A GENERALISED MATHEMATICAL MODEL OF ELECTRICITY CONSUMPTION FOR ELECTRIC DRIVES IN SMART CITIES APPLICATIONS

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Annotation**:** The generalised mathematical model of electricity consumption is developed by using electromechanical analogies and Lagrange's equations of 2-nd kind in the view of the system of two first order and one two order ordinary differential equations with the related initial conditions. It allows representing the dependency between supplied voltage and the electricity consumption of the electric drive. This mathematical model is for following researches directed to formulate and to solve the problem of the optimal control theory, so that it allows developing the energy efficient automated controls for electric drives required in smart cities applications.

Key words: electricity consumption, electric drive, mathematical modelling, smart sites.

RELEVANCE. Automated electric drives are principal structural elements in a lot of technical systems involved in households and industrial applications at present. The green transition ambitions of the EU make more significant importance of electric drives, because they have zero own carbon emission, and they can provide carbon neutrality together with the green power generating technologies like solar, wind, hydro and nuclear. Although automated electric drives are known and widely used a lot of times, they are traditionally designed without energy efficiency demands and they provide only the stability of exploitational modes. At the same time, energy efficiency demands require more careful mathematical modelling of the processes in the automated electric drive to represent dependencies between automated controls and energy consumptions, as it is discussed for an example in the published explorations [1,2]. So, developments of the improved mathematical modelling of the processes inherent for the automated electric drives are the relevant problem due to relationships with energy efficiency demands for infrastructures of smart cities. The purpose of this research is in development of a generalized mathematical model of the electricity consumption in electric drives providing the required detailing to represent dependencies between automated controls and energy consumptions.

RESULTS. An electric drive provides required operations due to electric voltages supplied on the electric motor, so that the corresponding electric power is consumed to have the required angular velocity of the output shaft of the electric drive with corresponding exploitation loads. If an electric drive is considered as an automation object, then the supplied voltage is chosen to provide the required angular velocity of the output shaft of the electric drive during an operation, so that to represent the dependence between controls and electricity consumptions it is necessary to represent the dependence between the supplied voltage and angular velocity of the output shaft of the electric drive with correspondent exploitation loads. Taking into account all these circumstances, to represent the dependence between controls and electricity consumptions it is required to consider automated electric drive as the electromechanical system, and the electromechanical analogies with Lagrange's equations of 2-nd kind are used to do it.

The electric drive with the direct current electric motor is considered [1, 2], so that the electric charge q_e in the rotor winding and the angle φ of rotation of the output shaft are considered as the generalised coordinates representing the electric drive as the electromechanical system. It is assumed, that the rotation angle of the rotor of electric motor is defined by the rotation angle of the output shaft are related

$$\varphi_e = \varphi_e\left(\varphi\right), \tag{1}$$

where φ_e is the rotation angle of the rotor of the electric motor.

The electromechanical analogies and the Lagrange's equations of the 2-nd kind lead to the systems of two differential equation of second order, but it is possible to exclude the electric charge q_e from these equations, so that after correspondent equal transformations these equations will have the following view with the related initial conditions:

$$L_e \frac{dI}{dt} = -R_e I - B_e \frac{d\varphi_e}{d\varphi} \frac{d\varphi}{dt} + U_e\left(t\right), \qquad (2)$$

$$\left(J_e \left(\frac{d\varphi_e}{d\varphi}\right)^2 + J_m\left(\varphi\right)\right) \frac{d^2\varphi}{dt^2} + \left(2J_e \frac{d\varphi_e}{d\varphi} \frac{d\varphi_e}{dt^2} + \frac{dJ_m}{d\varphi}\right) \frac{d\varphi}{dt} = B_e \left(\frac{d\varphi_e}{d\varphi}\right)^2 \frac{d\varphi}{dt} - M\left(t, \varphi, \frac{d\varphi}{dt}\right), \qquad (3)$$

$$I\left(t_0\right) = I_0, \quad \varphi\left(t_0\right) = \varphi_0, \quad \frac{d\varphi}{dt}\left(t_0\right) = \omega_0, \qquad (4)$$

where t is the time; $I = dq_e/dt$ is the electric current in the rotor winding; L_e, R_e, B_e and J_e are the inductance, the resistance of the rotor winding, the electrotechnical characteristic of the electric motor and the moment of inertia of their rotor; $U_e\left(t\right)$ is the voltage supplied to the electric motor; $J_m\left(\varphi\right)$ is the moment of inertia of the mechanical parts of the electric drive and loads; $M\left(t, \varphi, d\varphi/dt\right)$ is the generalised mechanical couple representing the loads relatively the rotation axis of the output shaft of the electric drive; t_0 is the given initial time, and I_0, φ_0, ω_0 are the given values of the electric current, rotation angle, rotation velocity at the initial time $t = t_0$.

To estimate the consumed electric energy, the mathematical model (2)–(4) of the electric drive must be complemented by the following differential equation and the related initial condition:

$$\frac{dE}{dt} = I\left(t\right)U_e\left(t\right), \quad E\left(t_0\right) = E_0, \qquad (5)$$

where $E = E\left(t\right)$ is the consumed electric energy before the time t and E_0 is the consumed electric energy before the initial time $t = t_0$.

The mathematical model (2)–(5) gives the representation of dependency between the voltage $U_e\left(t\right)$ supplied to electric motor and electricity consumption $E\left(t\right)$ of the electric drive. The energy efficient automated control for the electric drive must be due to the choice of the voltage $U_e\left(t\right)$ supplied to electric motor, and to have such energy efficient controls, it is necessary to formulate and to solve the related problem of the optimal control theory on the basis of the developed mathematical model (2)–(5).

CONCLUSIONS. The generalised mathematical model of electricity consumption in electric drives is developed in the view of the system of ordinary differential equations with the related initial conditions. It is shown, that estimations of the electricity consumptions require considering of the electric drive as the electromechanical system, so that at least two generalised coordinates are necessary. The view of the differential equations of the developed mathematical model is significantly defined by the relation between the rotation angles of the rotor of electric motor and the output shaft of the electric drive. Continuation of this research is planned to consider the particular examples of electric drives with different relations between the rotation angles of the rotor of electric motor and the output shaft.

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