

## Modification of measurement theory

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## Modification of measurement theory

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**Abstract.** To ensure the correctness of measurements in nonlinear dynamic systems in paper was the development the measurement theory. We study the radically new conditions of measurement problems related to the specific behavior of nonlinear dynamic systems. Shown, that between qualitative theory of differential equations and methods of evaluation of measurement uncertainty there is a link that provides an assessment of conditions of measurement.

### 1. Introduction

The basic postulate of the measurement theory is the assertion that the measured value should have a unique value [1]. Based on measurements results, this unique value can only be evaluated, while a measure of uncertainty. Better put a measuring experiment, ie c less than the uncertainty determined by the parameters used in the measurement equation, the less uncertainty is determined by the value measured. Thus, the basics of the existing theory of measurement is the principle of classical determinism - the accuracy of the initial conditions determine the accuracy of the knowledge of the characteristics of dynamic systems, regardless of time of observation for the latter.

In metrology, application of classical Laplace determinism is clear and transparent. The results of indirect measurements are fully determined by the initial conditions, which were recorded in the uncertainty values of initial parameters. Therefore, the overall approach to reporting measurement results as properties of a stable and sustainable state of the system, whose parameters are known with some accuracy, is the foundation of the modern theory of measurement.

The realities of the modern development of science, technology, medicine, economics and many other areas of social development require the use of measurements as the primary means of obtaining objective information in those cases for which the theoretical framework has not yet been developed. Therefore, for those cases for which the statement measuring problem goes beyond the existing theoretical framework, it is necessary to modify the basic principles of existing theory or develop new measurement frameworks.

In this context, the purpose of this paper was to further develop the theory of measurements [2-5] to ensure the correctness of measurements in nonlinear dynamic systems. We study the radically new conditions of measurement problems related to the specific behavior of nonlinear dynamic systems. Shown, that between qualitative theory of differential equations and methods of evaluation of measurement uncertainty there is a link that provides an assessment of conditions of measurement.

## 2. Basics of measurement theory

The measurement theory for correct measuring in dynamic systems based on the property of system behavior in times. This property lies in the fact that the behavior over time is predictable and regular. Since the behavior of dynamic systems described by the decisions of the relevant differential equations, the equation of measurement is based on these equations. Depending on the complexity of the dynamic system can be used by a differential equation with N parameters

$$\frac{\partial}{\partial t} X = f(X, Y_1, Y_2, \dots, Y_N). \quad (1)$$

The condition of steady state of this dynamical system, described by the equation

$$\frac{\partial}{\partial t} X = 0, \quad (2)$$

or equation

$$f(X, Y_1, Y_2, \dots, Y_N) = 0. \quad (3)$$

This is a stable stationary state, rewritten in a simpler form, is a measurement equation, which is analyzed in the Guide to the Expression of uncertainty in measurement [1]

$$X = f(Y_1, Y_2, \dots, Y_N). \quad (4)$$

The uncertainty of measured value X determined in accordance with the equation

$$\Delta X = \sum_{i=1}^N \left( \frac{\partial(f)}{\partial Y_i} \right) \Delta Y_i \quad (5)$$

The basics of the modern theory of measurements are physical models describing the conditions of performance and analysis of measurement results.

In accordance with the first model - any measured dynamic physical quantity has a single value. Mathematically, this condition can be represented only solution of equation (4). This uncertainty  $\Delta X$  is determined by the values  $\Delta Y_i$  and derivatives in equation (5) have a fixed value.

The second model considers the random scatter of measurement results as a random, ergodic process. Ergodicity registered variations X described by the equation

$$\langle X \rangle = \bar{X}. \quad (6)$$

For metrology ergodicity of random variations of the measured values of physical quantities, is the most satisfactory, if not the sole rationale for the simultaneous application, as the averaging time, and averaging over the probability law of distribution of possible states.

### 3. Modification of basics of measurement theory

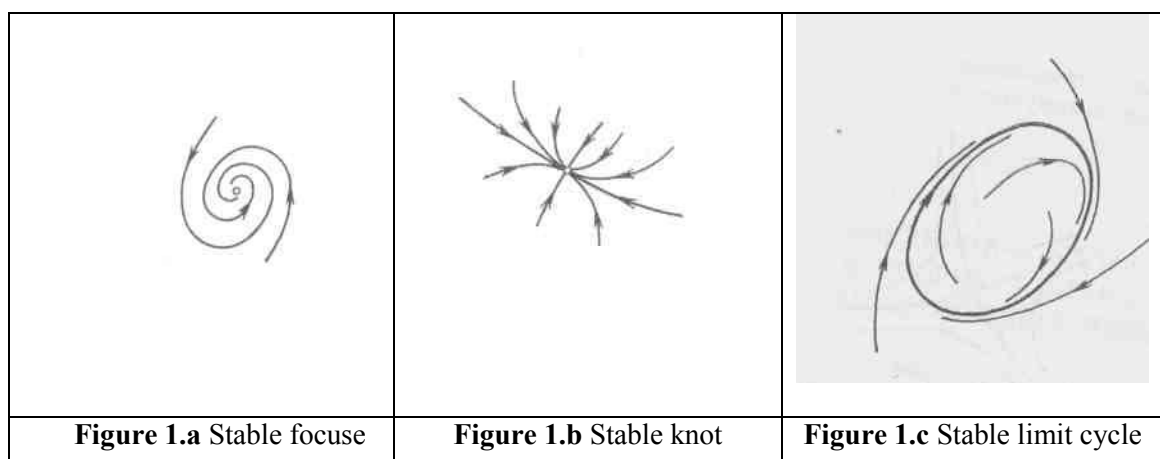
To describe the behavior of dynamical systems than analytical solutions to differential equations are also used qualitative methods, based on the qualitative theory of differential equations. These methods allow us to study the solution of differential in the phase space, i.e. in some mathematical multidimensional space, as coordinates of which are considered variable system of equations. However, until recently they were not used when describing the measurement process.

If the dynamical system described by the  $n$  - dimensional system of differential equations

$$\frac{\partial}{\partial t} X = f(X) \quad , \quad (7)$$

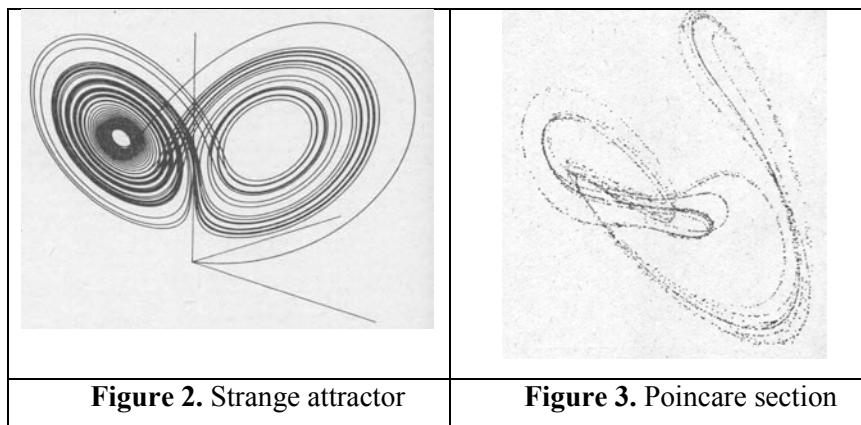
где:  $X = (X_1, X_2 \dots X_n)$ ,  $f = (f_1, f_2 \dots f_n)$ .

so physically significant for the measurements are only those of a dynamic system that describes the stable and steady-state solutions of the  $n$  - dimensional system of differential equations. Qualitative theory of differential equations on the plane suggests three specific points on the phase plane:-stable focus (1.a); stable knot (1.b); limit cycle (1.c). This theory can also explore the features of the structure of phase space subdivision and, very importantly, the domain of attraction of stable states (attractor). Assuming that the system is in one of the above-mentioned stable states, the results of measurements of variables will affect the value of the field and the speed of gravity. Nevertheless, the results of measurements aimed at establishing



of values corresponds to a single stable state. In three-dimensional and more the phase space, are not only simple attractors, but strange attractors [6]. In this case, the solution of differential equations in three-dimensional phase space may have the form shown in Figure 2. This strange attractor is stable and having attracting region. Peculiarities of behavior of dynamical systems in

the field of strange attractors is mathematically and physically well understood, which implies that the values of parameters during the time random change. This feature is explained by the fact that the trajectory on which the dynamical system moves in phase space never intersects with itself and is in a closed region of phase space. Therefore, it gets confused and behaves complex irregular manner. Poincare section of the strange attractor appears, as shown in Figure 3



The problem of assessing the real value of the measured quantity in this case lies in the fact that all the measurements characterize the actual state, which randomly varies over time. It would not have been fulfilled individual measurement the scatter of measurements will be characterized by dynamic behavior of the system. Therefore, the evaluation of uncertainty of measurement result will be determined not by the size and nature of the external random perturbations, and dynamic behavior of the system.

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