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MODELING OF PROCESSES OF SPONTANEOUS IGNITION USING ROTHE’S AND TWO-SIDED APPROXIMATION METHODS

The **aim** is to use the Rothe’s method in combination with the two-sided approximation method for the spontaneous ignition processes modeling. Self-ignition is inherent in bulk materials (grain, coal, etc.), since the heat accumulation occurs due to the oxidation reaction. So, it is possible to assume that there is an internal heat source in the embankment, therefore the mathematical model of self-ignition processes can be presented as an initial boundary value problem for a semilinear heat conduction equation, for solving which the above methods can be used.

Results. The spontaneous combustion process can be considered as heat transfer in a body with an internal heat source, but it is necessary to characterize this source by including its power as a term $f(T)$ in the heat conduction equation:

$$\frac{\partial T}{\partial t} = A\left(\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2}\right) + f(T).$$

Since the heat appears in the embankment as an oxidation reaction result and accelerates this reaction, we can use an approximation based on the Arrhenius equation, according to which $f(T)$ is proportional to $e^{-\frac{E_a}{R(T+273)}}$, or e^{θ}, if we proceed to consider the temperature change function $\theta(x_1, x_2; t)$ and use the exponential decomposition proposed by Frank-Kamenetskii.

To solve the initial-boundary value problem for the specified equation, we select a modeling time interval at which we introduce a grid with a fixed step. We search for a solution according to the Rothe's method along the nodes of this grid. Using finite differences approximation of the partial derivative of T with respect to time, we obtain a sequence of boundary value problems that can be solved sequentially. Using the Green's function, each problem is reduced to the Hammerstein integral equation, for which, according to the method of two-sided approximations, we construct a conical invariant segment, the ends of which become the initial values for the iterative process of two-sided approximations.

The approximation to the exact solution at the corresponding time layer at each iteration is half of the sum of upper and lower approximations, while it is possible to estimate the error as maximum of their difference, so the iterative process should be continued until this value is less than 2ε (ε is the predetermined required accuracy).

Conclusions. The result of applying the Rothe's method in combination with the two-sided approximation method to the self-ignition modeling processes is a set of functions for the nodes of introduced grid at the modeling time interval, which depend only on spatial coordinates. Using them, it is possible to determine whether, during the simulation time, enough heat will accumulate in the embankment under study to cause an ignition, and to take measures to prevent it.