS. Multidisciplinary Journal of Science and Technology, 5(5), 1187-1202.