MODELLING OF TRANSIENT PLASMONS DYNAMICS IN METALLIC CYLINDERS

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Abstract – **Transient pulsed beam excitation of the surface plasmon polaritons in metallic cylinder and hypothetical cylinder having both negative real parts permittivity and permeability is visualized and analyzed. The complex source point concept is used to simulate an incident transient beam. Rigorous mathematical method based on the Laplace transformation has been applied. Time domain field representation is obtained through the evaluation of the residues at singular points associated with the eigenvalues of the structure and integrals along the branch-cuts of a complex plane.**

I. INTRODUCTION

Recently, surface plasmon polaritons (SPPs) have attracted great amount of attention due to their potential use for the subwavelength field enhancement and localisation that are explored in single molecule detection, transmission through a subwavelength aperture, subwavelength imaging, and improvement in the performance of conventional photonics components such as modulators and switches and others [1-4].

It is known that SPPs can exist on a metal wire that can be considered as a plasma cylinder in the optical region. Cylinders with ordinary nonmagnetic metal is characterised by a negative permittivity $\varepsilon < 0$ and support only TE polarized SPPs. In a hypothetical left-handed material (LHM) cylinders, considered here as well, the permeability is also negative ($\varepsilon < 0$ and $\mu < 0$) and SPPs of the both polarisations can be excited.

In this paper we consider the excitation of SPPs on a cylindrical surface by a transient pulsed beam. The external beam is modelled by a pulsed complex source point [5-7] with complex coordinates. To find the excited fields we use a rigorous mathematical tool that allows analysing problems both in the frequency and in the time domains. By applying the Laplace transformation directly to a wave equation we derive an analytical solution in the frequency domain; the time dynamics of the electromagnetic field is recovered by the inverse Laplace transformation. In this way we evaluate the residues at singular points associated with the eigenvalues of the structure and the integral along the branch-cuts in the complex plane. This approach guarantees the calculation with controllable accuracy and allows us to extract and to interpret physical phenomena. This method was introduced by C. Baum (singularity expansion method) in the 1970-s, has been successfully used in variety of ultra-wide-band antenna and target identification problems [8-9],and has been successfully applied by authors to a variety of 1-3D time domain problems with nondispersive media [10-13]. In this paper this method is expanded to dispersive materials: (i) plasma and (ii) LHM.

II. MATHEMATICAL BACKGROUND

We consider an infinite cylinder of radius a with metallic plasma or LHM inside, surrounded by vacuum. The cylinder is excited by an external transient beam directed along the tangent line to the boundary. The pulse duration is τ and the central carrier frequency is ω_0, and TE- polarized field is considered. The Laplace transform of the *z*-component of the magnetic field satisfies the equation

$$\Delta H(p) - n^2 p^2 / c^2 \, H(p) = 0 \tag{1}$$

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where c the speed of light in vacuum is, p is the Laplace transform variable. This equation is the analogue of the Helmholtz equation in the Laplace transform domain if $p = i\omega$. We take the simplest form for the refractive index for the metallic plasma

$$n^2 = 1 + \omega_{pe}^2 \Big/ \left(p^2 + p\gamma_{pe}\right) \tag{2}$$

and for the LHM

$$n^2 = \left[1 + \omega_{pe}^2 \Big/ \left(p^2 + p\gamma_{pe}\right)\right]\left[1 + \omega_{pm}^2 \Big/ \left(p^2 + p\gamma_{pm}\right)\right] \tag{3}$$

where ω_{pe}, ω_{pm} and γ_{pe}, γ_{pm} denote the corresponding plasma and damping frequencies respectively. The solution is constructed in the form of the modified Bessel functions series with unknown coefficients which are to be found then from boundary conditions. The resulting field in the time domain is obtained through the inverse Laplace transformation.

III. RESULTS AND DISCUSSION

Figure 1 (left panel) represents the scattering cross section of the plasma cylinder for the size parameter $\omega_{pe}a/c$ =5, $\gamma_{pe}/\omega_{pe} = 10^{-3}$. Multipolar plasmons are seen in the spectrum where SPP_m represents a surface plasmon polariton with m angular field variations. The normalized values are used: the normalized frequency $ka = \omega a/c$ and the normalized time $T = tc/a$ where t is real time. Figure 1 (right panel) shows the transient dynamics of the plasmons exited by the pulsed beam. The normalized pulse duration is $2\pi a/c$, and the central frequency of the beam coincides with the eigenfrequency of the SPP_6, however interference of different SPPs resonances is also seen in form of beating in time domain in Figure 1 (right panel).

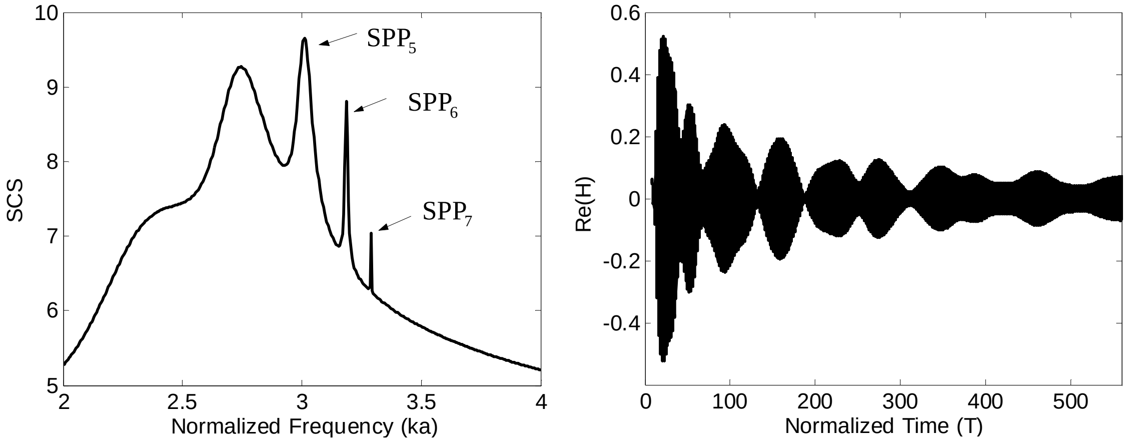

Fig. 1: The Scattering Cross Section (SCS) of the plasma cylinder (left panel), the real part of the magnetic field versus the normalized time (right panel), the size parameter is $\omega_p a/c$ =5, $\gamma/\omega_p = 10^{-3}$.

Figure 2 shows the snapshots of the transient SPP travelling around the plasma cylinder excited by the external transient beam with the normalized central frequency $\omega_0 a/c$ =3.5. The first image presents the field pattern at the moment when the source is just turned off. At the beginning of the transient process we see significant leakage that results in decreasing of the amplitude (see color bar). This pulse generates SPPs propagating in the anticlockwise direction (the white arrow indicates the direction of the pulsed beam incidence). Figure 3 represents snapshots of the absolute value of the magnetic field for the same problem. We see that beating of the simultaneously excited SPPs gives rise to asymmetric running field pattern.

Further consideration concerns the LHM cylinder with the refractive index given by (3). The values of the parameters are: $\omega_{pe}a/c = \omega_{pm}a/c$ =5; $\gamma_{pe}/\omega_{pe} = \gamma_{pm}/\omega_{pm} = 10^{-3}$. Figure 4 (left panel) represents spectrum of the step-like pulse (the Heaviside unit function time dependence). Both SPPs and left-handed whispering gallery modes are seen, SPPs are localized near the value ka =3.5 ($\mathrm{Re}(ka) = 3.46$, $\mathrm{Re}(ka) = 3.49$, $\mathrm{Re}(ka) = 3.51$ correspond to SPP_6 , SPP_7 , and SPP_8, respectively); the left handed modes belong to the interval $0 < ka < 2.7$. In contrast to the 'right-handed' cylinders, where the whispering gallery modes are confined due to the nearly total internal reflection, the modes of LHM cylinder are to some extend exotic ones and have no commonly accepted explanation [14].

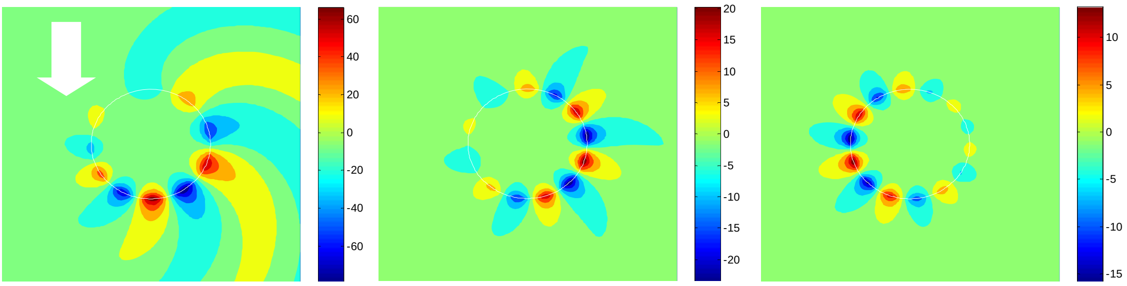

Fig. 2: Snapshots of the SPPs dynamics (real part of the magnetic field) propagation along the plasma cylindrical boundary. From the left to the right $T = 6\pi$, $T = 50\pi$, $T = 100\pi$. The white arrow indicates the direction of the incidence of pulsed beam.

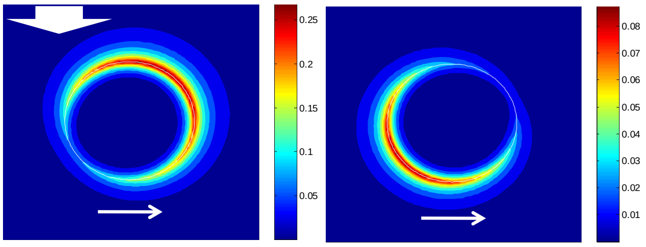

Fig. 3: Snapshots of the dynamic SPP (absolute value of the magnetic field) propagation along the plasma cylindrical boundary. From the left to the right: $T = 150\pi$, $T = 550\pi$.

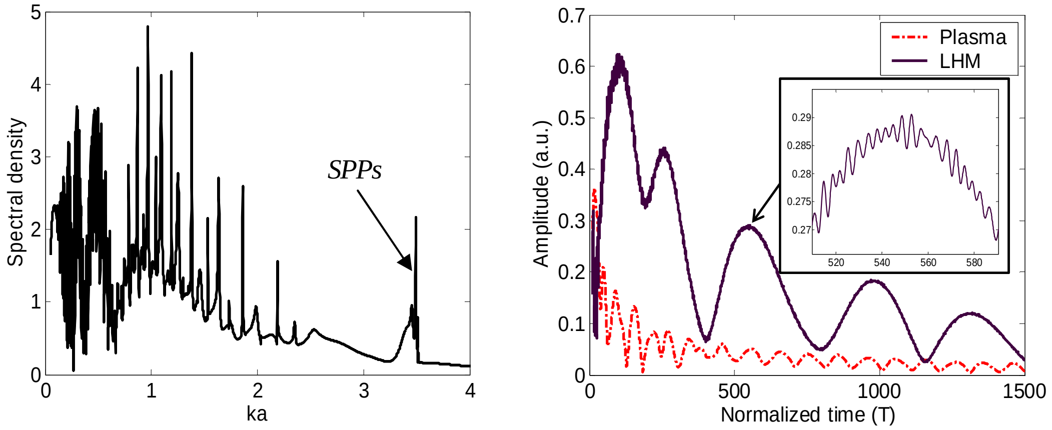

Fig. 4: The internal spectral density in LHM cylinder (left panel); the amplitude at inner point near the boundary versus the normalized time in LHM and in plasma cylinder (right panel).

Figure 4 (right panel) shows the intensity of the surface running waves excited by the beam directed along the tangent on the LHM boundary. The central pulse frequency ($\omega a/c$ =3.5) overlaps a few SPPs for the LHM. For these values the dielectric permittivity and the magnetic permeability of the LHM cylinder are both close to minus one. The phase velocities are directed opposite inside and outside the cylinder. Nevertheless, the incident beam generates the pulse running in the same direction as in the case of just plasma cylinder, considered above.

The time domain behaviour of module of the field would be very similar to that given in Fig. 3 for the plasma cylinder.

The time domain behaviour of the excited SPPs on the plasma boundary for the same incident beam is presented in Fig. 4 (right panel). Dramatic field enhancement is seen for the LHM cylinder. The distance between the nearest peaks indicates the asymmetric pulse round-trip time. Slowing the speed of the pulse on the LHM boundary in 3-4 times is detected. The inset in Fig. 4 (right panel) zooms the absolute value of the excited SPPs on the LHM boundary. 'Ringing' is visible due to the excitation of the left handed whispering gallery modes of LHM cylinder in addition to the surface waves.

VI. CONCLUSION

The problem of the transient excitation of SPPs on the surface of plasma or LHM cylinders is investigated by the rigorous and efficient mathematical method. This method allows obtaining analytical solutions in the Laplace transform domain. The time domain dynamics of the excited field is described by means of residues evaluation at singular points that correspond to the excited modes. An external beam excitation is modelled by the complex source point with the complex coordinates that generate running waves.

The transient dynamics of SPPs has been visualised and discussed. It was shown that the incident radiation is coupled to the several SPPs simultaneously, which interfere and produce an asymmetric propagating wave. Slowing of the group velocity on LHM boundary is shown as well.

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