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MULTI-FRACTAL ANALYSIS IN PROBLEMS OF THE APPLIED PHYSICS

Leonid F. Chernogor*, Oleg V. Lazorenko, Andriy A. Onishchenko*****
** School of Radiophysics, Biomedical Electronics and Computer Systems,*
V. N. Karazin Kharkiv National University, Kharkiv 61022, Ukraine e-mail: Leonid. F. Chernogor@gmail.com
*** School of Physics, V. N. Karazin Kharkiv National University, Kharkiv 61022, Ukraine,*
e-mail: Oleg.V.Lazorenko@karazin.ua
**** Faculty of Automatics and Computerized Technologies, Kharkiv National University of Radioelectronics,*
Kharkiv 61166, Ukraine, email: Andrey.Onishchenko@nure.ua

For the researchers, a review of the existing multi-fractal analysis methods applied in different branches of modern applied physics is proposed. For each multi-fractal analysis method, some actual references allowing to study the method and to develope corresponding numerical realization are given.

Introduction

According to the non-linear and the system paradigms, many processes generated in open, non-linear, dynamical systems under influence of a powerful source of energy release are appeared to be short-time, ultra-wideband, non-linear and fractal. In the paper [1] published one year ago, we have introduced a short review of the existing mono-fractal analysis methods, which can be useful and quitly applicable for solving of the applied physics problems, where there are a lot of such processes and such systems. To investigate their fractal properties more extended, the multi-fractal analysis methods should be applied. Unfortunately, similar the mono-fractal analysis methods, these methods are often appeared to be quite unknown for the most part of researchers too.

The purpose of this work is to present the multi-fractal analysis methods to the researchers. Due to volume limitations of this paper, the number of references for each method is strictly limited.

Multi-Fractal Analysis Methods

Multi-fractal analysis of the signals and processes is based on calculation of the set of numerical characteristics, namely the spectrum of generalyzed dimensions, which are known as the Renyi dimensions too, the scaling exponent, which is known as the mass index too, the multi-fractal spectrum function, which is known as the Hausdorf's multi-fractal spectrum, the singularities spectrum or the scaling spectrum too, and some other (see, for example, [2, 3]).

Now there are over ten multi-fractal analysis methods, which are regularly applied, in particular, in the applied physics (see, for example, [4]). Today there are the Wavelet Transform Modulus Maxima (WTMM) method [5 – 7], the Wavelet Coefficients method [8], the Wavelet Leaders Method [8, 9], the multi-fractal detrended fluctuation analysis (MF DFA) [10], the generalyzed Hurst exponent method [11, 12], the multi-fractal regime detecting method [13], the weighted generalyzed Hurst exponent method [14], the local method of the second moment [15], the method based on the Cohen's class non-linear transforms [16, 17], the multi-fractal diffusion entropy analysis (MF DEA) [18], the Large deviation multifractal spectrum method [19], the Cumulative Mass Method, which is known as The "Sandbox" Method too [20], the multi-fractal Detrended Moving Average (MFDMA) method [21] and other.

Besides methods listed above, there are the methods, which unite the multi-fractal and cross-correlations analyses [22], for example, Detrended Cross-Correlation Analysis (DCCA) та Multi-Fractal Detrended Cross-Correlation Analysis (MF-DXA). Most likely, it is already difficult to call them methods of multifractal analysis, but they allow to evaluate the cross-correlation of two signals using detrending and multifractal properties.

In the review paper [4], much more extended set of references and descriptions for the multifractal analysis methods listed above can be found.

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