RADIATION OF ACCELERATING PULSES WITH SPECIFIED ENVELOPES

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Abstract

A problem for radiation of time accelerating electromagnetic pulses is considered in paraxial approximation. It is shown that the pulse envelope moves in the time-spatial coordinates on the surface of a cylinder, the parabolic one for the pulse in the form of Airy function and for the hyperbolic one for Gaussian. Each of the pulse propagates in time with deceleration along the dominant propagation direction and drifts uniformly in the lateral direction stopping at infinity.

Keywords: antenna, pulse radiation, accelerating propagation, Airy pulse, Gaussian.

1. INTRODUCTION

Recent theoretical and experimental investigations of accelerating Airy beams [1-5] evoke wide interest to the pulses which move on curvilinear trajectories. In the majority of these investigations time is eliminated from the paraxial equation by using the predominant time harmonic dependence $E = F(x, y, z)e^{i(\omega t - kx)}$ for a wave propagating along the x axis. In this case a complex dependence exists between the lateral and the longitudinal coordinates. The parabolic dependence between time and the longitudinal coordinate with the leading dependence $\sim e^{i\omega t}$ is also considered in a number of publications. It is shown in [2] that a circular symmetric input field with the temporal behaviour according to the Airy function keeps this symmetry at later times propagating with time acceleration. Spatiotemporal Airy light bullets as the solution to a paraxial equation in a dispersive medium are investigated in [6]. The temporal analysis of Airy pulses in dispersive and nonlinear medium reveals such phenomena as the generation of solitons [7]. A fundamental remark concerning the causality effects of the phenomenon is made in [8]. It is noted that the spatial and temporal accelerations are physically qualitatively different. According to the authors' mathematical formulation of the problem, if the accelerated motion is considered in time, the solution requires the backward flow of time. Some physical ideas for overcoming this problem are proposed.

Here we suggest a different approach, considering a simple statement of the problem when electromagnetic pulses in nondispersive medium are generated by an external source. Our results provide a physically natural picture of the phenomenon and avoid the problem of backward flow of time. We do not make any assumptions on the temporal dependence of the field. We assume the dominant propagation along the x axis, $E = F(t, x, y, z)e^{-ikx}$, which is typical for solving problems in the paraxial approximation. The solution to the master equation is derived by a rigorous method of the Green's function. It has a clear physical meaning being free of exotic corollaries. This approach allows constructing other decelerating pulses (not Airy), the Gaussian is given as an example.

2. FORMULATION OF A PROBLEM

We consider an electromagnetic field radiated by an extrinsic source given by an electric current $\mathbf{j} = (0, j, 0)$ located in the plane $x = x_0$ perpendicular to the direction of the radiation propagation, Fig. 1. In this case the radiated electric field has the same orientation as the current, $\mathbf{E} = (0, E, 0)$ and it is controlled by the inhomogeneous wave equation

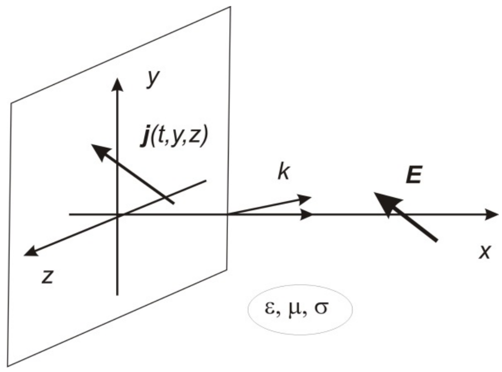

Fig. 1. The layout of the problem.

$$\left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}\right) - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} - \mu_0 \mu \sigma \frac{\partial E}{\partial t} = \mu_0 \mu \frac{\partial j}{\partial t} \quad (1)$$

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where $v^2 = 1/(\varepsilon_0 \varepsilon \mu_0 \mu)$ is the velocity of light in this medium, ε_0 and μ_0 are the permittivity and the permeability of vacuum, ε, μ and σ are the permittivity, the permeability, and the conductivity of the medium.

We do not assume any predefined temporal dependence of the field and take the dependence on the longitudinal coordinate $x > x_0$ dominant, $E = F(t,x,y,z)e^{-ikx}$. Here, $k > 0$ is a parameter characterizing the dominant spatial periodicity of the wave and F is the pulse envelope. With this assumption the well known paraxial approximation of a slow varying envelope $\left|F^{''}_{xx}\right| << \left|2ik_x F^{'}_x\right|$ is used.

The solution to the problem is obtained by the Green's function method

$$G = \frac{-(1-i)v}{8\pi x}\sqrt{\frac{k}{\pi x}}\theta(x)e^{i\frac{k}{2}x + i\frac{kv^2}{2x}[(t+i\frac{\mu_0\mu\sigma}{2k}x)^2 - y^2 - z^2]} \quad (2)$$

This function does not contain retardation and describes the propagation of a disturbance with the infinite velocity as it is inherent to solutions to parabolic type equations. Therefore, the processes described by this function do not contain the cause-effect relationship.

3. The Airy Pulse

We take the source current running in the plane $x = x_0$ in the form of a pulse described by the Airy function, Fig. 2,

$$j = \delta(x - x_0)\mathrm{Ai}\left((t - q_1 y - q_2 z)/T\right)e^{\alpha(t - q_1 y - q_2 z)/T}/T \quad (3)$$

Here, T is a normalized parameter and the parameter α is introduced for ensuring energy finiteness of the source, the idea proposed in [2] for a problem with harmonic temporal dependence of the phenomenon.
The calculation gives the radiated wave with the envelope described by the Airy function with complex argument:

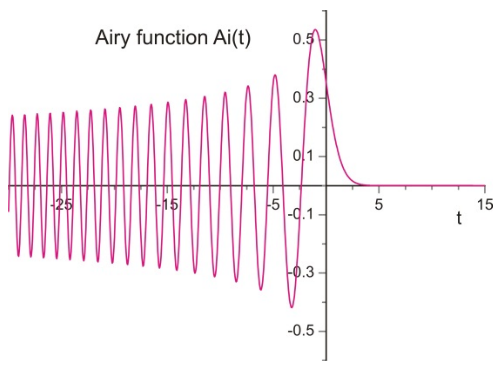

Fig. 2. The temporal dependence of the source current.

$$F(t,x) = \frac{i\mu_0\mu}{2kT} j_0 e^{ik(x+x_0)/2 + i\varphi(x)}\theta(x - x_0)\times$$
$$\frac{\partial}{\partial t}\left\{ e^{i\left(\bar{t} + i\frac{\mu_0\mu\sigma}{2k}(x - x_0))\right)\frac{1}{T}(m\frac{x-x_0}{2kv^2T^2} - i\alpha)} \times \right.$$
$$\left. \times \mathrm{Ai}\left[\frac{\bar{t}}{T} + i\frac{\mu_0\mu\sigma}{2kT}(x - x_0) - \left(\frac{m(x-x_0)}{2kv^2T^2}\right)^2 + i\frac{\alpha m(x - x_0)}{kv^2T^2}\right]\right\} \quad (4)$$

Here, the function

$$\varphi(x) = i\frac{2}{3}\left(m\frac{x-x_0}{2kv^2T^2}\right)^2 - 2\alpha\left(m\frac{x-x_0}{2kv^2T^2}\right)^2 + $$
$$+ i\alpha^2 m\frac{x-x_0}{2kv^2T^2}$$

$\bar{t} = t - q_1 y - q_2 z$ is reduced time, and $m = 1 - q^2 v^2$, $q^2 = q_1^2 + q_2^2$. The coefficient m shows the degree of paraxial approximation. If $m = 0$ then the parabolic approximation is inapplicable. The pure case of parabolic approximation is realised when $m \to 1$ that is when $q_1, q_2 \to 0$.

Note that following the Green's function (2) the radiated field (4) does not contain retardation and, therefore, it does not describe the relationship of cause-effect.

The trajectory of this Airy pulse envelope in time-spatial coordinates (t,x,y,z) lies on the surface that has the form of a parabolic cylinder, Fig. 3,

$$\left(t - q_1 y - q_2 z\right)\frac{1}{T} - \left(m\frac{x - x_0}{2kv^2T^2}\right)^2 = const \quad (5)$$

The pulse envelope moves on this surface decelerating along the longitudinal axis x and drifts uniformly with time in the lateral direction. The velocity of the envelope movement in the longitudinal direction is derived from (5) assuming that y and z are constants. This gives the envelope velocity along the propagation direction $\dot{x} = 2k^2v^4T^3m^{-2}/(x - x_0)$ as a function of the distance from the source or as a function of time $\dot{x} = kv^2T^{3/2}m^{-1}/\sqrt{t - const\,T}$. This velocity tends to zero with time as well as with the distance from the source. The acceleration of this movement $\ddot{x} = -\dot{x}^2/(x - x_0)$ is negative everywhere in the region of the pulse existence, $x - x_0 > 0$, and it also tends to zero with the distance from the source confirming the decelerating character of the movement.

4. The Decelerated Gaussian

For comparison, the source current of the Gaussian form running in the transverse plane is considered

$$j = j_0\delta(x - x_0)wT^{-1}\pi^{-1/2}e^{-w^2(t - q_1 y - q_2 z)^2/T^2} \quad (6)$$

The field radiated by this source is the running Gaussian

$$E=\frac{1}{2ik}j_0\mu_0\mu e^{-ik(x-x_0)/2}\frac{w\theta(x-x_0)}{2T\sqrt{\pi}\sqrt[4]{1+m^2u^2(x-x_0)^2}}\times$$
$$\frac{\partial}{\partial t}\left[\exp\left(-\frac{w^2}{T^2}\frac{\overline{t}^2}{1+m^2u^2(x-x_0)^2}\right)\times\right.$$
$$\left.\times\exp\left(i\frac{w^2}{T^2}\frac{\overline{t}^2mu(x-x_0)}{1+m^2u^2(x-x_0)^2}-\frac{i}{2}\arctan(mu(x-x_0))\right)\right] \quad (7)$$

where $\overline{t}=t-q_1y-q_2z$ is as above and the parameter $u=2w^2/(kv^2T^2)$ is determined by the beam waist radius.

The pulse envelope moves on the surface of the hyperbolic cylinder

$$(t-q_1y-q_2z)^2a^{-1}-m^2u^2(x-x_0)^2=1 \quad (8)$$

where $a=const$, Fig. 4. The projection of the hyperbolic cylinder onto the plane (t,x) illustrates the decelerating motion of the Gaussian envelope along the hyperbolic trajectories, but its velocity changes in different manner comparing with the Airy pulse.

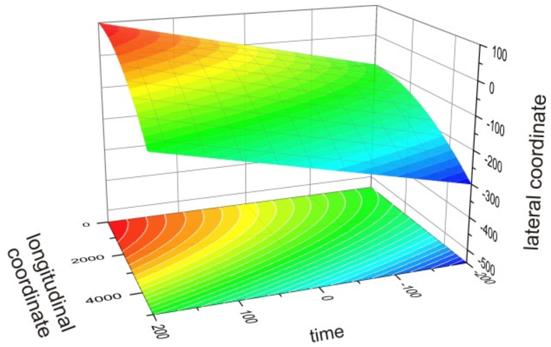

Fig. 3. The surface of the parabolic cylinder in the space time coordinates along which the Airy pulse propagates.

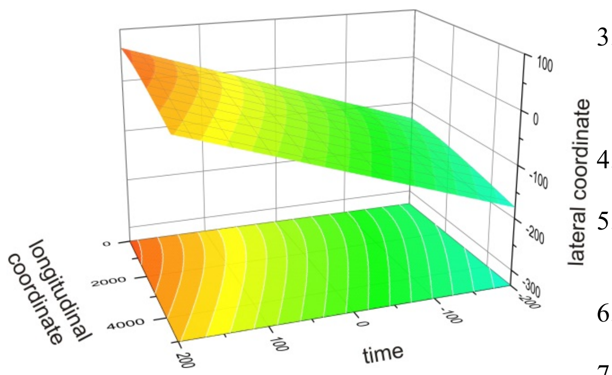

Fig. 4. The surface of the hyperbolic cylinder in the space time coordinates along which the Gaussian pulse propagates.

The dependence between the longitudinal and the transverse velocities on this surface is obtained by differentiating the equation (8). In the lateral direction (for a fixed coordinate x) the envelope movement, as in the case of the Airy pulse, is uniform and its velocity is constant: $q_1\dot{y}+q_2\dot{z}=1$. For given values of y and z the envelope movement is decelerating along the longitudinal axis x and its velocity changes as

$$\dot{x}=\frac{\sqrt{1+m^2u^2(x-x_0)^2}}{\sqrt{a}\,m^2u^2(x-x_0)}=\frac{t}{\sqrt{a}\,mu\sqrt{t^2-const}} \quad (9)$$

In contrast to the Airy pulse this velocity asymptotically tends to the nonzero value $\dot{x}_\infty=1/\sqrt{m^2u^2a}$ with time as well as with the distance from the source.

5. Conclusions

A time dependent electromagnetic field in paraxial approximation generated by a current running in a plane transverse to the propagation direction of the electromagnetic pulse is considered. It is shown that the Airy pulse envelope propagates in time with deceleration along the dominant propagation direction and drift uniformly in time and in the lateral direction staying on the parabolic cylinder surface. The longitudinal velocity of the Airy pulse envelope tends to zero with time and distance from the source. For comparison, the radiation of the decelerating Gaussian is investigated also. It is shown for comparison that the Gaussian propagates along the surface of the hyperbolic cylinder and the velocity of its envelope, as distinct from the Airy pulse, tends asymptotically to a nonzero value.

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