Transient Transformation of Surface Plasmon Due to Time Variations in Dielectric Permittivity of Nanowire Environment

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***Abstract*—In this paper, a 2D problem of a surface plasmon transformation due to a time variation of silver nanowire environment is analytically solved. Solution of the problem is derived in the form of the Laplace transform. Accurate time domain inversion makes it possible to analyze transient and steady state regimes.**

Keywords—time domain analysis; surface plasmons; time varying domain; nanowire

I. Introduction

Surface plasmons have been the subject of significant interest in recent years as they exhibit properties ideal for a wide range of potential applications. These include: plasmonic waveguides [1], subwavelength resonators [2] and optical nanoantennas [3]. Moreover, delocalized and localized surface plasmons have been explored for their potential in a single molecule detection including use of the Surface Enhanced Raman Scattering (SERS) effect [4], transmissions through the subwavelength apertures [5], etc.

The silver nanowire structure is a candidate for key components in future ultra-compact photonic devises [6]. It can be considered as a plasmon biosensor to monitor tiny biomolecular interactions [7] and as a novel modulator to control the intensity of the transmitted surface plasmon polaritons through a nanowire array [8].

Possible future nanophotonic technologies demand devices that can process optical signals through the excitation of the surface plasmons. However it is challenging problem due to the extremely strong absorption losses in metal at optical frequencies. The suggestion to compensate the losses by an optical gain using dye molecules in presence of metal nanoparticles [9] or using nanoparticles with gold core and dye-doped silica shell [10] has been tested in experiments recently. For these applications an accurate frequency and time domain modeling that provides a valuable insight into fundamental processes is of great importance.

A monochromatic electromagnetic field incident on a temporally invariant object is scattered with changes in its wavenumber and field pattern but its frequency remains unaltered. In contrast, temporal variations of the dielectric permittivity of an unbounded medium transform the frequency of an existing field. Presence of boundaries and time varying media leads to complicated transient process that can involve change of the frequency and field patterns [11].

Here we investigate, in an accurate mathematical manner, transient transformation of a localized surface plasmon excited on a silver nanowire embedded in a dielectric medium with timevarying parameters. This problem can be viewed, e.g. as a biosensor reaction to refractive index change of the environment caused by a change of the biomaterial concentration on the wire surface.

II. Mathematical background: formulation and solution

We consider a circular infinitely-long plasma cylinder of radius a embedded in a dielectric medium with permittivity ε_1. The frequency dependent plasma permittivity ε_p is described by the Drude model

$$\varepsilon_p = 1 - \omega_p^2 \cdot (\omega(\omega - i\gamma))^{-1}, \qquad (1)$$

where ω_p represents the plasma frequency, γ is the material absorption. The polar system of coordinates (ρ,φ) is introduced co-axially with cylinder.

Such kind of nanowires can support plasmons only in the H-polarized regime, the z-coordinate of their magnetic field can be expressed as [12-16]

$$h_0 = H_0 \begin{cases} b_k J_k(n_p k_0 \rho) \cos k\varphi \cdot e^{-i\omega t^*}, & \rho < a, \\ H_k^{(2)}(n_1 k_0 \rho) \cos k\varphi \cdot e^{-i\omega t^*}, & \rho > a, \end{cases} \qquad (2)$$

where $k_0 = \omega_0 / c$ is the wave number, c is the light velocity in vacuum, $n_p = \sqrt{\varepsilon_p}$, ε_p is defined by formula (1), $n_1 = \sqrt{\varepsilon_1}$, the time dependence is $e^{i\omega_0 t}$, $e^{i\omega_0 t^*}$ symbolizes the fact that the plasmon excited at some moment of time $t^* < 0$, and ω_0 is the corresponding plasmon eigenfrequency which is the solution of the equation

$$n_p J_k(n_p k_0 a) H_k'^{(2)}(n_1 k_0 a) - n_1 J_k'(n_p k_0 a) H_k^{(2)}(n_1 k_0 a) = 0, \quad (3)$$

coefficient b_k is

$$b_k = \frac{H_k^{(2)}(n_1 k_0 a)}{J_k(n_p k_0 a)}. \quad (4)$$

At zero moment of time, the dielectric permittivity outside the metal nanowire changes in value from ε_1 to ε_2 that is presented schematically in Fig. 1.

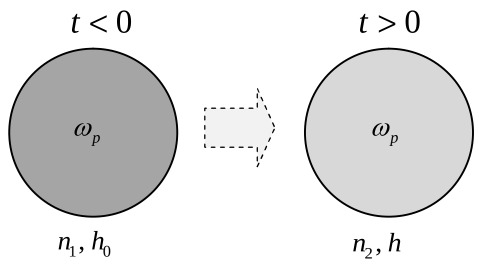

Fig. 1. Schematic diagram of the problem.

The main goal of this investigation is to study transient response of the surface plasmon to a change of the dielectric permittivity of outer space. This formulation of the problem allows deriving the analytical solution that can reveal physical phenomena in detail.

Non-stationary equation for the electromagnetic field in plasma with dielectric permittivity given by the Drude model (1) has the following form:

$$rot rot \vec{h} + \frac{1}{c^2}\varepsilon_\infty \frac{\partial^2}{\partial t^2}\vec{h} + \frac{1}{c^2}\frac{\omega_p^2}{\gamma}\frac{\partial}{\partial t}\vec{h} - \frac{1}{c^2}\frac{\omega_p^2}{\gamma}\frac{\partial^2}{\partial t^2}\int_0^t e^{-\gamma(t-t')}\vec{h}(t')dt' = 0. \quad (5)$$

Using the fact that magnetic field of the plasmon has only one nonzero component ($z-$ component), we can write equation for the transformed field as

$$\Delta h + \frac{1}{c^2}\varepsilon_\infty \frac{\partial^2}{\partial t^2}h + \frac{1}{c^2}\frac{\omega_p^2}{\gamma}\frac{\partial}{\partial t}h - \frac{1}{c^2}\frac{\omega_p^2}{\gamma}\frac{\partial^2}{\partial t^2}\int_0^t e^{-\gamma(t-t')}h(t')dt' = 0,\ \rho < a, \quad (6)$$

$$\Delta h - \frac{n_2^2}{c^2}\frac{\partial^2 h}{\partial t} = 0,\ \ \rho > a, \quad (7)$$

where h represents the z -component of the magnetic field,

$$\Delta = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2}. \quad (8)$$

Applying the Laplace transform $H(p) = \int_0^\infty h(t)e^{-pt}dt$ to equations (6) and (7), we obtain the following equations for the image functions:

$$\Delta H + q^2(\varepsilon_\infty + \frac{1}{c^2}\frac{\omega_p^2}{q(q+\gamma/\ )})H = \frac{q}{c^2}(\varepsilon_\infty(c + \frac{ik_0}{\ }) + \frac{\omega_p^2}{\gamma q})H_0 b_k J_k(n_p k_0 \rho)\cos k\varphi,\ \rho < a, \quad (9)$$

and

$$\Delta H + n_2^2 q^2 H = \frac{n_2^2}{c}(q + ik_0\frac{n_1^2}{n_2^2}) \times H_k^{(2)}(n_1 k_0 \rho)\cos k\varphi,\ \rho > a, \quad (10)$$

where $n_2 = \sqrt{\varepsilon_2}$, $q = p/c$.

The solution of (9) and (10) represents a superposition of the initial value problem and contributions of the boundary terms [17],

$$H(\rho < a) = H_0 \frac{b_k}{c}\frac{1}{q - ik_0}J_k(n_p k_0 \rho)\cos k\varphi e^{-i\omega t^*} + B_k(q) I_k(\tilde{n}_p q\rho)\cos k\varphi e^{-i\omega t^*}, \quad (11)$$

$$H(\rho > a) = \frac{1}{c}\frac{n_2^2 q + i n_1^2 k_0}{n_2^2 q^2 + n_1^2 k_0^2}H_k^{(2)}(n_1 k_0 \rho)\cos k\varphi e^{-i\omega t^*} + C_k(q) K_k(n_2 q\rho)\cos k\varphi e^{-i\omega t^*}, \quad (12)$$

Here $I_k(..), K_k(..)$ are the modified Bessel functions and

$$\tilde{n}_p = \sqrt{\varepsilon_\infty + \frac{1}{c^2}\frac{\omega_p^2}{q(q+\gamma/c)}}. \quad (13)$$

The coefficients B_k and C_k are found from the boundary conditions that involve continuity of the tangential components and have the following form:

$$B_k = \frac{n_p k_0 J_k(n_p k_0 a) K_k'(n_2 q a) + n_2 q J_k'(n_p k_0 a)\ K_k(n_2 q a)}{\tilde{n}_p K_k'(n_2 q a) I_k(\tilde{n}_p q a) - n_2 I_k'(\tilde{n}_p q a)\ K_k(n_2 q a)} \times N, \quad (14)$$

$$C_k = \frac{\tilde{n}_p q J_k'(n_p k_0 a)\ I_k(\tilde{n}_p q a) + n_p k_0 I_k'(\tilde{n}_p q a) J_k(n_p k_0 a)}{\tilde{n}_p K_k'(n_2 q a) I_k(\tilde{n}_p q a) - n_2 I_k'(\tilde{n}_p q a)\ K_k(n_2 q a)} \times N, \quad (15)$$

where $N = \dfrac{iq(n_2^2 - n_1^2)}{c(q^2 n_2^2 + k_0^2 n_1^2)(q - ik_0)}H_0 b_k e^{-i\omega t^*}$.

We find the inverse transformation to the time domain by virtue of the Mellin formula $h(t) = 1/(2\pi i)\int_{-i\infty}^{+i\infty} H(p)e^{pt}dp$. Expressions (11) and (12) have the singular points $q = ik_0$, $q = \pm ik_0\, n_1/n_2$ and also singular points associated with localized surface plasmons given by the equation

$$\tilde{n}_p K_k'(n_2 q a) I_k(\tilde{n}_p q a) - n_2 I_k'(\tilde{n}_p q a)\ K_k(n_2 q a) = 0, \quad (16)$$

where $(k = 0, 1, 2, ...)$.

Asymptotic expansions of (11) and (12) for large value of q are as follows:

$$B_k I_k(\tilde{n}_p q\rho) \sim \frac{H_0}{c} \cdot \frac{n_2 - \tilde{n}_p}{\tilde{n}_p} b_k J_k'(n_2 qa) \sqrt{\frac{a}{\rho}} e^{\sqrt{\varepsilon_\infty} q(\rho - a)}, \quad (17)$$

$$C_k K_k(n_2 q\rho) \sim \frac{H_0}{c} \cdot \frac{n_2 - \tilde{n}_p}{\tilde{n}_p} b_k J_k'(n_2 qa) \sqrt{\frac{a}{\rho}} e^{n_2 q(a - \rho)}. \quad (18)$$

Thus, these terms, when inverting to the time domain, demonstrate the time delay. It means that after the time change of the dielectric permittivity electromagnetic field inside the wire is represented by only the first term of (11) and outside the wire by the first term of (12). Keeping in mind the time dependence e^{pt} we see that the waves that describe the influence of the boundary appear with some time delay. This time delay that can be expressed in terms of the unit Heaviside functions $\Theta(ct/\sqrt{\varepsilon_\infty} + \rho - a)$ and $\Theta(ct/n_2 - \rho + a)$ inside and outside of the nanowire, respectively.

III. Numerical results

For numerical modeling, we consider a silver nanowire with radius $a = 25$ nm, and corresponding parameters of the Drude model are $\omega_p = 1.4525 \cdot 10^{16}$ Hz, $\gamma = 7.0656 \cdot 10^{13}$ Hz, $\varepsilon_\infty = 5.2573$, which in optical range fit silver parameters [18]. We also consider the normalized time, $T = t \cdot c/a$. The normalized frequency of the initial plasmon is $w_0 = \omega_0 \cdot a/c = 0.6822 + i*0.003$ ($n_1 = 1.45$, $k = 1$) that is found from (3). The magnetic field of the plasmon is represented by formula (1). Its field pattern is shown in the inset in Fig. 5.

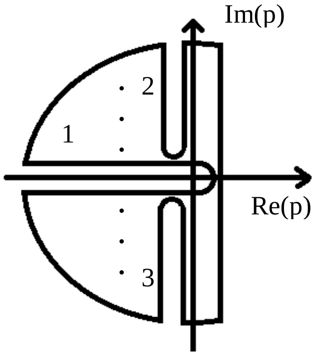

Fig. 2. The contour of the integration in calculation of the Mellin integral.

The time domain field is recovered using the Cauchy residue theorem. This approach guarantees accurate back transformation of the functions and allows us to gain understanding and insight into the transient processes. This method has been already successfully exploited for a variety of time domain problems [11, 19-21] with different media and geometries.

The contour of integration of the complex plane when referring to the time domain is shown in Fig. 2 (insets in the figure indicate the number of the branch cut). One can see three branch points: two branch points correspond to $\tilde{n}_p = 0$, and a branch point $p = 0$. The singular points $q = ik_0$ and $q = \pm ik_0\, n_1/n_2$ of the equations (11) - (12) are removable ones (the corresponding residues are zeros) and consequently do not contribute to the residue sum.

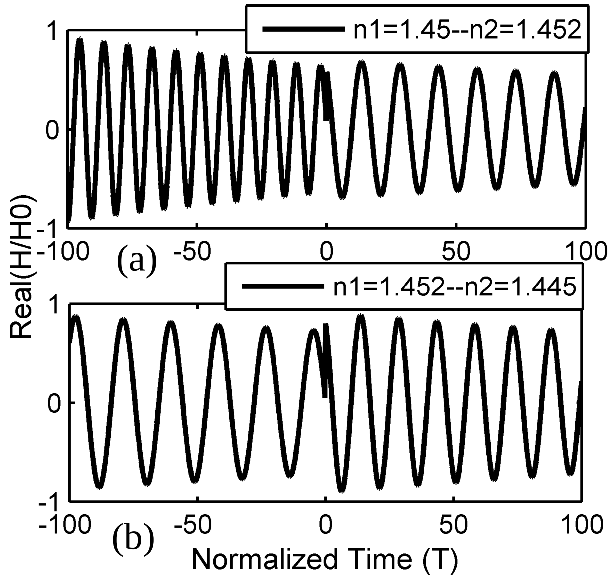

Fig. 3. The transformation of the magnetic field as a result of the change of the dielectric permittivity: (a) $n_1 = 1.45$, $n_2 = 1.452$; (b) $n_1 = 1.452$, $n_2 = 1.445$.

Nonzero contribution comes only from the solutions of equation (16), which correspond to the eigenfrequencies of the silver cylinder. All these frequencies are complex, their imaginary parts determine the velocity of decay of the oscillations. For each azimuthal index $k = const$ equation (16) has one solution, which corresponds to a surface plasmon, and an infinite number of singular points corresponding to the bulk plasmons.

Calculating residues at each bulk plasmon eigenfrequency, we observe that their contribution to the total field is negligibly small. Only an excited surface plasmon has amplitude comparable to that of the initial plasmon. Regarding to this, the overall effect of the abrupt change in refractive index transforms the initial plasmon to a transformed one with similar field pattern and a slight shift of the eigenfrequency.

Figure 3 shows the time dynamics of the transformation of the magnetic field inside the silver nanowire. Before zero moment of time the initial field is observable. Change in the permittivity at zero moment of time disturbs the field. The frequency shift of oscillation is seen. Fig. 4 presents the contribution of the integrals along the branch cuts in the complex plane. Their values, normalized by amplitude of the total field, are very small and can be neglected. Fig. 5 shows the total magnetic field pattern. Black dashed vertical line indicates the boundary of the silver nanowire. We observe that after the change of the permittivity the amplitude of the transformed field slightly decreases for $\Delta n = 0.005$ and increases for $\Delta n = -0.005$.

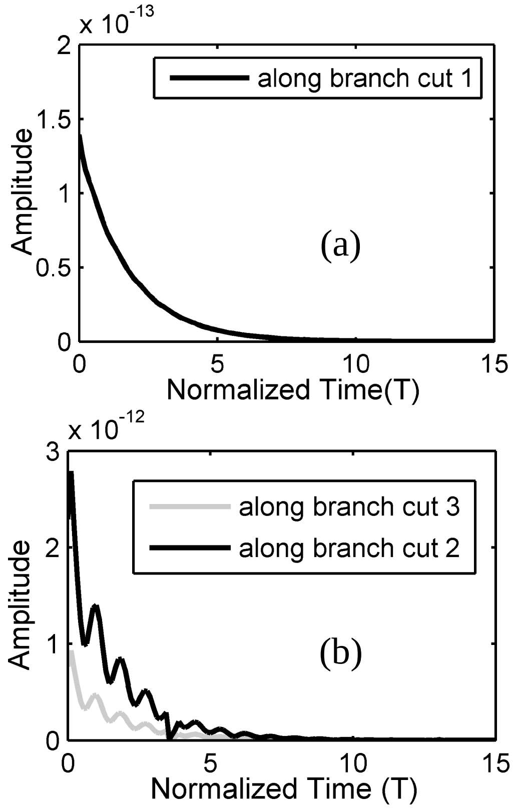

Fig. 4. The contribution of the integrals along the branch cuts (a) along $p = 0$ and (b) along $\tilde{n}_p = 0$ .in the complex plane ($n_1 = 1.45$, $n_2 = 1.452$).

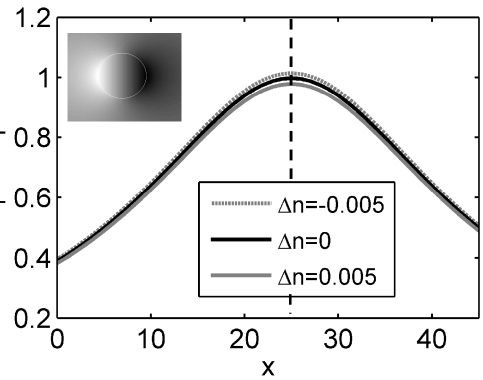

Fig. 5. The magnetic near field along the x coordinate for different values of the dielectric permittivity $n_2 = n_1 + \Delta n$ ($n_1 = 1.45$).

IV. CONCLUSIONS

The analytical solution of the problem of transient transformation of surface plasmon due to time variations in dielectric permittivity of nanowire environment has been derived. Accurate time domain inversion has made it possible to analyze transient and steady state regimes. It is concluded that the change of the dielectric permittivity leads to the frequency shift and the field pattern conservation.

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