UDC 519.2:004.9

L. Kirichenko

THE METHOD OF DISTINCTION MONOFRACTAL AND MULTIFRACTAL PROCESS FROM TIME SERIES

Abstract. Based on the numerical analysis of the sample multifractal characteristics obtained by method of multifractal detrended fluctuation analysis the statistical criterion for accepting the hypothesis of monofractal properties of the time series is proposed. Results of investigations to identify mono- and multifractal properties of the time series of different nature are given.

Keywords: monofractal and multifractal time series, multifractal detrended fluctuation analysis, generalized Hurst exponent.

Problem statement

It is now recognized that many information, biological, physical and technological processes have a complex fractal structure. Fractal analysis is used for modeling, analysis and control of complex systems in various fields of science and technology. Fractal analysis is applied to predict seismic activity and tsunami and to determine the age of geological rocks in geology; to diagnose diseases and physiological states of the records of ECG and EEG in medicine, to study the mutations and changes at the genetic level in biology; to predict the crisis and risk using financial series in economy; to study the turbulence and thermodynamic processes in physics. This list is not exhaustive. [1,2].

Processes that exhibit fractal properties can be divided into two groups: self-similar (monofractal) and multifractal. Monofractal processes are homogeneous in the sense that they have single scaling exponent. Their scaling characteristics remain constant on any range of scales. Multifractal processes can be expanded into segments with the different local scaling properties. They are characterized by the spectrum of scaling exponents. [2,3].

All of the above led to the emergence of the models of fractal stochastic processes. Note the lack of universal models that could be used to describe the fractal processes of various nature. And vice versa one and the same process can be described in several models depending on research objective. In the general case the choice of model is based on the characteristics of the studied time series.

Determination if the time series has monofractal or multifractal property is one of the important steps of selecting and constructing mathematical models. In the case of monofractality the mathematical models of processes can be fractional Brownian motion, fractal point processes, fractal process ARIMA and others. For modeling multifractal processes can use fractal stable Levy motion, stochastic multiplicative cascades, fractal motion in multifractal time, etc [3].

In practice the distinction between monofractal and multifractal properties of the stochastic process is a difficult task. This is due to errors of estimation of fractal characteristics of real time series [4-7]. Currently, there is no universally accepted criterion of distinguishing mono- and multifractal processes by their sample fractal characteristics. The purpose of work is to present the method to distinguish between mono- and multifractal processes from time series, and the study on this basis fractal properties of time series of different nature.

Estimation of multifractal characteristics of from time series

Stochastic process with continuous time $X(t),\ t \geq 0$ is called self-similar with parameter $H,\ 0 < H < 1$ if for any value $a > 0$ finite-dimensional distributions of $X(at)$ identical to finite-dimensional distributions of $a^H X(t)$ i.e. $\mathrm{Low}\{X(at)\} = \mathrm{Low}\left\{a^H X(t)\right\}$. Parameter H called parameter Hurst, is a measure of self-similarity of a stochastic process. Moments of self-similar stochastic process can be expressed as $\mathrm{M}\left[|X(t)|^q\right] = C(q) \cdot t^{qH}$, where the value $C(q) = \mathrm{M}\left[|X(1)|^q\right]$.

Multifractal processes have more complex scaling behavior: $\mathrm{Law}\{X(at)\} = \mathrm{Law}\{a^{H(a)} \cdot X(t)\},\quad a > 0$. For multifractal processes the following relation holds: $\mathrm{M}\left[|X(t)|^q\right] = c(q) \cdot t^{qh(q)}$, where $c(q)$ is certain deterministic function, $h(q)$ is generalized Hurst exponent, which is nonlinear function in the general case. Value $h(q)$ at $q = 2$ matches the value of measure of self-similarity H. For monofractal processes the generalized Hurst exponent does not depend on the parameter q: $h(q) = H$. [3].

There are many methods of parameter estimation of self-similar and multifractal processes from time series. When estimating of the multifractal characteristics one of the most popular is the method of multifractal detrended fluctuation analysis (MFDFA) [5,6]. When using the method MFDFA the cumulative time series $y(t) = \sum_{i=1}^{t} x(t)$ of the initial investigated one $x(t)$ is constructed. Then it is divided into

ISSN 1562-9945

N segments of length s. For each segment of $y(t)$ fluctuation function $F^2(s) = \frac{1}{s}\sum_{t=1}^{s}(y(t) - Y_m(t))^2$ is calculated, where $Y_m(t)$ is local m-polynomial trend within the limits of this segment of length s. By changing the time scale s at a fixed value q the dependence $F_q(s) = \left\{\frac{1}{N}\sum_{i=1}^{N}[F^2(s)]^{\frac{q}{2}}\right\}^{\frac{1}{q}}$ is found. Next, the dependence of the fluctuation function $F_q(s)$ of the parameter q is determined. If the investigated series have fractal properties, then the fluctuation function $F_q(s)$ has the power dependence $F_q(s) \propto s^{h(q)}$.

Method to distinguish between mono- and multifractal processes

Thus, having received the values of generalized Hurst exponent $h(q)$, it is theoretically possible to conclude about monofractality of the process, if this function is constant. However, in practice this is a difficult task. Estimate $\hat{h}(q)$ obtained from the time series is a curve which tends to a constant value with increasing length of the series [4,5,7]. Figure 1 (left) presents the sample values $\hat{h}(q)$ of monofractal process for realizations of different lengths. On the other hand, comparison with the range of values Δh for the multifractal processes (Fig. 1, right), shows that the difference between mono- and multifractal realizations is quite significant.

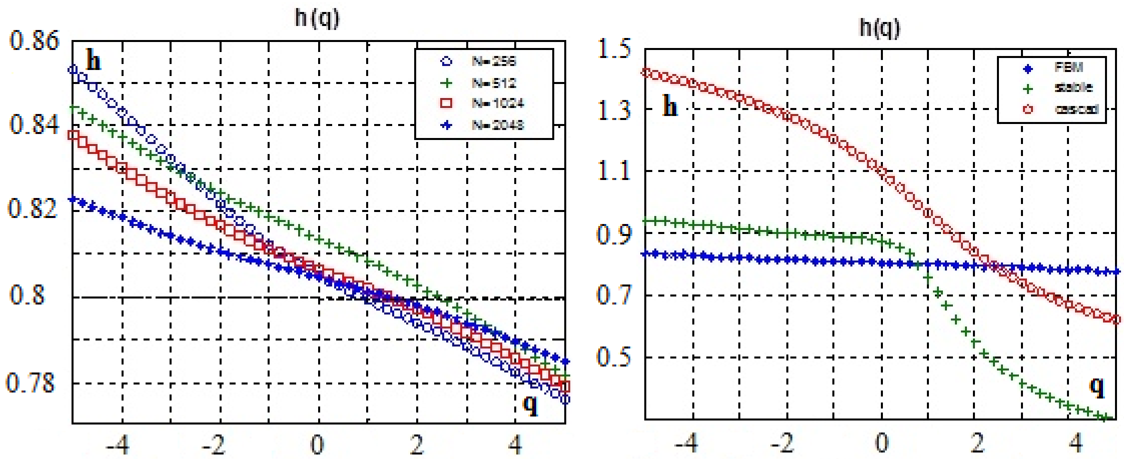

Figure 1 – Values $\hat{h}(q)$ for monofractal series with different lengths (left) and multifractal series (+, 0), and monofractal one (x) of length 1000 values (right)

The results of multifractal analysis of model time series allowed to develop and test the method proposed in [8], which allows to accept or reject the hypothesis about presence of monofractal properties of time series based on studies of sample values of the generalized Hurst exponent obtained by MFDFA.

Numerical analysis showed that a random variable $\Delta h = h(q1) - h(q2)$ at $q > 0$ has normal distribution $N(m_h, s_h)$ the parameters of which depend on the realization length and values q. The proposed criterion considers the magnitude $\Delta h = h(0.1) - h(5)$. By numerical simulation of mono- and multifractal processes sample values m_h and s_h for the time series of length N were obtained.

The value of a random variable Δh can be used as a statistical criterion for acceptance or rejection of the hypothesis of monofractal properties of time series. In this case, the null hypothesis is the assumption of monofractality. After obtaining estimate of function $\hat{h}(q)$ by MFDFA observed value $\Delta\hat{h} = \hat{h}(0.1) - \hat{h}(5)$ is calculated. Hypothesis is accepted with a significance level α if the resulting value falls in the range of acceptable values $\Delta\hat{h} < m_h(N) + t_\alpha s_h(N)$, where N is length of time series, m_h and s_h are the corresponding values calculated for monofractal process, α is the significance level, t_α is quantile of the standard normal distribution.

Research results of time series

In this work, a number of studies to identify mono- and multifractal properties of time series of different nature were carried out. As studied time series the electroencephalograms, financial series and temperature series have been chosen. The left side of Fig. 2 shows time series of electroencephalograms of laboratory animals for different phases of sleep: wakefulness state (awake), rapid eye movement (REM) sleep and slow-wave sleep (SWS). Financial series are Dow Jones index and gold prices in 2004-2008. Also series of average daily temperatures in Kiev in 2000-2006 which possess a cyclic component were investigated.

For each series generalized Hurst exponent were obtained by MFDFA. The table shows the results of the analysis of sample values of the generalized Hurst exponent $\hat{h}(q)$. The sample values $\Delta h = h(0.1) - h(5)$ of the series shown in the fig. 2 are presented in the third column of the Table. Critical values Δh based on estimated data for the significance level α =0.05 and the corresponding length of the series are given in the fourth column. On the basis of the obtained values the hypothesis of monofractal properties of time series has been accepted or rejected.

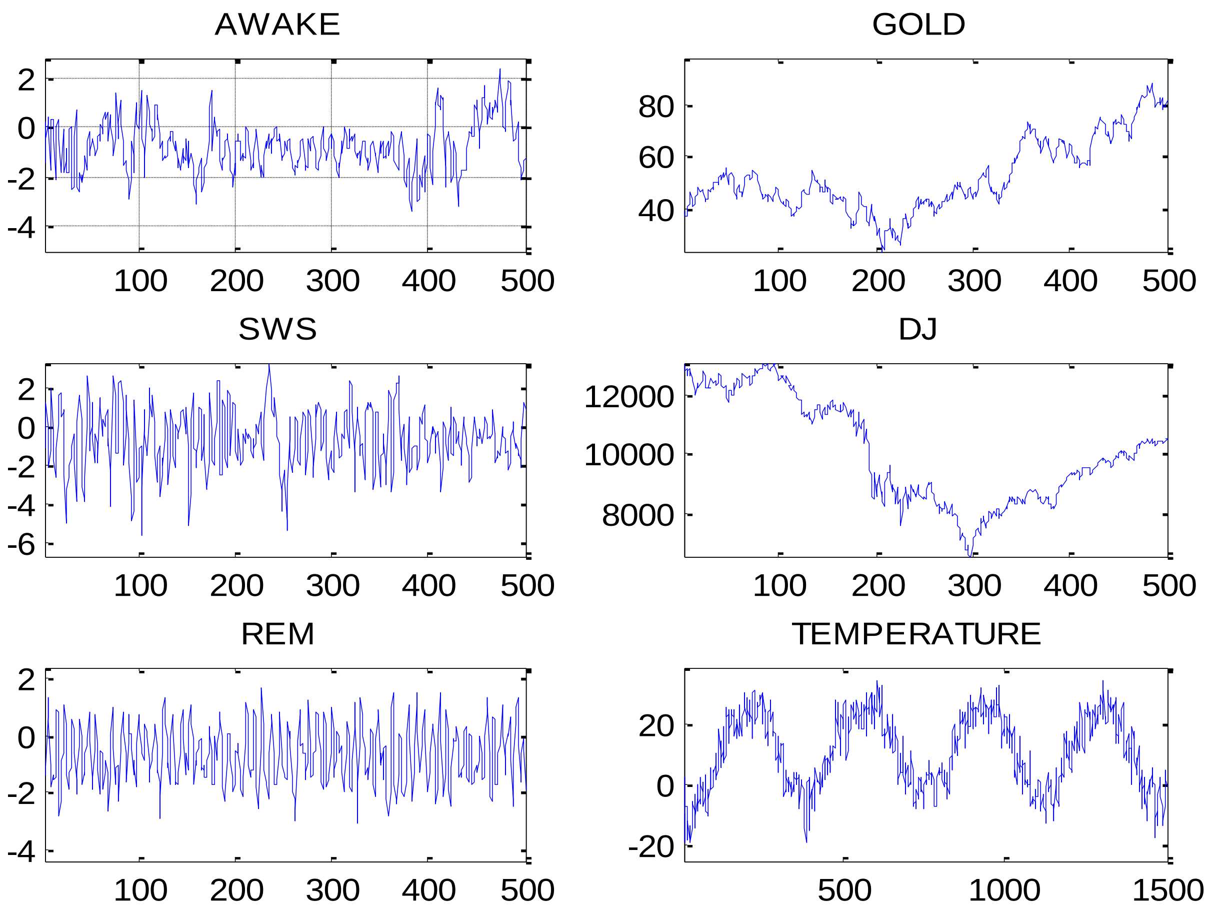

Figure 2 – Investigated time series: electroencephalograms, financial series and temperature series.

Table. Test of hypothesis of monofractality.

Time series	Length	Sample values $\Delta\hat{h}$	Critical value Δh	Fractality
EEG (awake)	1000	0.1743	0.0859	multi
EEG (REM)		0.169		multi
EEG (SWS)		0.068		mono
Dow Jones index	500	0.2416	0.1248	multi
Gold prices		0.0991		mono
Temperature	2000	0.0264	0.07312	mono

Conclusion

Based on the numerical analysis of the sample multifractal characteristics the statistical criterion to distinguish between mono- and multifractal processes from time series is proposed. Results of investigations to identify mono- and multifractal properties of the time series of different nature are given.

Studies indicate that the heterogeneity of the fractal structure of time series depends not only on the nature of the series, but also its local characteristics. Identifying the presence or absence of multifractal properties, we can find a more efficient mathematical model of a stochastic process that generates a similar time realizations. It should also be noted that usually monofractal mathematical models have less cumbersome mathematical apparatus and are simpler to implement than multifractal models.

REFERENCES

1. Шредер М. Фракталы, хаос и степенные ряды. Миниатюры из бесконечного рая / М. Шредер. – Ижевск: НИЦ «Регулярная и хаотическая динамика», 2005. – 528 с.

2. Федер Е. Фракталы / Е. Федер. – М.: Мир, 1991. – 254 с.

3. Riedi R.H Multifractal processes, in Doukhan P., Oppenheim G., Taqqu M.S. (Eds.), Long Range Dependence: Theory and Applications, Birkhuser. – 2002. – P. 625–715.

4. Oswiecimka P. Wavelet versus detrended fluctuation analysis of multifractal structures / P. Oswiecimka, J. Kwapin, S.Drozdz // Physical Review E: Statistical, Nonlinear, and Soft Matter Physics. –2006. – Vol. 74. – P. 161-203.

5. Kantelhardt J.W. Multifractal detrended fluctuation analysis of non-stationary time series / J.W. Kantelhardt, S.A. Zschiegner, A. Bunde, S. Havlin, E. Koscielny-Bunde, H.E. Stanley // Physica A. – 2002. – № 316. – P. 87–114.

6. Kantelhardt J. W. Fractal and Multifractal Time Series /J. W. Kantelhardt [Электронный ресурс]: 2008.– Режим доступа: http://arxiv.org/abs/0804.0747.

7. Кириченко Л. О. Исследование выборочных характеристик, полученных методом мультифрактального флуктуационного анализа / Л. О. Кириченко // Вісник НТУУ «КПІ». Інформатика, управління та обчислювальна техніка: зб. наук. пр. – 2011. – № 54. – С.101–111.

8. Кириченко Л. О. Сравнительный мультифрактальный анализ временных рядов методами детрендированного флуктуационного анализа и максимумов модулей вейвлет-преобразования / Л. О. Кириченко // Автоматизированные системы управления и приборы автоматики. – 2011. – Вып. 157. – С.66–77.

УДК 519.2:004.9

Кириченко Л.О. **Метод распознавания монофрактальных и мультифрактальных процессов по временным рядам //** Системные технологии. Региональный межвузовский сборник научных работ.- Выпуск?(??).-Днепропетровск, 2015. –С. ??-??.

На основе численного анализа выборочных мультифрактальных характеристик, полученных методом мультифрактального детрендированного флуктуационного анализа, предложен статистический критерий для принятия гипотезы о монофрактальных свойствах временного ряда. Представлены результаты исследований по выявлению моно- и мультифрактальных свойств временных рядов различной природы.

Библ.8 , табл.1, рис 2.

УДК 519.2:004.9

Кіріченко Л.О. **Метод розрізнення монофрактальних і мультифрактальних процесів за часовими рядами** // Системні технології. Регіональний міжвузівський збірник наукових робіт.- Випуск?(??).-Дніпропетровськ, 2015. –С. ??-??.

На основе чисельного аналізу вибіркових мультифрактальних характеристик, отриманих методом мультифрактального детрендірованного флуктуаційного аналізу, запропонований статистичний критерій для прийняття гіпотези про монофрактальні властивості часового ряду. Представлено результати досліджень з виявлення моно- і мультифрактальних властивостей часових рядів різноматної природи.

Бібл.8 , табл.1, рис. 2.

UDC 519.2:004.9

Kirichenko L.O. **The method of distinction monofractal and multifractal process from time series //** System technologies.- N(??).-Dnipropetrovsk, 2015. –P. ??-??.

Based on the numerical analysis of the sample multifractal characteristics obtained by method of multifractal detrended fluctuation analysis the statistical criterion for accepting the hypothesis of monofractal properties of the time series is proposed. Results of investigations to identify mono- and multifractal properties of the time series of different nature are given.

Ref.8, tab.1, fig.2.

Кириченко Людмила Олеговна – д.т.н., профессор кафедры прикладной математики Харьковского национального университета радиоэлектроники.

 ISSN 1562-9945