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**GOLDEN SECTION AND SOLAR CYCLES IN NATURAL PHENOMENA AND BIONICS**

In development of Auric Time Scale based on the Golden section power series with the unity equal to the average Solar cycle length, new acknowledgements are presented that widen the synchronism between the ATS and planetary and terrestrial phenomena periods up to seconds and dozens of millennia. It is shown that distribution of Solar cycle energy emission centers also follows these series.

**1. Introduction**

Solar activity (SA) presents interest from both practical and theoretical points of view. Solar events may disrupt or damage such Earth-Space activities as satellite operations, high-altitude polar flights, electric power distribution, pipeline operations, climate, biological objects, etc. [www.sel.noaa.gov, science.nasa.gov]. That is why forecasting of SA levels presents an important problem not only for astrophysicists, but for a variety of applications. However, though the physical causes of correlations between some manifestations of SA are explained, and their use together with empirical models allow us to obtain more or less reliable short-termed forecasts, the origin and mathematical description of the basic 11-year SA cycles (Solar cycles, for short) remain obscure.

For these reasons, study of these cycles continues to attract attention of astronomers [1], mathematicians [2] and experts in other branches of knowledge [3]. The basic aspects of these studies are associated with detection of cycles in nature and society that correlate with the Solar cycles, and revealing of supercycles and subcycles of the basic Solar cycle both within the sequence of these cycles, and in relation with the planetary revolutions. The former aspect of this problem provides us with a lot of well-grounded correlations [3–6], but it is evidently a sphere of applications. On the contrary, the latter one presumes mathematical study in search of those laws which may be used for more exact forecasting and understanding of this phenomenon.

From mathematical point of view, the principal difficulties in study of Solar cycles consist in small amount of sampling and absence of stable criterion specifying their critical points. Thus, only less than two dozens of Solar cycles were observed on a regular basis relative to sunspots, and only few ones – with regard to other Solar indices, whereas subjective criteria (cycle minimum and maximum [7]) are used for treating these values. Besides, we must allow for noticeable variation in cycle parameters and noise pollution of Solar data which, to some extent, are attributed to planetary influence [2, 6] and, for this reason, called modulation.

All these factors hamper quantitative study of Solar cycles; first of all – due to a lack of numerical tools for

testing of obtained empirical dependencies. Hence, it becomes clear that only a system treatment of all these obstacles may allow us to reveal more or less general laws that stand beyond the development of Solar.

General model of Solar cycle presumes description of SA magnitude (basically – the sunspot number) in time. Not only this problem, but even its sub-problem consisting in forecasting of cycle minimum and maximum for the forthcoming cycle is far from its solution. Therefore, specifying an unbiased and stable criterion for selection of Solar cycle reference points, to be taken instead of empirical maxima and minima [7], as well as studying of their properties on the grounds of error analysis presents an important scientific problem being crucial for many applications.

**2. Analysis of the problem**

**2.1. Solar activity** is understood as origination of active formations in Solar atmosphere; the main of them are as follows [www.sel.noaa.gov, science.nasa.gov].

*Sunspot* is an area seen as a dark spot on the photosphere of the Sun. Sunspots are concentrations of magnetic flux, typically occurring in bipolar clusters or groups. According to historical records, Theophrastus was the first who identified them in 325 BC. In Europe they were discovered with the invention of telescope in 1610, independently by G. Galilei, J. Fabricius, C. Scheiner, and T. Harriot.

Daily observations and counts of sunspots were started at the Zurich Observatory in 1749, after R. Wolf has devised in 1848 a daily index of sunspot activity – Sunspot Number (SSN) as  $R = k(10g + s)$ , where  $S$  is the number of individual spots,  $g$  – the number of sunspot groups, and  $k$  – an observatory factor. Today Wolf sunspot counts continue, since no other index of the sun activity reaches into the past as far and as continuously. The determination of International Sunspot Numbers  $R_i$  results from a statistical treatment of data originating from an international network to guarantee continuity with the past Zurich series  $R_z$ . These numbers may be obtained at [8, 9]. Below, *monthly mean  $R_i$  numbers* are considered, if not specified otherwise.

Side by side with SSN, the 10.7cm Radio Flux (Flux, for short) index is supplied which represents the level of Sun's radio emissions. This index is closely correlated with SSN, but its values are recorded from 1947.

Solar flare is an intense eruption of energy on the Solar disk. Flares are our solar system's largest explosive events; they occur near sunspots. Often above sunspot groups, the solar atmosphere can release tongues of gas and magnetic fields called *Coronal Mass Ejection* (CME).

A large CME can contain billion tons of matter that can be accelerated to several million miles per hour. CMEs produce disturbances that strike the Earth with sometimes catastrophic results. The frequency of CMEs varies with the sunspot cycle.

Thus, though the origin of Sunspots is not well understood, it is known that the magnetic field in sunspots stores energy that is released in solar flares and other events. As a result, they usually occur in a cycle that mimics the eleven-year sunspot cycle. Therefore, for the reasons of its continuity, long prehistory and close correlation with other factors of SA, we can take the SSN for a representational index of Solar activity as a whole.

**2.2. Solar cycle.** During an 11-year SA cycle, the number and the magnitude of Solar effects vary from minimum to maximum, and back – to minimum. In average, the duration of this cycle makes 11 years, though its actual values varied from 7 to 17 years. In parallel with it, the magnetic sunspot cycle of 22.1 years (the *Hale* cycle) is generally accepted. Since the start of daily observations, the cycles carry the Zurich series numbers [7]. The current 23<sup>rd</sup> cycle has started in 1996 and now approaches its end.

The monthly mean SSN [8] are obtained for filtering of daily variations in this index. In order to decrease the swing of monthly mean SSN, the *smoothed monthly mean SSN* [7] is defined as the arithmetic average of two sequential 12-month running means of monthly mean numbers. However, though use of smoothed values benefits to visual presentation of development of SA, this approximation distorts the monthly counts. Nevertheless, namely these values are used in the definition of the conventional maxima and minima epochs of Solar cycle: «When observations permit, a date selected as either a cycle minimum or maximum is based in part on an average of the times when extremes are reached in the monthly mean sunspot number, in the smoothed monthly mean sunspot number, and in the monthly mean number of spot groups alone. Two more measures are used at time of sunspot minimum: the number of spotless days and the frequency of occurrence of «old» and «new» cycle spot group.» [9].

This subjectivity leads to an unacceptable, for mathematical study, uncertainty in estimation of extrema: «May 1996 marks the mathematical minimum of Cycle 23. October 1996 marks the consensus minimum determined by an international group of solar physicists. April 2000 marks the mathematical maximum of Cycle 23. However, several other solar indices (e.g., solar radio flux) recorded a higher secondary maximum in late 2001.» [9]. Fig. 1 illustrates this situation, where use of smoothing polynomial instead of smoothed SSN clearly shows a bimodal nature in development of SA.

**2.3. Solar Cycle Prediction.** With the aim to work out a forecast for the amplitude and phasing of the most common indicators of solar and geomagnetic activity for Solar Cycle 23 and to recommend it for operational use,

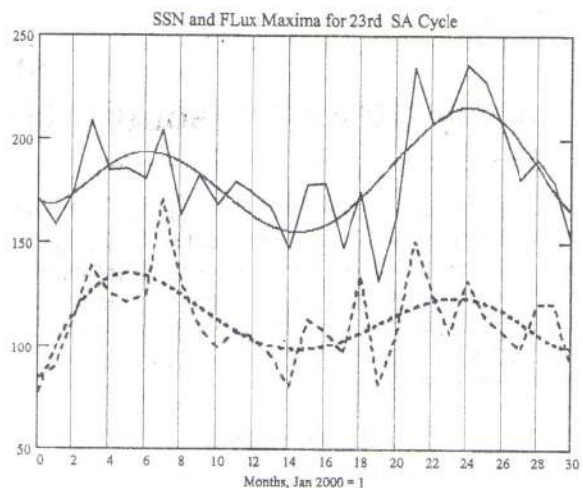


Fig. 1. Monthly Mean SSN and Radio Flux for cycle 23 (January 2000 to June 2002)

Upper lines: Polygon — Monthly mean Flux (divided by 10)  
Smooth line — Smoothing polynomial for Flux  
Lower lines: Polygon — Monthly mean SSN  
Smooth line — Smoothing polynomial for SSN

the NOAA and NASA recruited a scientific panel of 12 scientists from 10 agencies that issued the report [10] in which it summarized the findings of the panel. It ascertains «lack of a successful quantitative, theoretical model of the Sun's magnetic cycle» and states that criticisms of long-term solar cycle prediction focus on the thin physical foundation beneath such predictions. The panel largely agrees with these criticisms, and encourages scientific community to develop a fundamental understanding of the solar cycle that would provide the basis for physical rather than the present empirical prediction methods. In the summary [11] of the Second Meeting, the panel notes that the epoch tables specifying cycle maxima and minima included in [12] are not consistent with a strict determination based on smoothed sunspot number.

This means that the existing level of physical knowledge and the empirical techniques allow us to predict the magnitude and the phase (extrema epochs) of the forthcoming SA cycle, but with a great error and for the smoothed SSN. Indeed, an error of the forecasted magnitude of maximum makes about  $\delta_{Magn} \approx 33\%$ , and the model suggestion that the Solar cycle has a single maximum must evidently be rejected due to the observed SA dynamics. Thus, monthly mean SSN was growing in Cycle 23 until it reached the cycle maximum of 170.1 (July 2000). Then, after a decline up to the value of 80.6 (February, 2001), it starts to grow and reaches the second pronounced peak of 150.7 (September, 2001) — see fig. 1. Moreover, the Solar flux has its cycle maximum at the second comb of SSN — at September and December of 2001. In smoothed values being used in that model an error for the second maximum makes 20 months, or 40% relative to 4-year period of forecasting, which even does not fit the interval estimation. Note also

that Cycle 22 shows a similar bimodal behaviour in SSN and Solar flux.

Therefore, we may conclude that a *unimodal concept* of Solar cycle is not adequate; neither the Solar cycle *maximum (minimum)* epoch grounded on the *smoothed* SSN reflect the actual status of SA, though it is expected to await that a characteristic with such name must reflect its meaning and not with a 40% error even for one cycle.

**2.4. Interaction models.** Apart from Solar-index ones, the models which show correlation between the Solar cycle extrema and planetary periods and positions exist.

T. Landscheidt [3] suggested that the major instability events (on Sun and Earth) took place when the Centre of masses (CM) of the Solar system remained in or near the Sun's surface. For a quantitative model of this phenomenon he considers the torque  $T$  acting on the Sun and shows that strong impulses of the torque are initiated when Jupiter, the CM and Sun's centre CS (JU-CM-CS) are in line, and major instability events caused by these conjunctions nearly always are accompanied by energetic eruptive activity. This allows to efficiently predict weather and geomagnetic storms.

Though the *Golden Section* is not detected in the Torque-Fractal Model (TFM), Landscheidt implies that «it could be that the pentadactyl pattern created by the Sun and the outer planets hints to a special function of the *Golden section* in the *Solar system*. ... If we want to understand cycles in depth, we must consider these root elements and their connections. This is valid, too, for planetary cycles.» [13]. However, the TFM does not engender neither an accurate estimate for the average Solar cycle length  $T_o$ , nor a TFM-correlation with Solar cycles. In this sense the TFM can be taken as an analytical model that describes how the JU-CM-CS modulates, but not governs, the development of a Solar cycles.

The synchronism between the SA cycle harmonics and planetary periods was also studied with the use of Fourier spectrum of Zurich series [1]. It turned out that several dozens of peaks (viz. periods) on the spectrum have been identified with the orbital periods  $P_i, P_j$  of planets according to the formula  $1/P_j = 1/P_i \pm 1/P_j$ . Though the obtained synchronism between some Solar cycle harmonics and planetary paired periods presents an outstanding result by itself, it does not provide us neither with the basic 22-year period, nor with a structure of harmonics.

Many other dependencies are now discovered between the Solar cycle harmonics and a wide spectrum of periods in nature and society [5, 14]. But with the exception of Auric Time Scale (ATS) [15] grounded on the Golden section number, no other system structures a significant part of these periods.

**2.5. Statement of the problem.** The provided analysis shows that until now there exists neither explanation, nor numerical description of trends in Solar cycles, apart from the commonly accept presence of 11- and 22-year

cycles. From this viewpoint, the *average duration of Solar cycle*  $T_o$  is of fundamental importance. Since the SSN statistics has several order larger prehistory than any other Solar index and this number is closely correlated with other indices, the *monthly mean SSN* is taken as the source data in study of Solar cycles. In order to obtain reliable conclusions from these «polluted» data, we must analyze them on the grounds of *error analysis*. As far as a suggestion relative to the *Golden section* does actually take place for the Solar system periods [15] and historical spiralling of time [16], it also presents an interest to reveal, whether the Golden section is seen in Solar cycles. Last, not least, is the necessity to define an effectual substitute (*reference point*) for a cycle maximum.

With the due regard of these requirements, we come to a general problem of structural and parametric identification of Solar cycles. In treating this problem, (1) a critical epoch of Solar cycle must be formally defined and (2) a model of distribution of these epochs (cycle structure) is to be proposed. For this, the concept of Regular model of Solar cycle maxima distribution [17, 18] is disseminated onto the Solar cycle medians with the use of ATS and threshold error analysis for adequate testing of synchronism (correlation) between epochs and periods.

### 3. Auric Time Scale and error analysis

At present, a wide spectrum of correlations of periods has been established for the diverse phenomena in various spheres of knowledge: from biology and economy – up to geology, the most part of which are associated with the Solar cycle and its harmonics. However, the trustworthiness of correlations between some of these periods is frequently called in question, since speculations in most cases dominate a quantitative analysis. As a result, we come to a situation when absence of consolidated criterion for accepting a synchronism between periods on the one hand, and lack of generally accepted structure of periods, on the other hand, put the impediments in study of cycle structures and correlations. For settling this problem, the concepts of error analysis and Auric Time Scale were put forward.

**3.1. Auric Time Scale** [15, 16]. The Golden Section number  $\Phi = 1.6180339\dots$  is known from the antiquity. The inverse value  $\varphi = \Phi^{-1} = 0.6180339\dots$  also bears this name, whereas coincidence of fractions is not occasional as  $\Phi = 1 + \varphi$ . The following two-sided progression with unitary initial term is called [15] the *Auric series*  $\Gamma$

$$\dots, \Phi^{-2}, \Phi^{-1}, \Phi^0 = 1, \Phi^1, \Phi^2, \dots \quad (1)$$

Without negative powers, it takes the form  $\dots, \varphi^2, \varphi^1, \varphi^0 = \Phi^0 = 1, \Phi^1, \Phi^2, \dots$ . Series  $\Gamma_n$  presents the *natural replenishment* of series  $\Gamma$  or order  $n$ , if a natural value  $n$  is assigned to the initial term of series  $\Gamma$ . Among all these series, the greatest interest presents the series  $\Gamma^* = \Gamma_2$ , or

$$\dots, 2\Phi^{-2}, 2\Phi^{-1}, 2\Phi^0 = 2, 2\Phi^1, 2\Phi^2, \dots$$

The *Fibonacci numbers (series)* are the terms of infinite series  $u$  of natural numbers  $u_1, u_2, \dots$  defined by initial terms  $u_1 = u_2 = 1$  and recurrent equation  $u_{n+1} = u_n + u_{n-1}$ .

Within a scale factor, the series (1) is the only one that also satisfies the Fibonacci's property  $\gamma_{n+2} = \gamma_n + \gamma_{n+1}$ ; besides, the Golden section and series  $u$  are connected as follows:

$$u_n = [\Phi^n - (-\varphi)^n] / \sqrt{5}, \tag{2}$$

$$\Phi^n = u_n \cdot \Phi + u_{n-1}. \tag{3}$$

In parallel with the series  $u$ , consider the series  $v$  which is generated as the series  $u$ , but with other initial terms  $v_1 = 1, v_2 = 3$ . The connection between this series and series  $u$  may be described as follows:

$$v_n = u_{n-1} + u_{n+1}, \quad n > 1; \quad v_n = \Phi^n + (-\varphi)^n. \tag{4}$$

Consider also the *United series*  $z$  composed of their terms as  $u_1, v_0, u_2, v_1, u_3, v_2, u_4, v_3, \dots$ , where  $v_0 = 0$ . Other properties of these series are given in [15]. The bulk of Auric series terms that are used below are given in tab. 1.

Table 1  
Fibonacci Numbers and Basic Auric Series

Index $k$	Series $u$ $u_k$	Series $v$ $v_k$	Series $\Gamma$ $\Phi^k$	Series $\Gamma^*$ $2\Phi^k$
.	.	.	.	.
-3	-	-	0.24	0.47
-2	-	-	0.38	0.76
-1	-	-	0.62	1.24
0	-	-	1.00	2.00
1	1	1	1.62	3.24
2	1	3	2.62	5.24
3	2	4	4.24	8.47
4	3	7	6.85	13.71
5	5	11	11.09	22.18
6	8	18	17.94	35.89
7	13	29	29.03	58.07
8	21	47	46.98	93.96
9	34	76	76.01	152.03
10	55	123	122.99	245.98
11	89	199	199.00	398.01
12	144	322	322.00	643.99
13	233	521	521.00	1042.00
14	377	843	843.00	1686.00
15	610	1364	1364.00	2728.00
16	987	2207	2207.00	4414.00
17	1597	3571	3571.00	7142.00
18	2584	5778	5778.00	11556.00
19	4181	9349	9349.00	18698.00
20	6765	15127	15127.00	30254.00
21	10946	24476	24476.00	48952.00
22	17711	39603	39603.00	79206.00
.	.	.	.	.
.	.	.	.	.

Note, that (3) gives a convenient rule for calculation of powers of  $\Phi$  and it follows from (3), (4) that  $\Phi^n$

converges to  $v_n$ , and  $2\Phi^n$  - to  $(2\sqrt{5}/\Phi^3) \cdot u_{n+3} \approx 1.056 \cdot u_{n+3}$ .

In other words, the Golden section series  $\Gamma$  engenders the series  $\Gamma^*$ , whereas the series  $v$  and  $u$  give an integer presentation for the former ones. This unity of multiplicativity and additivity presents a deep basis of series  $\Gamma$  and Fibonacci numbers, and engenders a close connection between them which is important as a structure presenting a system of basic periods [15]. For this reason the system of series  $\Gamma$  and  $\Gamma^*$ , in parallel with  $u, v$ , and  $z$ , is further considered as the *Auric Time Scale* in a wide sense, the time unity of which (viz. the term  $\Phi^0 = 1$  of series  $\Gamma$ ) corresponds to the selected period of physical process (e.g. Tropical year, Solar cycle length, or other period). For convenience, the Earth year is taken for this unity. Namely this algebraic system, the core of which is specified by the series  $\Gamma$ , is used as a *discrete* structure of a set of benchmarks which correlate the bulk of basic periods from planetary to terrestrial ones.

**3.2. Threshold Error Analysis** [18]. The actuality of error analysis for the considered problem corresponds fully to that how frequently it is ignored, when the values obtained from observations are taken for the "exact" ones. The more so because a short source data prehistory results in obtaining of relatively wide confidence intervals for the solutions to have practical sense.

For the *threshold error*  $\delta^*$  we take a relative error pertaining to the least exact, but important quantity out of the considered ones. This allows us to explicitly adjust the decision making to the source data accuracy and, further on, to supply the result with its error for subsequent use.

Let  $\delta^*$  present a threshold relative error for the given system of periods  $Z$ , and  $T, t$  - the estimates for its elements  $X, Y \in Z$ . Define the error of equality for these values as the ratio presenting an analogue of relative error

$$\delta_{T,t} = |T - t| / \min(|T|, |t|). \tag{5}$$

In this case we accept the periods  $X, Y$  equal to within the error  $\delta_{T,t}$ , and denote this as  $X \cong Y (\delta_{T,t})$ . If  $\delta_{T,t} \leq \delta^*$ , we accept these periods equal  $X \cong Y$  to within the system  $Z, \delta^*$ . Thus, out of relation  $2\Phi^n \approx 1.056 \cdot u_{n+3}$  for the terms of series  $u$  and  $\Gamma^*$  we get  $2\Phi^n \cong u_{n+3}$  (5.6%) and regard them as equal for  $\delta^* = 6\%$ .

Together with considering of relative error being appropriate for periods, in analysis of proximity of absolute time moments we must take into account the origin of time coordinate. In this case we consider an absolute error

$$\Delta_{xy} = |x - y|, \tag{6}$$

and compare it against the threshold error  $\Delta^*$  to be defined in compliance with  $\delta^*$ , though in some cases the value  $\delta^*$  is obtained from  $\Delta^*$ , if the latter one defines the degree of a source data uncertainty.

Comparing of errors (5), (6) against the threshold value  $\delta^*$  ( $\Delta^*$ ) specifies a *binary relation*  $X \cong Y$  of synchronism for elements of system  $Z$  with respect to the given accuracy  $\delta^*$  ( $\Delta^*$ ). Though transitivity for this relation may not be guaranteed for all possible chains of periods for a continuous scale, this does not impede from revealing of classes of equivalences of periods (in a sense synchronism) in case of ATS due to its *discrete* structure, if we specify these classes by  $\delta^*$ -deviations from the exact ATS values.

When considering a system of Solar and planetary revolution periods, the Solar parameters are evidently to be taken for estimating the value of  $\delta^*$  (or  $\Delta^*$ ) as the least exact, but most important one. The more so the 11-year Solar cycle period and its harmonics correlate not only with the majority of both planetary and terrestrial phenomena periods, but also with a probable «master generator» of the Solar System – the nearby hypernova Eta Carinae [19].

#### 4. Solar cycle length estimation and its accuracy

Thus, the average length of the 11-year Solar cycle  $T_o^*$  presents special interest for this study. The basic estimates for it are varying around the value of  $T_o' = 11.1$  yr, but the principal question remains as to estimation of two decimal digits of mantissa. Although a Solar cycle is considered now as an interval between two sequential minima, in estimating the average length  $T_o^*$  the epochs of maxima are more useful, as the pre-telescopic observations are associated with phenomena originated at Solar maxima, while inexact definition of minimum hampers use of even existing SSN data.

For the given epochs of maxima  $\tau_n = \{t_1, t_2, \dots, t_n\}$ , the value of  $T_o^*$  can be estimated by the mean cycle length

$$T_o = \frac{1}{n-1} \sum_{i=1}^{n-1} (t_{i+1} - t_i) = \frac{1}{n-1} (t_n - t_1), \quad (7)$$

that presents a *stable and unbiased estimate* of  $T_o^*$ , the accuracy of which the higher, the longer the series  $\tau_n$ .

A new catalogue [20] of sunspot observations estimates this period as  $T_{o,C} = 11.116 \pm 0.007$  and provides an approximation for maxima epochs:  $Year = 4.0 + 11.116N$ , where  $N$  is a cycle number ( $N = 178$  for maximum of 1980/81). However, this value exceeds significantly other estimates which are in good correspondence with a series of cross-correlations, and, even for the specified  $N$ , gives the value 1982.648 against the official year of 1979.9 [7]. J. Schöve [21] has made a mainly auroral analysis back to 649 BC and calculated that between 215BC and AD 1947 there were 195 sunspot cycles thus giving  $T_{o,S} = 11.08$  yr as the average length. Due to a significant variation in periods he also gives an interval estimate of 11.04 to 11.09 yr and considers the question to be not definitively settled [6]. T. Cole, in development of his work has obtained an estimate of

11.06 yr [22]. T. Niroma [6] supports an estimate of  $T_o = 11.07$  yr with an interval of 11.05 to 11.09 years. The same point estimate was obtained in [17, 18]. As far as these grounded estimates oscillate around the latter one, this value

$$T_o = 11.07, \quad (8)$$

is considered below as the most grounded point estimate for the average length of the 11-year Solar cycle  $T_o^*$ .

Absolute error for estimates (7), (8) can be evaluated so

$$\Delta_T = |T_o - T_o^*| = \left| \frac{1}{n-1} (t_n - t_1) - T_o^* \right|, \quad (9)$$

It depends on the errors in defining the epochs  $\Delta_{E,1}$ ,  $\Delta_{E,n}$  of the first and last maxima in the series  $\tau_n$ , and in the deviations  $\Delta_{D,1}$ ,  $\Delta_{D,n}$  of length of the first and last cycles from the average value. Therefore,  $\Delta_T$  does not exceed the following upper estimate for this error

$$\Delta_T \leq \frac{|\Delta_{E,n}| + |\Delta_{D,n}| + |\Delta_{E,1}| + |\Delta_{D,1}|}{n-1}, \quad (10)$$

where an average deviation in defining an epoch of maximum may take a value of up to 2 years, if they present a bimodal distribution of SSN, and up to 3 years for the ancient cycles being estimated by Aurorae which indicate maxima with such a delay [6]. Since the duration of Solar cycle may have a length of 7-17 yr, the worst estimate for  $\Delta_T$  makes  $\Delta_{T,w} \approx 17/(n-1)$  yr, while an average estimate for  $\Delta_T$  makes  $\Delta_{T,a} \approx 5/(n-1)$  yr.

At present, three principal series of SSN exist. For brevity, call them the *short, medium* and *long samples*. The first one presents the Zurich series. The medium sample contains telescopic observations since 1610, where all SSN maxima are specified [8], the current one is 36<sup>th</sup>. The long sample presents about two hundred Solar maxima epochs restored by observations of Aurorae and <sup>14</sup>C.

The estimate  $\Delta_{T,w}$  makes about 0.75, 0.5, and 0.085 yr for short, medium, and long sample, whereas the average estimate  $\Delta_{T,a}$  makes 0.23, 0.14 and 0.025 yr, respectively. However, the accuracy of estimate (7) can actually be even higher, since the values  $\Delta_{E,1}$ ,  $\Delta_{E,n}$ ,  $\Delta_{D,1}$ ,  $\Delta_{D,n}$  may compensate each other, which is typical for statistical data. Hence, the *average error*  $\Delta_a$  for the long sample can be estimated as a mean of the latter two values  $\Delta_a \approx 0.01$  yr, but hardly less than 0.01 yr. Meanwhile, even if the epochs  $t_1$ ,  $t_n$  are defined exactly, the deviations  $\Delta_{D,1}$ ,  $\Delta_{D,n}$  remain effective (they may compensate by chance only). In this case the estimate  $\Delta_{T,w}$  makes an order of 0.06 yr for a long sample, or  $\Delta_w \approx 0.07$  as a mean for this and the above value, while for the short and medium samples this effect is seen in a significant variation of reported estimates for  $T_o^*$ .

Thus, use of long sample gives a mean limiting error in estimate (8) as  $\Delta_w \approx 0.07$ , which corresponds to

Niroma's intervals; by rounding this value to the tenth we obtain the searched *absolute* and *relative threshold errors*

$$\Delta^* = 0.1 \text{ yr}, \quad \delta^* = 0.1/11.07 \approx 1 \%. \quad (11)$$

### 5. ATS as a Structure of Periods and Evolutional Time

**5.1. ATS:** algebraic structure of Solar System periods [16]. Consider the Mercury to Pluto orbital periods together with the average periods of Solar Equator rotation, asteroid belt (AB) revolution, 11-year Solar cycle (with its 22-year Hale cycle), and the *main Solar System's period* — a minimal number that engenders the planetary periods as its natural harmonics; for short, call a hypothetic object with the latter period to the name of Proserpine — a planet being searched until now. As far as the Solar cycle length presents the most important, while the least exact period of these data, the value (11) is taken for a threshold error in acknowledging the below correlations.

Relative frequency  $\Omega_{ij} = T_i / T_j$ ,  $T_i > T_j$ , of periods  $T_i$ ,  $T_j$  defines the period of object  $j$  as harmonic with «number»  $\Omega_{ij}$  relative to the period of object  $i$  in ideal case, when  $\Omega_{ij}$  is an integer; call this situation a synchronism of period  $T_j$  and harmonic  $\Omega_{ij}$  of period  $T_i$ . We know that absence of mathematically exact synchronism does not yet mean absence of its physical manifestation, but presumes that the synchronism between periods the more pronounced, the lesser the deviation of actual value of  $\omega_{ij} = T_i / T_j$  from some integer frequency  $\Omega_{ij}$ . In compliance with this approach, three main period  $T_{Ju}$ ,  $T_{Ur}$  and  $T_{Pr}$  were revealed, the harmonics of which come to a good agreement with the periods of remaining objects.

By taking account of importance of the structures defined by these main periods, call them planetary series. Namely, a planetary series  $Q$  is a sequence of objects and their periods (or harmonics), where the root object  $Q$  presents the largest (root) period  $T_Q$ , while the fundamental periods of the remaining objects coincide with harmonics of this root period.

Denote the series of Jupiter, Uranus and Proserpine  $J$ ,  $U$ , and  $P$ ; then, the series  $J$ , for example, comprises the following objects:  $J = \{Jupiter (1), SA (1), AB (3), Mars (6), Earth (12), Venus (20), Mercury (50), SR (174)\}$ . With respect to the definition of the main Solar System's period  $T_{Pr}$ , its harmonics must be close to the planetary ones. By allowing for their multiplicative nature, estimate this period as geometrical mean of known root planet periods,  $T_{Pr} = \sqrt{(\omega_{Ju}^{Pr} \cdot T_{Ju}) \cdot (\omega_{Ur}^{Pr} \cdot T_{Ur})} = 501.144$ . Analysis of these series allows us to conclude the following.

(1) The considered periods, to within an accuracy of 1-2%, are distributed over the three *planetary series*  $J$ ,  $U$ ,  $P$ , whereas the factors of these periods common to all series are presented in the series  $P$  with the least error.

(2) The period  $T_{Pr}$  is the minimal main period for Pluto, Neptune and other planets. Hence, whether the

hypothetical planet Proserpine exists, or not, the quantity  $T_{Pr}$  presents the fundamental period for Solar-planetary interaction.

(3) The system of planetary series  $J$ ,  $U$ ,  $P$  describes the Solar-planetary synchronism on a level of harmonic relations with the accuracy which, by order of magnitude, corresponds to the source data error  $\delta^*$ .

(4) In a differential form [15], we come to a *sequentially enclosed planetary series*  $J \subset U \subset P$ : the planetary periods form three series  $J$  (Mercury to Jupiter),  $U$  (Mercury to Uranus) and  $P$  (Mercury to Proserpine) that satisfy the *Rule of multiplicativity and additivity* of harmonics:

— harmonic of a planet in the external series equals to the product of its internal series harmonic and the internal series root planet ( $Ju$ ,  $Ur$ , or  $Pr$ ) harmonic taken from the external series:  $\omega_i^{(n+1)} = \omega_i^{(n)} \cdot \omega_{R_n}^{(n+1)}$ ;

— planet harmonic multiples at the external series comprise those in the internal series and the internal series root planet multiples taken from the external series:

$$c_i^{(n+1)} = c_i^{(n)} \cup \omega_{R_n}^{(n+1)},$$

where  $\omega_i^{(n)}$ ,  $\omega_i^{(n+1)}$  — harmonics of object  $i$  in the internal and external series, respectively;  $n=1, 2, 3$  relates to  $J, U, P$ ;

$\omega_{R_n}^{(n+1)}$  — harmonic of the root  $R_n$  of the series  $n$  in the external series with number  $(n+1)$ ;

$\omega_{R_n}^{(n)}$  equals to 1 and specifies main period of  $n$ -th series;

$c_i^{(n)}$  — set of harmonic multiples of object  $i$  in the series  $n$ .

(5) In an integral form [5], we come to a *Golden section algebraic structure of mean periods*. Let  $\omega(Q)$ ,  $\Gamma(Q)$ , ( $Q \in \{J, U, P\}$ ) denote arithmetic and geometric mean harmonic for series  $Q$  and  $Q_a$  denote the series  $Q$  replenished with the object(s)  $SR$ ,  $SA$ , and/or  $AB$ . Then, with an average error  $\delta_a \approx 2.5 \%$  all arithmetic and geometric mean harmonics of series  $J, U, P$  and their replenishments coincide with the terms of united series  $z$ ; namely:

— replenishing of any series  $Q \in \{J, U, P\}$  with the element  $SR$  increases its mean  $\omega(Q)$  in  $\Phi^2$  times, thus shifting the value  $\omega(Q_{SR})$  by 2 terms in the series  $z$ ;

— in case of  $\Gamma(Q)$ , the replenished mean is increased in  $\Phi$  times and shifted in  $z$  by 1 term.

For all elements and series, the replenishment diagram for the arithmetic means is given in fig. 2.

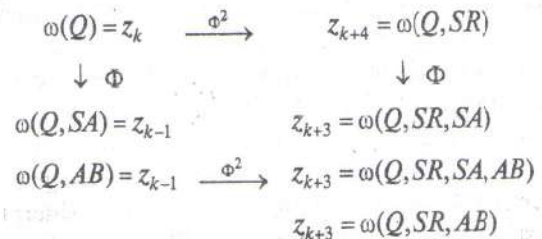


Fig. 2. Algebraic diagram of Golden section structure of mean periods. For series  $J, U, P$  the values  $z_k$  are  $z_{14}, z_{21}, z_{27}$

## 5.2. ATS: correlation of Solar and Terrestrial periods.

If we take the Earthy period  $T_{Ea}$  for the unity of series  $\Gamma$ , we obtain the sequence of periods  $T_i = \Phi^i$  being presented in col. 4 of tab. 1; in parallel with it, consider its basic replenishment — the series  $\Gamma_2 = \Gamma^*$  with terms  $2\Phi^i$  (col. 5), and their correlatives — the Fibonacci series  $v$  and  $u$  (cols. 2 and 3). Then, analysis [15] of  $\Phi$ -correlations of considered periods results in the following conclusions.

1. All the objects, including the comet Halley, or their main harmonics specified by their place in the planetary series (e.g. 6 for Uranus in series  $P$ ) coincide with terms of series  $\Gamma$  or  $\Gamma^*$  with an average error of 0.9%. In this sense the ATS describes the Solar-planetary synchronism with the use of both natural and  $\Phi$ -harmonics.

2. As far as  $\Phi^5 \approx 11.09$  (viz.  $\Phi^5 \approx T_o$  (0.2%)) it becomes clear, that if the value  $T_o$  is accepted for the time unity instead of Earth year, then, within the accepted accuracy, all these periods decrease their absolute values in  $\Phi^5$  times, but remain within the ATS (viz. continue to take the terms of  $\Gamma$  or  $\Gamma^*$ ). This allows us to suggest that *the average length of the Solar cycle  $T_o$  should be considered as the true time unity of the Solar system*. The more so because both of the Solar rotation periods fit these series:  $t_o \approx 2 \cdot \Phi^7$  (0.3%),  $\tau_o \approx \Phi^{12} T_o$  (0.06%).

3.  $\Phi$ -harmonics retain exclusive stability in contrast to natural ones. Thus, within the specified error  $\delta_*$ , the increase in ratio of  $k = T_a / T_b$  results in a situation when harmonic resonance  $T_a = k T_b$  takes place at a number of harmonics:  $T_a = k_i \cdot T_b$ , ( $i = 1, 2, \dots, n$ ); e.g. for the fundamental period  $T_P = 520$  yr and  $T_{SR} = 0.0344$  yr at  $\delta_* = 1\%$  we obtain that  $k = 520 / 0.0344 = 15116$ ,  $k_1 = k - k\delta_* \approx 14965$ ,  $k_2 = k + k\delta_* \approx 15267$ . This means that the period  $T_{SR}$  might be corresponded with  $N = k_2 - k_1 = 303$  harmonics of  $T_P$ . For  $\Phi$ -harmonics such spreading of resonance is impossible; for the above periods we obtain that  $k = 15116$  fits  $\Phi^{20} = 15127$  within the comparison error of  $\delta = 0.07\%$ , whereas  $\Phi^{19} \approx 9349$ ,  $\Phi^{21} \approx 24476$ . This stability allows us to consider the ATS terms as the equally exact unities of time, since the equality  $\Phi^n = \mu_n \Phi + \mu_{n-1}$  allows us to reveal a synchronism between the given period and  $\Phi$ -harmonic of any order  $n$ .

4. Due to its physical influence and Golden section position among the planetary periods in  $\Gamma$ , the basic Solar cycle period  $T_o$  can be considered as occupying the heart of this structure; the more so that it comes to close natural and  $\Phi$ -harmonical synchronism with the main Solar system, Solar rotation and Eta Carinae (see para. 6.2) periods.

5. By taking into account physical actuality of the Hale cycle and allocation of sunspots at the meridian of the Sun on its opposite side, we can conclude that the series  $\Gamma^*$  containing the periods  $t_o$ ,  $2 \cdot T_o$  is as actual as  $\Gamma$ .

Apart from the considered synchronism in Solar-planetary periods, within the accepted accuracy of

$\delta_* = 1\%$  the basic periods of a large amount of phenomena in Nature and society with the periods of up to hundred years also coincide [15] with the terms of series  $\Gamma$  or  $\Gamma^*$ , as well as with the associate Fibonacci series  $u$  and  $v$ . But some new results are worth mentioning as they widen the range of these correlations to seconds and to dozens of millennia.

**ATS as a clock cycle of brain waves.** By paying special attention to use of powers of Golden section and value  $2\Phi$  for describing harmonics of Electroencephalogram (EEG), Weiss H. and Weiss V. have shown that «The metric of brain waves can always be understood as a superposition of  $n$  harmonics times  $2\Phi$  ... From this Liberson had drawn the conclusion that «all significant channels in EEG could be  $n$  multiples of one fundamental frequency of about 3.3 Hz» [5]. However, much more exact approximation  $n \cdot f$  for EEG harmonics can be obtained, if one uses for the fundamental frequency  $f$  the term  $\Phi^{38}$  of  $\Gamma$  instead of an arbitrary value  $2\Phi \approx 3.24$  (Hz). Indeed, as the term  $\Phi^0 = 1$  is taken in  $\Gamma$  for a year, in seconds it makes  $365.24 \times 24 \times 60 \times 60$  and, hence,  $\Phi^{38}$  yr  $\approx 0.36105$  s., which is corresponded to the frequency  $\eta_{38} = 1 / \Phi^{38} \approx 2.77$  (Hz). Show now that this value as a main frequency  $f$  makes significantly better approximation for the basic EEG harmonics than  $2\Phi$ .

Firstly, the linear regression line  $L$  for the considered model [5] of EEG harmonics (without the last outlier EEG harmonic 29, which significantly deviates from a linear model) goes almost exactly through the ATS-engendered values  $f_n^* = \eta_{38} \times n$ , whereas the values of  $f_n = 2\Phi \times n$  diverge noticeably from  $L$  (all values of  $f_n^*$  are closer to  $L$ , than the values of  $f_n$ , except of the last one). This is so because the use of value  $\eta_{38}$  for the fundamental period gives more exact approximation than the value  $2\Phi$  (5% vs. 11% in error), whereas considering of more exact regression (without the outlier 29) gives even more contrasting result — 1% and 16% respectively. Comparing of variances for these approximants supports the same conclusion: F-ratio for variances makes 1.89 and 5.41 (without the last harmonic of 29) for the critical value of 3.8 at 5% confidence level.

Therefore, although the value  $\eta_{38} = 1 / \Phi^{38}$  (Hz) does not «resemble» digits of the Golden section, it approximates the brain frequency distribution with significantly greater accuracy than the suggested value  $2\Phi$  (Hz), comes from  $\Phi$  as a term of series  $\Gamma$ , and sets up direct correlation with the Earth year and Solar cycle. Therefore, the term  $\Phi^{38}$  can be considered as the *true fundamental period of brain waves*.

**ATS as a reference scale for supercycles.** Consider now how the most reliable and quite accurately evaluated periods of greater length being found in Solar and terrestrial cycles correlate with the ATS terms. To this end, the concluded set of well-acknowledged periods [6] provides us with such spectrum of cycles (only six ATS periods of 48 that cover the range of 1 to 2207 years are not met in this spectrum; this may have a clear

explanation: they are large enough to be revealed in the existent short-prehistory samples).

All these values are given in tab. 2 (cols. 1, 3); they are obtained while studying cosmogenic Carbon-14, aurorae, geological laminations, and other millennia-long samples. Besides, some duplicating values are given with asterisk, if they are specially marked in bibliography, as well as periods exceeding the value 2289 yr, which were revealed in analysis of precession of Earth orbit and other effects [3].

Table 2

Most reliable periods in Solar and Terrestrial cycles

Period	ATS term	Period	ATS term
43- 45	47	305-314	322
51- 52	55	415-425, 400*	398
57- 59	58	554-569*, 500*	521
63- 67	123/2	600	610
78.8 ± 3.3	76	955*	987
85- 90, 89.4*	89	1020-1070*, 1100*	1042
104-105 (?)	94, 123	1325*	1364
143-148	144	2289*, 2200*	2207
154-157	152	3600*, 3780*	3571
177-227	199	19 000*	18698
260-280, 256*	246	23 000*	24476
		40 000*	39603

Tab. 2 provides us with systematic and evident confirmation of ATS consistency in the capacity of a discrete scale of Solar cycle periods and their Terrestrial correlatives. In particular, we may conclude the following.

1. An attempt to harmonically correlate the revealed Solar and Terrestrial cycles has evidently failed, even with treating of summation of periods. On the contrary, we see that each actual period quite accurately fits some term of ATS. Though this synchronism with  $\Phi$ -harmonics is inexplicable from the viewpoint of the conventional resonance theory, as a matter of fact we can state that ATS synchronizes the major part of the basic periods in Nature and society.

2. Variations in Solar cycle lengths, planetary modulation and other effects result in fuzziness of Solar cycle harmonics and Terrestrial periods. Nevertheless, we see, especially for the supercycles, that with a high accuracy the revealed actual periods «go» just by the ATS «steps», and the longer the cycle, the higher the accuracy of this synchronism.

3. Close ATS-term/natural-period correlations leave us very little hope to await that all these coincidences are random for such a great interval of values.

**5.3. ATS: the benchmarks of evolutionary time.** Like ATS describes a wide spectrum of fundamental periods in nature and society, the finite part of the same Auric Scale (1), in contrast to conventional consideration of fixed length cycles, describes the «evolutional» time as the sequence of periods of decreasing duration so that

the major events occur within the vicinities of the *separation epochs* between the successive intervals of lesser and lesser length. In this sense, the «historical» time could be considered as «compressed». In order to verify this statement, a synchronism was established between the separation epochs defined by the Mayan Calendar and the most prominent events in Nature (all known geomagnetic inversions, flashes of Supernovae, most destructive earthquakes, etc.) and society (coming of Greatest Reformers, millennia trend in growth of population of China, et al) for the available data covering the last 13 millennia [16]. This way, the ATS might be likened to the Auric Spiral of Time, the turns of which, as fractals, are better described by the Aurically structured trends, whereas at each turn the phenomena are more adequately seen via the harmonic cycles of equal duration, unless this spiral approaches the point of bifurcation.

**5.4. Resume.** *The discrete nature of ATS, the range of Solar and Terrestrial periods it covers, and the accuracy of correlations between its terms and the periods considered in applications and differed by orders allow us to consider this scale as the searched structure that «quantifies» time both over the spectrum of known periods and stretch of «historical» time, at least for a dozen of millennia.*

## 6. Regular Model for Solar cycle maxima distribution

**6.1. Direct and Regular models.** By taking into account that general level of SA is not formally defined, the correlation between the Solar radio flux and the Wolf number is astoundingly good on both monthly and yearly basis, the SSN statistics is provided with the largest prehistory and sunspots precede other Solar effects, the SSN index is considered below as the general index of SA. At the same time, as far as almost nothing is known about what makes the cause of development of Solar cycles, we are not required to consider them as a sequence of totally dependent processes.

In order to find a model describing the integral behaviour of Solar cycles, the accepted cycle maxima epoch since 1605 were considered [17, 18]. The problem was stated as follows: find an approximating model that gives unbiased and consistent point and interval estimates for Solar cycle maxima epochs (peaks, for short) for decades, which is stable and statistically preferable to the known ones. Source data (*L-sample*) for development of such model are presented by  $n=36$  SSN maxima epochs [7] with absolute error not less than 0.1 yr, which cover all telescopic observations over 17 to 20 centuries; *S-sample* covers the Zurich series of observations. At this, the «conventional» and proposed models of SSN peak distribution were considered:

**Direct model.** Let  $\tau > 0$  be a random variable and  $t_{i-1}$  be the last actual peak. Then, the value  $\tau_i$  taken by  $\tau$  defines the *next* actual peak as follows  $t_i = t_{i-1} + \tau_i$ .

**Regular model.** Let a sequence  $\dots, t_j^*, t_{j+1}^*, \dots$  satisfy to

$$t_{j+1}^* = t_j^* + T, T = \text{Const.} \quad (12)$$

Then, the value  $\delta_j$  defines the actual peak with number  $j$

$$t_j = t_j^* - \delta_j. \quad (13)$$

We imply that  $T = T_0$  and use the estimate (8),  $T_0 = 11.07$ , which gives [18] the following epochs for *model peaks*

$$t_k^* = 1605.27 + 11.07 \cdot k, (k = 0, \pm 1, \pm 2, \dots), \quad (14)$$

where  $k=0$  corresponds to the first observed maximum, and  $k=35$  – to maximum of cycle 22; negative values provide extrapolation in prehistory of 1600. With this model, our interest focuses on the distribution of deviations  $\{\delta_j\}_j$ .

**6.2. Basic properties of Regular Model.** The deviations  $\delta_j = t_j^* - t_j$  of (13) show double-sided Relay distribution, one side of which is flatter. Due to F-criterion the variances for L and S sample deviations can be adopted as equal, while t-criterion shows that the difference in 0.2 year between S and L model peaks can be accepted as insignificant; this difference is insignificant from practical point of view as well. But much more important conclusions that are obtained without statistical concepts and in spite of rough source data specify that the Regular model:

(i) allows us to forecast SSN maxima over a time period of up to two centuries (which is incomparably exceeds any existing approach based on a direct-type model) with less error than it does the direct model for just one Solar cycle;

(ii) shows clustering of maxima, which reveals an inexplicable discrete factor in Solar cycles. This means that 67 % (viz. 24 of 36 actual peaks) of observed SSN maxima epochs form the pairs (*clusters*) which present the same deviations from model peaks within insignificant error of 1-3 %; in other words, for the epochs  $t_i, t_j$  of the same cluster the ratio  $(t_i - t_j)/T_0 = e_{ij}$  gives an *integer* (supercycle multiple) with an error of  $10^{-3}$ . This event may be regarded as random, but with a ravishingly small probability of  $10^{-11}$ .

The Regular model gives us a lucky chance to trace through it a *Solar-Stellar correlation*. The extreme members of supernovae might produce “*hypernovae*”, cataclysms hundreds of times more energetic still. At least one possible member of this putative class of hypernova is Eta Carinae [23]. Flashes of supernova are very important for this study, all the more that all of them that have been registered in our Galaxy over the last millennia are synchronous with the considered Auric epochs of the Mayan Calendar [16]. Even more so for the Hypernova. That is why the following correlations [19] are so important.

(i) In terms of cycle duration being inverse to the frequency of the events, the average period  $T_0 = 11.07$  yr of Solar cycle presents the second harmonics of the basic

Eta Carinae radiation event cycle period of  $T_{EC} = 5.5306$  yr with the accuracy of  $\delta_0 = 0.08\%$ .

Further on, by taking the accepted reference point estimation 2003.5, we obtain the Eta Carinae event distribution

$$\tau_i^* = 2003.5 + T_{EC} \cdot i, (i = 0, \pm 1, \pm 2, \dots). \quad (15)$$

Comparing of events (14) and (15) results in the following:

(ii) Within the existing two-centennial observations, the Solar cycles and Eta Carinae events present a synchronous, in the narrow sense, processes where the model peaks  $t_k^*$  of (14) present the most prominent Eta Carinae events with not less accuracy than the peaks  $\tau_i^*$  defined by (15).

*Hypothesis.* A relative stability of the Regular model over the two millennia allows us to assume that Eta Carinae presents a Master-Clock for the Solar Cycles; namely, that: (i) the highly exact coincidence of the average duration  $T_0$  Solar cycles and doubled value  $T_E$  of the basic Eta Carinae’s period  $T_{EC}$ , as well as the *synchronism* between the events they define are *not random*; (ii) the huge emissive power of *Eta Carinae*, in comparison with the Sun, assumes it to be a *master generator for the Sun* for this pair of stars.

**6.3. Resume.** In (12)–(14) we firstly meet a discrete component in a system which is conventionally considered as a continuous one. In other words, *the Solar maxima epochs show a trend to develop at the discrete distances of the model epochs  $t_k^*$ , while the same deviations  $\delta_k$  of actual maxima repeat in an integer number of  $T_0$ .* However, the Regular model by itself has not yet allowed us to reveal the internal structure of Solar cycles. For disclosing it, we are required to take into account more detailed data than smoothed SSN and subjective criteria.

## 7. Solar cycle median

With the aim to increase the resolving capacity of Regular model, we are required to *define a quantity* that can reflect definite physical property of SA with the use of a *sample* presenting the *actual values of some index of SA*. For the considered reasons, this criterion must not rely upon a *single-modal concept* of Solar cycle; the more so that during an 11-year cycle not one, but *two SSN maxima* are frequently developed (M. Gnevishev), relative to latitudinal distribution of sunspots, which are separated by two-tree years.

As to a Solar index, no substitute exists for sunspot number. However, use of monthly mean sunspots in standard *Ri* units [7] (SSN, for short) instead of smoothed values allows us to increase a source data accuracy by an order of values, since the former ones present epochs with an error of half a month (instead of tenth of a year) and do not distort the SA magnitude in contrast to latter ones that may shift a maximum by months. On the contrary, turning down of daily SSN allows us to filter out those variations in SSN which are insignificant for analysis of a cycle.

**7.1. Definition of Solar cycle median.** The average cycle duration  $T_o = 11.07$  presents a stable and unbiased integral characteristic of Solar cycles. For obtaining a function  $F(S)$  that specifies an epoch of cycle energy emitting centre as a consistent characteristic of a cycle, consider a sample  $S = (s_1, s_2, \dots, s_n)$  of monthly mean SSN, with the first and last values associated with the cycle bounds. We demand for this function to satisfy the following requirements.

1.  $F(S)$  is stable in mathematical sense: small variations in bounds of  $S$  affect the epoch  $F(S)$ , but insignificantly.

2.  $F(S)$  reflects a Solar cycle energy distribution centre.

3. The accuracy of  $F(S)$  must be estimated.

For the basic claimants to this characteristic the following concepts may be considered: cycle maximum,  $M_r$  [7]; SSN monthly maximum,  $M_x$ , – the epoch of maximal value in  $S$ ; mode,  $M_o$ , – the epoch of maximum of a least power unimodal smoothing polynomial for sample  $S$ , which is used as an estimation for a cycle «maximum» (6<sup>th</sup> power suffices this goal); median,  $M_e$ , – the median for  $S$ .

Though the first three quantities reflect one or another integral property of a Solar cycle, none of them satisfies all the above requirements. Thus, the shortcomings of  $M_r$  were considered above. The value  $M_x$  presents basically a local feature of Solar cycle and can lie far from any other integral «maximum».  $M_o$  is sometimes used in predictive purpose (e.g. [10]), but its basic disadvantage lies in arbitrariness in specifying the «best» smoothing polynomial. For this reasons, they are considered below, but as the subjects for comparison.

Consider now the concept of median in more detail. Let a sample  $S$  present monthly mean SSN for months  $M_S = (t_1, t_2, \dots, t_n)$ . Since SSN reflects general level of SA, a value  $s_i$  presents a monthly averaged daily Solar emissive power, or a monthly averaged daily intensity of energy radiation. If  $z(t)$  presents a continuous analogue of  $s$ , the total Solar cycle energy emission  $E$  can be approximated on a monthly basis as follows:

$$E = \int_{t_1}^{t_n} z(t) \cdot dt \approx \sum_{i=1}^n s_i \cdot 1 = \sum_{i=1}^n s_i. \quad (16)$$

In continuous case, the epoch  $t^*$  is called *median*, if this point specifies the integral centre of distribution  $z(t)$

$$\int_{t_1}^{t^*} z(t) \cdot dt = \int_{t^*}^{t_2} z(t) \cdot dt = \frac{E}{2}. \quad (17)$$

So, in the considered context, median presents the centre of total Sun's energy radiation over a Solar cycle.

**7.2. Stability of median.** For a discrete analogue  $S$  of  $z(t)$ , estimate the epoch of median  $M_e$  by the middle of the month  $i^*$  where  $t^*$  resides. So, if

$$E_M = \sum_{i=1}^n s_i, \quad \sum_{i=1}^{i^*-1} s_i < \frac{E_M}{2}, \quad \text{and} \quad \sum_{i=1}^{i^*} s_i > \frac{E_M}{2} \quad (18)$$

then  $i^*$  is taken for the median  $M_e$ . It is stable in a sense of selection of time unity. Indeed, let  $\{\zeta_k\}_{k=1,2,\dots,K}$  present  $K$  daily values for the same time interval, and, for simplicity, all months be of equal duration  $m$ . For these values the sum, similar to (18), after grouping by months makes

$$E_D = \sum_{k=1}^K \zeta_k = \sum_{i=1}^n [\text{sum of daily values over month } i] \approx \sum_{i=1}^n [m \text{ times the monthly mean } s_i] = m \cdot \sum_{i=1}^n s_i. \quad (19)$$

Hence,  $E_D$  differs from  $E_M$  by a scale factor  $m$  (average number of days in a month). At this, if the median day  $\zeta^*$  is close to first (last) days of the month, it may get off the month  $i^*$  due to a difference between  $m_i$  and  $m$ , but with an insignificant probability, since the left and right sides of sample  $S$  (with respect to  $i^*$ ) are long enough (up to 5 years) and, in average, compensate for this small error.

As far as the median  $\zeta^*$  can be taken for the true one, we can conclude that, almost for sure, the true median  $t^*$  fits the month  $i^*$  and deviates from its centre by no more than 0.5 (month). Therefore, the error for the median estimate  $i^*$  does not exceed half a month, or

$$\Delta_{Me} \approx 0.5 \frac{1}{12} \approx 0.042 \text{ yr}. \quad (20)$$

Note, that this stability and accuracy cannot be attributed to a median, if *smoothed* monthly mean SSN is used.

Show now that median is stable with respect to selection of bounds of Solar cycle (the epochs of minima). As in existing approaches [7], in obtaining a median  $i^*$  in (18) we also have to define the epochs of minima, but this time the influence of sample bounds is not significant. For concreteness, consider the SSN over the last 50 years. Their values, in a several month vicinities of Solar maxima, were from 100 to 254; say 150 in av. For months of minima, these values were from 0 to 12; say 5 in av. Assume for simplicity that median  $M_e$  hits the middle of month  $i^*$  exactly. Then, new median  $M_e$  corresponds to the accumulated value

$$\sum_{i=1}^{i^*-1} s_i + 150/2 = \sum_{i=1}^{i^*-1} s_i + 75 = \frac{E_M}{2} \text{ (units).}$$

Hence, an error of  $\pm 6$  months in defining of the left minimum (month  $t_1$ ) will change the value of  $E_M$  by  $\pm 30$  units:  $(\pm 6 \text{ months}) \times (5 \text{ units a month}) = \pm 30$  units; in its turn, this shifts the former median to a position of  $E_M/2 \pm 30/2$ . This means that new median is  $\pm 15$  units shifted relative to the former one. But the value  $75 \pm 15$  still remains within the month  $i^*$ , and only an

error in  $x = \pm 30$  months, which can be obtained from the equation  $(\pm x \cdot 5) / 2 = 75$ , may push out the median from the original month. Of course, cycles with such low SSN do not observed normally, and only at random a median may hit the centre of month. But this qualitative proof allows us to exclude mathematics for the sake of clarity. Hence, a *monthly mean SSN median* provides us with a *stable characteristic of Solar cycle* (with respect to variation of cycle bounds, transfer to daily SSN and other Solar indices), which presents the *centroid of Solar cycle Energy emission*; by order of value, its error is  $\Delta_{Me} \approx 0.04$  year. Therefore, as it satisfies the stated requirements, the median  $M_e$  can be taken for the searched consistent characteristic of Solar cycle.

**7.3. Medians for Zurich series.** The medians for the Zurich series, together with cycle modes and monthly maxima are given in tab. 3. They are obtained as described above, with the use of monthly mean SSN [8]. Sample bounds correspond to cycle minima [7]; by allowing for its incompleteness, for the current, 23<sup>rd</sup> cycle, a symmetric time span is taken. For convenience, these values are completed with the official cycle maxima [7].

Table 3

Epochs of basic characteristics of Solar cycles

Zurich No.	Maximum $M_r$	Median $M_e$	Mode $M_o$	Month.max. $M_x$
1	1761.5	1761.29	1761.46	1761.38
2	1769.7	1770.54	1770.13	1769.79
3	1778.4	1779.29	1778.92	1778.38
4	1788.1	1789.38	1788.40	1787.96
5	1805.2	1803.96	1803.96	1804.79
6	1816.4	1817.13	1817.14	1817.21
7	1829.9	1829.38	1829.79	1830.29
8	1837.2	1837.96	1837.33	1836.96
9	1848.1	1849.04	1848.47	1847.79
10	1860.1	1861.04	1860.25	1860.54
11	1870.6	1871.38	1871.20	1870.38
12	1883.9	1883.63	1883.72	1882.29
13	1894.1	1894.38	1893.83	1893.63
14	1907.0	1906.96	1906.75	1907.13
15	1917.6	1918.04	1917.85	1917.63
16	1928.4	1928.13	1927.95	1929.96
17	1937.4	1938.54	1938.01	1938.54
18	1947.5	1948.63	1948.28	1947.38
19	1957.9	1958.54	1958.19	1957.79
20	1968.9	1969.79	1969.09	1969.21
21	1979.9	1980.88	1980.52	1979.71
22	1989.6	1990.71	1990.3	1990.63
23	2000.3	2001.21	2001.16	2000.54

**8. Regular Model for Solar Cycle Medians (RMM)**

**8.1. RMM.** Apply the Regular model concept (12) with the same value  $T_o = 11.07$  yr of average cycle length, but this time – to 23 medians of Zurich series. This gives the following epochs for Zurich cycle ( $i \geq 1$ ) model medians

$$M_j^* = 1760.396522 + T_o \cdot (i - 1), (i = 0, \pm 1, \pm 2, \dots). (21)$$

For pre-Zurich medians (for retaining Zurich numbers), the conventional maxima [7] are taken for the purpose of comparison; in (21) they are corresponded by model medians with numbers  $i = -12, \dots, 0$ . The basic values associated with this model are presented in tab. 4. The calculated (18) medians  $M_e$  are given in col. 2, the model (21) ones – in col. 3, and their difference – in col. 4.

Table 4

Calculated vs. Model Medians

Cycle N. $i$	Median $M_i$	Model Median $M_i^*$	Deviation $d_i = M_i - M_i^*$
1	2	3	4
-12	1615.5	1616.487	- 0.987
-11	1626.0	1627.557	- 1.557
-10	1639.5	1638.627	0.873
-9	1649.0	1649.697	- 0.697
-8	1660.0	1660.767	- 0.767
-7	1675.0	1671.837	3.163
-6	1685.0	1682.907	2.093
-5	1693.0	1693.977	- 0.977
-4	1705.5	1705.047	0.453
-3	1718.2	1716.117	2.083
-2	1727.5	1727.187	0.313
-1	1738.7	1738.257	0.443
0	1750.3	1749.327	0.973
1	1761.29	1760.397	0.893
2	1770.54	1771.467	- 0.927
3	1779.29	1782.537	- 3.247
4	1789.38	1793.607	- 4.227
5	1803.96	1804.677	- 0.717
6	1817.13	1815.747	1.383
7	1829.38	1826.817	2.563
8	1837.96	1837.887	0.073
9	1849.04	1848.957	0.083
10	1861.04	1860.027	1.013
11	1871.38	1871.097	0.283
12	1883.63	1882.167	1.463
13	1894.38	1893.237	1.143
14	1906.96	1904.307	2.653
15	1918.04	1915.377	2.663
16	1928.13	1926.447	1.683
17	1938.54	1937.517	1.023
18	1948.63	1948.587	0.043
19	1958.54	1959.657	- 1.117
20	1969.79	1970.727	- 0.937
21	1980.88	1981.797	- 0.917
22	1990.71	1992.867	- 2.157
23	2001.21	2003.937	- 2.727

**8.2. Statistical analysis of Regular Model.** Analysis of tab. 3 acknowledges close time correlation for the considered characteristics. Thus, all correlation coefficients exceed 0.999. With respect to squared and Chebyshev's deviation, the closest of them are cycle medians and modes with rms 0.446 yr and maximal deviation 0.98 yr. Second in turn is the concordance between the cycle maxima and modes, with rms 0.525 yr and maximal deviation 1.24 yr.

Show now that RMM is stable to choice of samples. By applying the model (12) with  $T_0 = 11.07$  yr to other characteristics of tab. 3, we obtain the model epochs that are given in tab. 5. Their accuracy is estimated by rms for the actual and model epochs. In parallel, these epochs were calculated for the sample means instead of  $T_0$  (the respective mean and rms values are given in brackets).

It is clear from tab. 5, that use of «own» means instead of  $T_0$  coarsens the results. At the same time, we observe sufficient stability of the model epoch of 1760 for  $T_0 = 11.07$  yr for all characteristics, as well as for the reference epoch 1760.25 of model (14) that allows for estimate maximum of 1604. At this, the RMM, even though the conventional maxima substitute the pre-Zurich values in E-series of medians, shows the least variance, which probably can be attributed to exclusion of subjective factors. For these reasons, and with taking into account that growth of amount of sample increases the accuracy, we may choose the average of these most significant dates for a common reference epoch, at least – for cycle maxima and medians

$$M_{Av} = \frac{1}{3}(1760.397 + 1760.21 + 1760.547) = 1760.385. \quad (22)$$

Regular model epochs for Z- and E-series

Table 5

RM for:	Zurich series (n=23)		Extended series (N = 36)	
	Model epoch	rms	Model epoch	rms
1	2	3	4	5
Maximum, $M_r$	1759.87	2.26 (10.86, 2.6)	1760.21	2.02 (10.99, 2.0)
Median, $M_e$	1760.40	1.87 (10.92, 2.2)	1760.55	1.72 (11.02, 1.7)
Mode, $M_o$	1760.09	2.03 (10.90, 2.3)	–	–
Monthly max. $M_x$	1759.97	2.34 (10.87, 2.7)	–	–

**8.3. Error-bound analysis of RMM.** For the RMM of (21), correlate Z-series median deviations  $d_i = M_i - M_i^*$  (tab. 4, col. 4) with respective terms of series  $\Gamma, \Gamma^*$  as shown in tab. 6, and for E-series – in tab. 7, where error  $\delta_i = |\gamma_k - |d_i||$  gives absolute deflection of  $d_i$  from respective ATS term  $\gamma_k$ . All values in these tables, except of col. 3, are given in years.

Tab. 6, tab.7 present the distribution of Z- and E-series median absolute deviations  $\{|d_i|\}_i$  over the terms  $\gamma_k \in S = \{\varphi^6, \dots, \Phi^3\}$  of the series  $\Gamma, \Gamma^*$  (median distribution, for short); if the terms of series  $\Gamma^*$  are not included, denote this series  $S^*$ . The largest value of series  $S$  corresponds to the greatest deviation, whereas

Z-median distribution over the series  $\Gamma, \Gamma^*$

Term $\gamma_k$ of ATS	Deviation $d_i$		Error $\delta_i$	
	Sign	Value		$i$
1	2	3	4	5
$\varphi^6$	0.056	18	0.043	0.013
		8	0.073	0.017
$\varphi^5$	0.090	9	0.083	0.007
$2 \cdot \varphi^4$	0.292	11	0.283	0.009
$2 \cdot \varphi^2$	0.764	5	-0.717	0.047
$\varphi^0 = \Phi^0$	1.000	1	0.893	0.107
		21	-0.917	0.083
		2	-0.927	0.073
		20	-0.937	0.063
		10	1.013	0.013
		17	1.023	0.023
		19	-1.117	0.117
$2 \cdot \varphi^1$	1.236	13	1.143	0.093
		6	1.383	0.147
$\Phi^1$	1.618	12	1.463	0.155
		16	1.683	0.065
$2\varphi^0 = 2\Phi^0$	2.000	22	-2.157	0.157
$\Phi^2$	2.618	7	2.563	0.055
		14	2.653	0.035
		15	2.663	0.045
		23	-2.727	0.109
$2 \cdot \Phi^1$	3.236	3	-3.247	0.011
$\Phi^3$	4.236	4	-4.227	0.009

the smallest one – to the source data error (20) since in this direction the terms of ATS series converge to zero.

It is clearly seen from these tables that modulus of each deviation fits some term of series  $S$ , whereas absence of deviations for the remaining terms of these series can be attributed to a small amount of observations. Indeed, we see that increase in number of observations (tab. 7 vs. tab. 6) leads to eliminating of these gaps. Evaluate now the closeness of this correspondence.

Firstly, estimate an error of discrepancy between the deviations and  $S$ -series terms. Thus, the average errors for col.5 of both tables make 0.063 and 0.058, resp., or  $\delta_A = 0.06$  yr for Z- and E-series. However, for the series  $S^*$  these errors are 0.11 and 0.13, or  $\delta_A^* = 0.12$  yr for both ones. This twice as much reduction in error caused by allowing for both series,  $\Gamma$  and  $\Gamma^*$ , supports the suggestion that the latter series is as actual as  $\Gamma$ .

As well, in continuation of the above analysis of closeness of various Solar cycle characteristics we obtain that the similar average errors for cycle mode ( $M_o$ ), monthly ( $M_x$ ) and official ( $M_r$ ) maxima deviations from the Regular model medians for the same  $S$  series are small enough and show almost the same values (tab. 8).

Therefore, as far as the average error  $\delta_A$  of discrepancy between the deviations of medians (against

Table 7

E-median distribution over the series  $\Gamma, \Gamma^*$

Term $\gamma_k$ of ATS		Deviation $d_i$		Error
Sign	Value	$i$	Value	$\delta_i$
1	2	3	4	5
$\varphi^6$	0.056	18	0.043	0.013
		8	0.073	0.017
$\varphi^5$	0.090	9	0.083	0.007
$2 \cdot \varphi^4$	0.292	11	0.283	0.009
		-2	0.313	0.021
$2 \cdot \varphi^3$	0.472	-1	0.443	0.029
		-4	0.453	0.019
$2 \cdot \varphi^2$	0.764	-9	-0.697	0.067
		5	-0.717	0.047
		-8	-0.767	0.003
		-10	0.873	0.109
$\varphi^0 = \Phi^0$	1.000	1	0.893	0.107
		21	-0.917	0.083
		2	-0.927	0.073
		20	-0.937	0.063
		0	0.973	0.027
		-5	-0.977	0.023
		-12	-0.987	0.013
		10	1.013	0.013
		17	1.023	0.023
$2 \cdot \varphi^1$	1.236	19	-1.117	0.119
		13	1.143	0.093
		6	1.383	0.147
$\Phi^1$	1.618	12	1.463	0.155
		-11	-1.557	0.061
		16	1.683	0.065
$2 \cdot \Phi^0$	2.000	-3	2.083	0.083
		-6	2.093	0.093
		22	-2.157	0.157
$\Phi^2$	2.618	7	2.563	0.055
		14	2.653	0.035
		15	2.663	0.045
		23	-2.727	0.109
$2 \cdot \Phi^1$	3.236	-7	3.163	0.073
		3	-3.247	0.011
$\Phi^3$	4.236	4	-4.227	0.009

Table 8

Error for  $M_o, M_x, M_r$  against the RMM (21)

$M_o$	$M_x$	$M_r$
0.09 yr	0.12 yr	0.11 yr

the Regular model values) and respective terms  $S$  of series  $\Gamma, \Gamma^*$  corresponds to the source data error (20), and this error is stable with respect to increase in amount of observations, we can conclude that this distribution of Solar cycle median deviations over the ATS series reflects some objective, or functional dependence. With taking into account tab. 8 we may resume that cycle modes, monthly and official cycle maxima also follow this dependence.

**8.4. Probability of randomness of median distribution.**

For each term  $\gamma_k \in S$  consider a segment  $I_k$  that contains both this value and those deviations which are referred to it. Let  $K$  be the number of such segments and  $N$  – the number of observations (23 for  $Z$ -series, and 36 – for  $E$ -series).

Assume that the deviations are uniformly distributed over the time interval  $R = \Phi^3 \approx 4.2$  (yr) specified by series  $S$ , and  $i_k$  gives the length of interval  $I_k$ . Then, a single median fits one of intervals  $I_k$  by chance with a probability of  $P_1 = (\sum_{k=1}^K i_k) / R$ , whereas all  $N$  observations fit these intervals accidentally with a probability of  $P_N = (P_1)^N$ .

For series  $S$  consider three types of intervals which reflect average, actual and maximal dispersion of deviations around the terms  $\gamma_k$ . Define the former one as  $\Delta_A = 2 \cdot \delta_A$ . The actual interval  $\Delta_D$  presents the maximal difference between the lowest and largest deviations, including  $\gamma_k$  itself. The maximal interval  $\Delta_M$  makes twice the maximal difference between the term  $\gamma_k$  and deviations being referred to this term. The similar intervals for the series  $S^*$  are marked by asterisk.

For the considered samples (tab. 6, 7), the probabilities of randomness of median distribution are given in tab. 9. The last line contains the geometrical mean probability for the above values (arithmetical mean is not indicative as it retains the maximum of these values).

Table 9

Probability of randomness of median distribution

Interval	Z-series	E-series
$\Delta_A$	$8 \times 10^{-12}$	$5 \times 10^{-18}$
$\Delta_A^*$	$4 \times 10^{-9}$	$2 \times 10^{-9}$
$\Delta_D$	$6 \times 10^{-14}$	$5 \times 10^{-19}$
$\Delta_D^*$	$5 \times 10^{-9}$	$4 \times 10^{-9}$
$\Delta_M$	$2 \times 10^{-10}$	$3 \times 10^{-13}$
$\Delta_M^*$	$2 \times 10^{-6}$	$2 \times 10^{-5}$
for $S$	$4 \times 10^{-12}$	$9 \times 10^{-17}$
for $S^*$	$3 \times 10^{-8}$	$5 \times 10^{-8}$

Here, as well, we see that transfer from series  $S$  to  $S^*$  decreases the probability in 4 orders for  $Z$ -series and by 9 orders for sample  $E$ -series, which gives an additional argument in support of use of both basic Auric series.

**9. Conclusion**

As it was established earlier, the Golden section power series  $\Gamma, \Gamma^*$  with the time unity equal to the average Solar cycle length  $T_o$  form the Time Scale whose discrete values define both the basic structure of Solar system and Terrestrial periods, and the crucial epochs in historical time. New evidences are given in this work that testify to the trustworthiness of this thesis, which include

a wide spectrum of phenomena with periods from 30 yr to several dozens of millennia, and periods of brain waves, as well as more support for the chosen value of  $T_0$ .

Absolutely new result of this study is the revealed Golden section structure of Solar cycles in the sense that medians deviate from the regularly distributed epochs by the discrete values being equal to the terms of series  $\Gamma$ ,  $\Gamma^*$ . Moreover, even cycle modes, monthly and official maxima also follow this distribution. This result acquires greater importance since the median presents a stable characteristic of Solar cycle energy emission centre. The presented conclusions are based on error and probabilistic analyses which show that the accuracy of the obtained correlations corresponds to the source data precision, whereas the probability of randomness of median distribution is negligibly small and makes  $10^{-12} \div 10^{-17}$ .

Thus, in contrast to employing of existing approaches to detection of Solar cycle harmonics and critical epochs, the use of ATS terms and medians allows us to reveal and qualify the respective Solar cycle features with a significantly greater accuracy and, therefore, confidence.

However, although we can say now that  $\delta_j$  in (13) takes the values of terms of series  $\Gamma$ ,  $\Gamma^*$ , it still remains unclear what underlies the choice of this or that term.

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