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Figurate Numbers (Arithmetic Progression) and Electromagnetic Wave Scattering on Spatial Lattices of Resonant Magnetodielectric Spheres*

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Considered is the problem of electromagnetic wave scattering on complex spatial lattices of special kind, consisting of resonant-size spheres whose spatial distribution is controlled by figurate numbers. Expressions for the scattered fields are derived.

The interest in figurate numbers arose in V century B.C. in Ancient Greece in connection with the development of Pythagorean arithmetic. The number structures had profound influence on the medieval arithmetic and continue influencing the mathematics and other sciences of today. There are various structures of figurate numbers, specified through functions or numerical tables.

Consider a specific structure of figurate numbers, defined through a numerical table [1] and the function

$$1+(|s|+1)|t|,$$

where $|s|$ = 0,1,2,3, …, and $|t|$ = 0,1,2,3, … . A specific table column corresponds to each value of $|s|$. The column $|s|$=0 corresponds to the arithmetic natural series.

Table

$\|t\|$ \ $\|s\|$	0	1	2	3	4	5	6	7	–
0	1	1	1	1	1	1	1	1	–
1	2	3	4	5	6	7	8	9	–
2	3	5	7	9	11	13	15	17	–
3	4	7	10	13	16	19	22	25	–
4	5	9	13	17	21	25	29	33	–
5	6	11	16	21	26	31	36	41	–
6	7	13	19	25	31	37	43	49	–
7	8	15	22	29	36	43	50	57	–
–	–	–	–	–	–	–	–	–	–

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The magnitude $1+(|s|+1)|t|$ is the arbitrary term of the arithmetic progression $a_1; a_1+d$, a_1+2d, a_1+3d … when the first term of the progression equals $a_1 = 1$ and the common difference d assumes the values d =1,2,3, …etc. ($d = |s|+1$).

Of considerable interest are the lattices in which the spatial distribution of nodes is controlled by numerical structures.

This paper is aimed at solving the problem of electromagnetic wave scattering on special complex spatial lattices consisting of small-size homogeneous resonant magnetodielectric spheres whose spatial distribution is controlled by figurate number structures (arithmetical progression) [1]. The scattered wavelength can be commensurate with the lattice constants, which enables studying the effect of structural lattice resonances of the electromagnetic interaction between the spheres on internal resonances of the lattice spheres and their fine structure. The solution describes the domains of abnormal dispersion of the lattices.

1. Problem formulation and solution

Consider a complex spatial lattice consisting of C sublattices ($c \in C$) . The sublattices are generated by the coordinate representation, whose Cartesian (rectangular) form is

$$\begin{aligned} x_{c,s} &= [s-0.5\{(-1)^s-1\}]d-(-1)^{s-1}x_{c,s=0} \qquad (s=0,\pm1,\pm2,...), \\ y_{c,t} &= [t-0.5\{(-1)^t-1\}]h-(-1)^{t-1}y_{c,t=0} \qquad (t=0,\pm1,\pm2,...), \\ z_{c,p} &= [p-0.5\{(-1)^p-1\}]l-(-1)^{p-1}z_{c,p=0} \qquad (p=0,\pm1,\pm2,...,\pm[|s|+1]|t|), \end{aligned} \tag{1}$$

where the values of d, h, and l are determined by the conditions $x=0$, $x=d$; $y=0$, $y=h$; $z=0$, $z=1$ while $x_{c,s=0}, y_{c,t=0}$ and $z_{c,p=0}$ are coordinates of the node generating the sublattice c and located inside the domain (Fig.1).

$$0 \le x_{c,s=0} \le d,\ 0 \le y_{c,t=0} \le h,\ 0 \le z_{c,p=0} \le l. \tag{2}$$

The coordinates $x_{c,s}, y_{c,t}$ and $z_{c,p}$ define positions of nodes of the sublattice c outside the domain Eq.(2) and are functions of the coordinates of $x_{c,s=0}, y_{c,t=0}$ and $z_{c,p=0}$. By considering $x_{c,s=0}, y_{c,t=0}$ and $z_{c,p=0}$ as certain functions of time, we can introduce a time dependence in the coordinate representation Eq.(1). Each node of the spatial lattice c Eq.(1) is associated with an ordered triple of numbers, $u = c(p,s,t)$; the selected node will be denoted $u' = c'(p',s',t')$, while the node within the domain Eq.(2) as $c(0=0, s=0, t=0)$. Setting maximum limiting values for the numbers p, s, t in Eq.(1), we can consider finite and infinite lattices.

The necessary type of the elemental lattice cell (primitive, body-centered, face-centered or other) is formed from C nodes within the domain Eq.(2), which is replicated by the coordinate representation Eq. (1) beyond the domain Eq.(2) in the form of a spatial lattice of specific form.

Fig. 1 shows spatial distributions of lattice nodes when the generating node is at the center of domain Eq. (2) for the cases $p = 0, \pm 1;\ s = 0, \pm 1;\ t = 0, \pm 1$ and $p = 0, 1, 2, 3, 4, 5, 6;$ $s = 0, \pm 1, \pm 2;\ t = 0, \pm 1, \pm 2$.

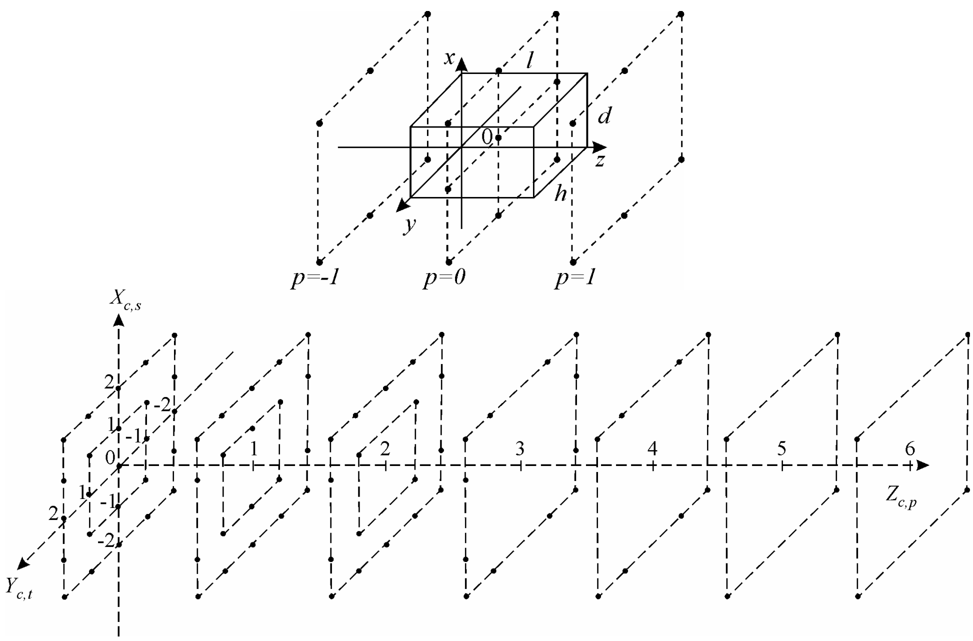

Fig. 1 Spatial lattice of nodes and problem geometry.

The node distribution along the z axis is governed by the table of figurate numbers (Fig.2). Each triple of numbers $p = 0, s, t$ of the plane $x_{c,s}, y_{c,t}, z_{c,p=0}$ is associated with a specific number in the table, for example: point $p = 0$, $s = -3$, $t = -3$ corresponds to the number 13; point $p = 0$, $s = -5$, $t = 4$ to the number 25, and point $p = 0$, $s = 4$, $t = 3$ is associated with the number 16.
These numbers determine the number of nodes along the z-axis Eq.(1) for the given node $p = 0, s, t$ (Fig. 2).

The interaction between the triple of numbers $p=0,s,t$ of the plane $x_{c,s}, y_{c,t}, z_{c,p=0}$ and the number in the table (Fig.2) is determined by the function

$$1 + (|s| + 1)|t| .$$

Hence, the numbers p, determining the node coordinates along the z-axis Eq.(1) are set by the sequence

$$0, \pm 1, \pm 2, \ldots, \pm([1 + (|s| + 1)|t|] - 1) ,$$

where $|s|, |t| = 0,1,2,3, \ldots$.

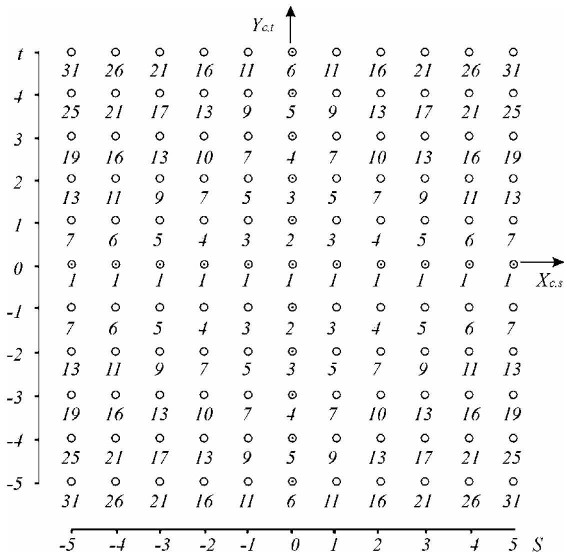

Fig. 2 Table of figurate numbers on plane $X_{c,s}$, $Y_{c,t}$

If the node coordinates in the domain Eq.(2) are varied, the positions of nodes outside the domain Eq.(2) will shift correspondingly, reflecting cell rearrangement and formation of the spatial configuration of the lattice. The node separation can be determined from Eq.(1),

$$r_{c'(p',s',t'),c(p,s,t)} = \sqrt{\left(x_{c',s'} - x_{c,s}\right)^2 + \left(y_{c',t'} - y_{c,t}\right)^2 + \left(z_{c',p'} - z_{c,p}\right)^2}. \tag{3}$$

In case the center of the domain Eq.(2) is occupied by a single lattice generating node, then for $p, s = 0, t$ we obtain from Eq.(1) a plane lattice whose nodes are distributed along the *z*-axis following the law of the natural number series (Fig.3).

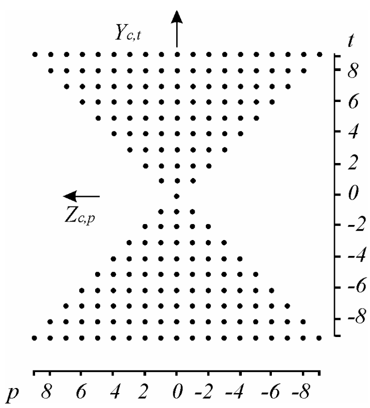

Fig. 3. Plane lattice of nodes ($p, s = 0, t$)

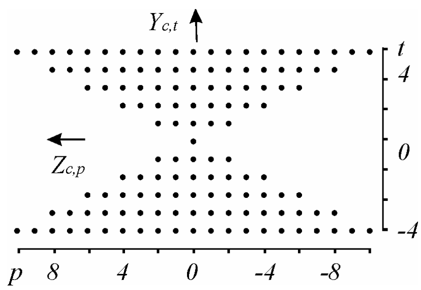

Fig. 4. Plane lattice of nodes $p, s = 1, t$

With $p,s=1,t$ we obtain a plane lattice with the nodes distributed along the *z*-axis according to the number sequence (1, 3, 5, 7, 9, 11, …) (Fig.4).

For $p,s=3,t$ we obtain a plane lattice with the node distribution dictated by the sequence (1, 5, 13, 17, 21, …) (Fig.5).

If $p=0,s,t=0$ we obtain a linear lattice with nodes distributed along the *x*-axis.

From the proper sections of the node distribution Eq. (1), one can obtain rearrangeable plane lattices of various forms. If an elemental cell is formed in the domain Eq.(2), then the lattices of Figs. 3-5 will contain cells from the domain Eq.(2) instead of single points.

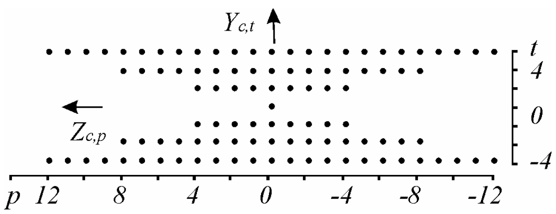

Fig. 5. Plane lattice of nodes *p*,*s*=3,*t*

The nodes of the sublattices Eq.(1) accommodate centers of material spheres of radii $a_{c(p,s,t)}$, characterized by the permittivities $\varepsilon_{c(p,s,t)}, \mu_{c(ps,s,t)}$ (denoted below as $\varepsilon_c, \mu_c, a_c$). The lattice spheres are in free space.

We will assume that $a_c / \lambda << 1$ out of the spheres, while inside a sphere the resonant case $a_c / \lambda_g \sim 1$ is possible (here λ is the wavelength out of the sphere and λ_g inside the sphere [2]).

To solve the problem, we will use the formalism of integral equations [3] and perform the analysis in two stages. First, we will find the internal field in the scattering spheres, and find the field scattered by the spatial lattice of spheres at stage two. The fields will be represented as follows:

$$\vec{E}(\vec{r},t)=\vec{E}(\vec{r})e^{i\omega t} \quad , \quad \vec{H}(\vec{r},t)=\vec{H}(\vec{r})e^{i\omega t} .$$

The scattered field can be determined from the known internal field in terms of the electrical, $\vec{\Pi}^e$, and magnetic, $\vec{\Pi}^m$, the Hertz potentials:

$$\begin{aligned} \vec{E}_{scat} &= \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\Pi}^e - ik\mu_0\left[\nabla,\vec{\Pi}^{\text{м}}\right], \\ \vec{H}_{scat} &= \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\Pi}^m + ik\varepsilon_0\left[\nabla,\vec{\Pi}^{\text{э}}\right]. \end{aligned} \tag{4}$$

The Hertz potentials of the scattered field have the form:

$$\begin{aligned} \vec{\Pi}^e &= \frac{1}{4\pi}\int_V\left(\frac{\varepsilon}{\varepsilon_0}-1\right)\vec{E}^0\left(\vec{r}'\right)f\left(\left|\vec{r}-\vec{r}'\right|\right)dV, \\ \vec{\Pi}^m &= \frac{1}{4\pi}\int_V\left(\frac{\mu}{\mu_0}-1\right)\vec{H}^0\left(\vec{r}'\right)f\left(\left|\vec{r}-\vec{r}'\right|\right)dV, \end{aligned} \tag{5}$$

where $\vec{E}^0(\vec{r}\,')$ and $\vec{H}^0(\vec{r}\,')$ are the internal fields of the scatterer; V the scatterer volume; ε_0 and μ_0 are the free space permeabilities and the function $f(|\vec{r}-\vec{r}\,'|)$ is the solution of

$$\Delta f(|\vec{r}-\vec{r}\,'|)+k^2\varepsilon_0\mu_0 f(|\vec{r}-\vec{r}\,'|)=-4\pi\delta(|\vec{r}-\vec{r}\,'|).$$

It satisfies the radiation condition at infinity and has the form

$$f\left(|\vec{r}-\vec{r}'|\right)=\frac{e^{-ki\sqrt{\varepsilon_0\mu_0}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \tag{6}$$

First we will evaluate the internal field of the scatterers for the case of $a_c/\lambda_g << 1$ inside the sphere and $a_c/\lambda << 1$ outside, and then extend the results of the calculations to the resonant case when $a_c/\lambda_g \sim 1$ inside the sphere. It can be shown that for external points of the sphere, $r > r'$, the Green function Eq. (6) of free space, integrated over the space volume gives:

$$W\left(\vec{r}\right)=\int_V \frac{e^{-ik\sqrt{\varepsilon_0\mu_0}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dV=\frac{4\pi}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\frac{e^{-ik_1 r}}{r}, \tag{7}$$

where $k_l = k\sqrt{\varepsilon_0\mu_0}$; $k = 2\pi/\lambda$ and r determines the distance from the center to an external point of the sphere.

The internal field of the sphere at $c'(p',s',t')$ can be found from the set of quasistationary non-uniform equations, which can be constructed based on integral equations [3]. The non-uniform equations for an arbitrarily selected sphere are

$$\begin{aligned}
&\vec{E}_{0c'(p',s',t')}\left(\vec{r}',t\right)=\left(\left\{1+\frac{1}{3}\left(\frac{\varepsilon_{c'}}{\varepsilon_0}-1\right)\right\}\vec{E}^0_{c'(p',s',t')}\left(\vec{r}',t\right)-\sum_p\sum_s\sum_t\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\times\right.\right.\\
&\times\frac{1}{4\pi}\left(\frac{\varepsilon_{c'}}{\varepsilon_0}-1\right)W^e_{c'(p,s,t)}\left(\vec{r}\right)\vec{E}^0_{c'(p,s,t)}\left(\vec{r}',t\right)-ik\mu_0\left.\left.\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_{c'}}{\mu_0}-1\right)W^m_{c'(p,s,t)}\left(\vec{r}\right)\vec{H}^0_{c'(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right)-\\
&-\sum_{c=1}^{C}\left(\sum_p\sum_s\sum_t\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\varepsilon_c}{\varepsilon_0}-1\right)W^e_{c(p,s,t)}(\vec{r})\vec{E}^0_{c(p,s,t)}\left(\vec{r}',t\right)-ik\mu_0\times\right.\right.\\
&\times\left.\left.\left[\nabla,\frac{1}{4\pi}\left(\frac{\mu_c}{\mu_0}-1\right)W^m_{c(p,s,t)}\left(\vec{r}\right)\vec{H}^0_{c(p,s,t)}\left(\vec{r}',t\right)\right]\right\}\right),\\
&\qquad\qquad c'\left(p,s,t\right)\neq c'\left(p',s',t'\right);\ \left(c\neq c'\right),
\end{aligned} \tag{8}$$

$$\vec{H}_{0c'(p',s',t')}(\vec{r}',t)=\left(\left\{1+\frac{1}{3}\left(\frac{\mu_{c'}}{\mu_0}-1\right)\right\}\vec{H}^0_{c'(p',s',t')}(\vec{r}',t)-\sum_p\sum_s\sum_t\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\times\right.\right.$$

$$\left.\left.\times\frac{1}{4\pi}\left(\frac{\mu_{c'}}{\mu_0}-1\right)W^m_{c'(p,s,t)}(\vec{r})\vec{H}^0_{c'(p,s,t)}(\vec{r}',t)+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_{c'}}{\varepsilon_0}-1\right)W^e_{c'(p,s,t)}(\vec{r})\vec{E}^0_{c'(p,s,t)}(\vec{r}',t)\right]\right\}\right)-$$

$$-\sum_{c=1}^{C}\left(\sum_p\sum_s\sum_t\left\{\left(\nabla\nabla+k^2\varepsilon_0\mu_0\right)\frac{1}{4\pi}\left(\frac{\mu_c}{\mu_0}-1\right)W^m_{c(p,s,t)}(\vec{r})\vec{H}^0_{c(p,s,t)}(\vec{r}',t)+\right.\right.$$

$$\left.\left.+ik\varepsilon_0\left[\nabla,\frac{1}{4\pi}\left(\frac{\varepsilon_c}{\varepsilon_0}-1\right)W^e_{c(p,s,t)}(\vec{r})\vec{E}^0_{c(p,s,t)}(\vec{r}',t)\right]\right\}\right),$$

where $\vec{E}_{0c'(p',s',t')}(\vec{r}\,',t)$, $\vec{H}_{0c'(p',s',t')}(\vec{r}\,',t)$ and $\vec{E}^0{}_{c'(p',s',t')}(\vec{r}\,',t)$, $\vec{H}^0{}_{c'(p',s',t')}(\vec{r}\,',t)$ are, respectively, the field of the incident wave and the internal field of the sphere $c'(p',s',t')$, while $\vec{E}_{c(p,s,t)}(\vec{r},t)$, $\vec{H}_{c(p,s,t)}(\vec{r},t)$ are the exterior fields of the rest of the spheres.

The values $W^e_{c(p,s,t)}(\vec{r}\,')$ and $W^m_{c(p,s,t)}(\vec{r}\,')$ are given by Eqs. (3), (7) and (8).

$$W^e_{c(p,s,t)}(\vec{r}')=\frac{4\pi}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\times\frac{e^{-ik_1r_{c'(p',s',t'),c(p,s,t)}}}{r_{c'(p',s',t'),c(p,s,t)}},$$
$$W^m_{c(p,s,t)}(\vec{r}')=-\frac{4\pi}{k_1^3}\left(\sin k_1a_c-k_1a_c\cos k_1a_c\right)\times\frac{e^{-ik_1r_{c'(p',s',t'),c(p,s,t)}}}{r_{c'(p',s',t'),c(p,s,t)}}.$$

The first terms in the right-hand parts of Eqs. (8) relate to the internal field $c'(p',s',t')$ of the sphere without account of the effect of all other spheres, while the rest of the terms make allowance for the effect of all other spheres on the scatterer $c'(p',s',t')$.

Eqs. (8) represent a set of $2N=2\sum_{c=1}^{C}N_c$ non-uniform vectorial equations , where N is the total number of the lattice spheres and N_c is the number of spheres of sublattice c. The solution for a selected sphere has the form

$$\vec{E}^0_{c'(p',s',t')}(\vec{r}',t)=\frac{1}{\Delta^{\text{эм}}}\sum_{c=1}^{C}\left(\sum_u\left[\widehat{g}^{eu'}_u\vec{E}_{0c(p,s,t)}(\vec{r}',t)+\widehat{\beta}^{eu'}_u\vec{H}_{0c(p,s,t)}(\vec{r}',t)\right]\right),$$
$$\vec{H}^0_{c'(p',s',t')}(\vec{r}',t)=\frac{1}{\Delta^{\text{эм}}}\sum_{c=1}^{C}\left(\sum_u\left[\widehat{\beta}^{mu'}_u\vec{H}_{0c(p,s,t)}(\vec{r}',t)+\widehat{g}^{mu'}_u\vec{E}_{0c(p,s,t)}(\vec{r}',t)\right]\right),\qquad(9)$$

where Δ^{em} is the determinant of the principal matrix of the equation set Eq. (8).

The solutions Eq.(9) are valid when $a_c / \lambda << 1$ outside the sphere and $a_c / \lambda_g << 1$ inside. However, they can be extended to the resonant case $a_c / \lambda_g \sim 1$ if we introduce effective permeabilities [2, 4] instead of ε_0 and μ_0 (see Fig.6),

$$\begin{aligned} \varepsilon_{ceff} &= \varepsilon_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right), \\ \mu_{ceff} &= \mu_c F\left(ka_c\sqrt{\varepsilon_c\mu_c}\right), \end{aligned} \tag{10}$$

where $F(ka_c\sqrt{\varepsilon_0\mu_0})$=...

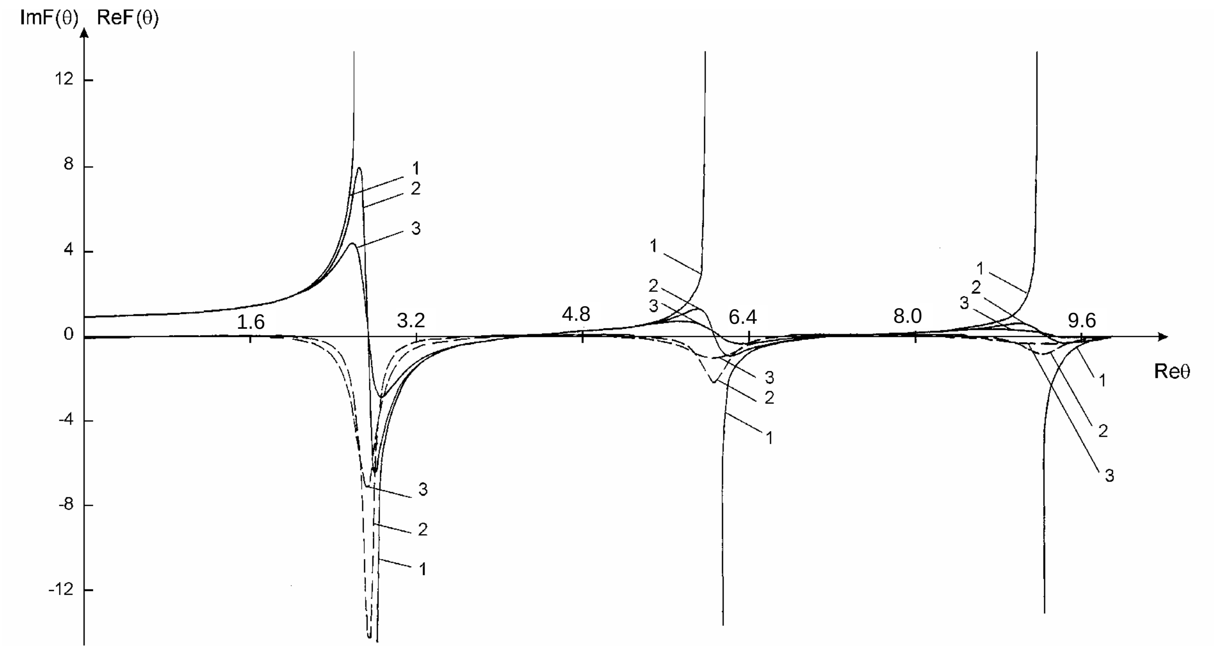

Fig. 6 The $F(\theta)$ function.

Fig.6 shows the behavior of Re $F(\theta)$ (solid curve) and Im $F(\theta)$ (broken curve) depending on Re θ for a variety of magnitudes of the dielectric loss tangent, $\tan\delta_\varepsilon$ (specifically, 1) $\tan\delta_\varepsilon = 0$; 2) $\tan\delta_\varepsilon = 0.05$; 3) $\tan\delta_\varepsilon = 0.1$) and $\mu_c = 1$; here $\theta = ka_c\sqrt{\varepsilon_c\mu_c}$.

In case the electromagnetic interaction between the spheres can be neglected, then general expressions for the internal field of an arbitrary sphere in the lattice Eq.(9) take the form (10)

$$\vec{E}^0_{c(p,s,t)}(\vec{r}',t) = \frac{3\varepsilon_0 e^{i\theta_{1c}}}{(\varepsilon_{ceff} + 2\varepsilon_0) + \theta_{1c}^2\varepsilon_{ceff} + i\theta_{1c}(\varepsilon_{ceff} + 2\varepsilon_0)}\vec{E}_{0c(p,s,t)}(\vec{r}',t),$$

$$\vec{H}^0_{c(p,s,t)}(\vec{r}',t) = \frac{3\mu_0 e^{i\theta_{1c}}}{(\mu_{ceff} + 2\mu_0) + \theta_{1c}^2\mu_{ceff} + i\theta_{1c}(\mu_{ceff} + 2\mu_0)}\vec{H}_{0c(p,s,t)}(\vec{r}',t),$$

with $\theta_{1c}^2 = k^2 a_c^2 \varepsilon_0 \mu_0$.

The Hertz potentials Eq.(5) of the field scattered by the spheres of the lattice, can be represented as a superposition of the Hertz potentials of individual spheres of the lattice, taking into account Eqs. (9) and (10),

$$\vec{\Pi}^e(\vec{r},t) = \sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t \frac{1}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\left(\frac{\varepsilon_{cэф}}{\varepsilon_0}-1\right)\vec{E}^0_{c(p,s,t)}(\vec{r}',t)\frac{e^{-ik_1 r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right],$$
$$\vec{\Pi}^m(\vec{r},t) = -\sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t \frac{1}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\left(\frac{\mu_{cэф}}{\mu_0}-1\right)\vec{H}^0_{c(p,s,t)}(\vec{r}',t)\frac{e^{-ik_1 r_{c(p,s,t)}}}{r_{c(p,s,t)}}\right]. \tag{11}$$

Here $r_{c(p,s,t)} = \sqrt{(x-x_{c,s})^2+(y-y_{c,t})^2+(z-z_{c,p})^2}$, where the coordinates (x,y,z) represent the observation node of the scattered field out of the lattice spheres; the $(x_{c,s}, y_{c,t}, z_{c,p})$ coordinates correspond to the point of location of the scattering sphere center (the lattice of Eq.(1)). Then, taking account of Eqs.(10) and (11) we can find from Eq.(4) the field scattered by the lattice spheres,

$$\vec{E}_{scat} = \sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t \frac{1}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\left\{\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)\widehat{L}_c\vec{E}^0_{c(p,s,t)}(\vec{r}') - \right.\right.$$
$$\left.\left. -ik\mu_0\left(\frac{\mu_{ceff}}{\mu_0}-1\right)(-1)\widehat{P}_c\vec{H}^0_{c(p,s,t)}(\vec{r}')\right\}e^{i\left(wt-k_1 r_{c(p,s,t)}\right)}\right],$$
$$\vec{H}_{scat} = \sum_{c=1}^{C}\left[\sum_p\sum_s\sum_t \frac{1}{k_1^3}\left(\sin k_1 a_c - k_1 a_c \cos k_1 a_c\right)\left\{\left(\frac{\mu_{ceff}}{\mu_0}-1\right)(-1)\widehat{L}_c\vec{H}^0_{c(p,s,t)}(\vec{r}') + \right.\right.$$
$$\left.\left. +ik\varepsilon_0\left(\frac{\varepsilon_{ceff}}{\varepsilon_0}-1\right)\widehat{P}_c\vec{E}^0_{c(p,s,t)}(\vec{r}')\right\}e^{i\left(wt-k_1 r_{c(p,s,t)}\right)}\right], \tag{12}$$

Here $\widehat{L}_c$ and $\widehat{P}_c$ are functional matrices of the form

$$\widehat{L}_c = \begin{bmatrix} \Psi_{xxc} & \Psi_{xyc} & \Psi_{xzc} \\ \Psi_{yxc} & \Psi_{yyc} & \Psi_{yzc} \\ \Psi_{zxc} & \Psi_{zyc} & \Psi_{zzc} \end{bmatrix}; \qquad \widehat{P}_c = \begin{bmatrix} 0 & \Psi_{zc} & \Psi^0_{yc} \\ \Psi^0_{zc} & 0 & \Psi_{xc} \\ \Psi_{yc} & \Psi^0_{xc} & 0 \end{bmatrix}.$$

The values contained in the functional matrices Eq. (12) have the form

$$\Psi_{xxc} = \frac{1}{r_{c(p,s,t)}}k^2\varepsilon_0\mu_0 + \frac{3\left(x-x_{c,s}\right)^2 - r^2_{c(p,s,t)}}{r^5_{c(p,s,t)}} - \frac{k_1^2\left(x-x_{c,s}\right)^2}{r^3_{c(p,s,t)}} + ik_1\frac{3\left(x-x_{c,s}\right)^2 - r^2_{c(p,s,t)}}{r^4_{c(p,s,t)}},$$
$$\Psi_{yyc} = \frac{1}{r_{c(p,s,t)}}k^2\varepsilon_0\mu_0 + \frac{3\left(y-y_{c,t}\right)^2 - r^2_{c(p,s,t)}}{r^5_{c(p,s,t)}} - \frac{k_1^2\left(y-y_{c,t}\right)^2}{r^3_{c(p,s,t)}} + ik_1\frac{3\left(y-y_{c,t}\right)^2 - r^2_{c(p,s,t)}}{r^4_{c(p,s,t)}},$$

$$\Psi_{zzc} = \frac{1}{r_{c(p,s,t)}} k^2 \varepsilon_0 \mu_0 + \frac{3\left(z - z_{c,p}\right)^2 - r_{c(p,s,t)}^2}{r_{c(p,s,t)}^5} - \frac{k_1^2 \left(z - z_{c,p}\right)^2}{r_{c(p,s,t)}^3} + ik_1 \frac{3\left(z - z_{c,p}\right)^2 - r_{c(p,s,t)}^2}{r_{c(p,s,t)}^4},$$

$$\Psi_{xyc} = \Psi_{yxc} = \frac{3\left(x - x_{c,s}\right)\left(y - y_{c,t}\right)}{r_{c(p,s,t)}^5} - k_1^2 \frac{\left(x - x_{c,s}\right)\left(y - y_{c,t}\right)}{r_{c(p,s,t)}^3} + ik_1 \frac{3\left(x - x_{c,s}\right)\left(y - y_{c,t}\right)}{r_{c(p,s,t)}^4},$$

$$\Psi_{xzc} = \Psi_{zxc} = \frac{3\left(x - x_{c,s}\right)\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^5} - k_1^2 \frac{\left(x - x_{c,s}\right)\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^3} + ik_1 \frac{3\left(x - x_{c,s}\right)\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^4},$$

$$\Psi_{xyc} = \Psi_{zyc} = \frac{3\left(y - y_{c,t}\right)\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^5} - k_1^2 \frac{\left(y - y_{c,t}\right)\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^3} + ik_1 \frac{3\left(y - y_{c,t}\right)\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^4},$$

$$\Psi_{xc} = \frac{\left(x - x_{c,s}\right)}{r_{c(p,s,t)}^3} + ik_1 \frac{\left(x - x_{c,s}\right)}{r_{c(p,s,t)}^2}, \ \Psi_{xc}^0 = -\Psi_{xc},$$

$$\Psi_{yc} = \frac{\left(y - y_{c,t}\right)}{r_{c(p,s,t)}^3} + ik_1 \frac{\left(y - y_{c,t}\right)}{r_{c(p,s,t)}^2}, \ \Psi_{yc}^0 = -\Psi_{yc},$$

$$\Psi_{zc} = \frac{\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^3} + ik_1 \frac{\left(z - z_{c,p}\right)}{r_{c(p,s,t)}^2}, \ \Psi_{zc}^0 = -\Psi_{zc}.$$

The field at an arbitrary node of the space outside the sphere can be represented as

$$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r},t) + \vec{E}_{scat}(\vec{r},t),$$

where $\vec{E}_0(\vec{r},t)$ is the undisturbed field of the incident wave.

The determinant of the equation set Eq. (8) allows deriving the resonance conditions. Provided the permittivites ε_c and μ_c of the lattice spheres are real-valued and $a_c / \lambda_g \sim 1$, the conditions can be found from

$$\det RE\left\|a_{sj}\right\| = 0 \tag{13}$$

where $\left\|a_{sj}\right\|$ is the principal matrix of the equation set Eq. (8) [5]. If the electromagnetic interaction of the spheres in Eq.(13) can be neglected, then by solving it relative to the function $F(\theta_c)$ (Fig.6), we can derive the condition for internal magnetic-type resonances of sphere c in the form

$$F(\theta_c) = -\frac{2\mu_0(\cos\theta_{1c} + \theta_{1c}\sin\theta_{1c})}{\mu_c\left[(1+\theta_{1c}^2)\cos\theta_{1c} + \theta_{1c}\sin\theta_{1c}\right]},$$

where $\theta_c = ka_c\sqrt{\varepsilon_c\mu_c}$, $\theta_{1c} = ka_c\sqrt{\varepsilon_0\mu_0}$.

Conclusions

Wave scattering by a lattice of spheres whose spatial distribution is governed by figurate numbers (arithmetic progression) has been considered for the first time. This solution for lattices with an anisotropic topological structure controlled by an arithmetic progression can be useful for developing devices to control the radiation field of electromagnetic emitters; creating highly dispersive composite materials with the use of regions of abnormal lattice dispersion, and for studying the effect of lattice defects on electromagnetic wave propagation.

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