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A RETROSPECTIVE ANALYSIS OF THE OPTIMAL PARAMETRIC CONFIGURATION OF THE BROWN PREDICTIVE MODEL IN THE NONCLASSICAL RANGE

This study is aimed at the theoretical justification, formalization, planning, and conduct of an experiment involving a retrospective analysis of the optimal parametric configuration of Brown's forecasting model. The relevance of this work stems from the acute problem of forecasting short time series, particularly those characterizing the dynamics of business processes in production and logistics systems. The object of the study is the process of parametric synthesis of Brown's forecasting model, **the subject** is the conditions for its optimal parametric configuration. **The goal** is to assess the relevance of theoretical recommendations regarding the parametric synthesis of the model through a statistical retrospective analysis of its optimal parametric configuration. **The tasks** are: analyzing the feasible set of the smoothing parameter; designing a statistical experiment for the retrospective analysis of the optimal configuration; conducting a practical experiment and analyzing the results; and formulating recommendations for improving the parametric synthesis technology based on the experimental results. **Methods** of mathematical and statistical analysis were used to prove the existence, on the plane of internal parameters, of two symmetric characteristic regions with extreme properties. **As a result**, a method for retrospective analysis of the optimal parametric configuration of a model, suitable for analyzing the characteristics of the plane of its internal parameters, was proposed; the reliability of the analytical results was statistically confirmed using real data. Conditions are formulated under which the concept of the model's effective memory length extends to the out-of-bounds set of the smoothing parameter. The main **conclusion** is the substantiated necessity of taking into account the configuration of root clusters and separately considering even and odd sample lengths when forming a point forecast.

Keywords: Brown's forecasting model; statistical analysis; inverse parametric identification; parametric synthesis; statistical experiment design.

1. Introduction

1.1. Relevance of the Topic

Modern management involves performing a predictive function, namely identifying trends in the dynamics of business process indicators and, in particular, predictively assessing their future values. Successfully addressing this challenge opens up the fundamental possibility of implementing so-called proactive business process management in practice [1]. The tools for implementing this function are typically decision support systems, which enable the differentiation of management objects and the optimization of their development scenarios in a dynamic environment [2].

The relevance and value of predictive models and methods are determined, in particular, by the scale of management objects – namely, the dimensionality of the vector of indicators that characterize the state of their own business processes or the state of their environment [3]. A typical example of multidimensional objects are production and logistics systems with a significant number of business processes, the state of which is measured by dozens or even hundreds of indicators [4]. Information about some of these

indicators may be present in the information space in the form of short time series, which require appropriate analysis to identify and account for hidden patterns [5].

Predictive models can be applied to certain classical logistics problems, such as the rational allocation of resources, which also arise in the modeling of information systems [6]. An example of a modern problem involving predictive tools is the analysis of user behavior in computer systems [7]. Risk management, particularly in the formation of modern investment portfolios [8], also involves developing an investment strategy based on the predictive assessment of indicators. Financial markets can also be considered traditional subjects of predictive models today [9].

The rapid growth in data volumes – which are relevant to business processes to varying degrees – has led to the emergence and development of fields such as Big Data [10], Product Analysis [11], Data Analysis [12], the Internet of Things [13], and others. The field of Predictive Analytics [14] has combined methods for analyzing and interpreting data, their statistical processing, and forecasting, developing – based on classical forecasting – a whole spectrum of modern

applied methods for analyzing and forecasting time series. Hybrid adaptive models – such as those based on fuzzy logic and neuro-fuzzy networks (ANFIS) – have become widely used, as they allow for the combination of expert knowledge with automated learning based on real-world data [15].

Modern researchers have gained a powerful set of tools for solving a wide variety of forecasting problems. In addition, there are numerous software implementations of forecasting models [16]. Nevertheless, in practice, when available statistical samples are small, situations often arise where it is difficult to find a viable alternative to simple single-parameter forecasting models. One of the best-known of these is the exponential smoothing model, or Brown's forecasting model (BFM), which has served as the basis for a whole series of structurally more complex models that implement a mechanism for smoothing a time series or its individual components.

1.2. Literature Review and Problem Definition

Despite the apparent simplicity of the BFM, the process of its effective parametric synthesis is the subject of research by many scientists [17]. It is essential to understand that familiarity with the specifics of the BFM tuning process – and forecasting models in general – is a prerequisite for their effective and repeated use. The Predictive Analytics toolkit requires a proper understanding of the specifics of the internal parametric synthesis of predictive models. Without it, there is a risk of losing the effectiveness of such tools. In particular, in production and logistics operations, this leads to the adoption of insufficiently informed decisions and, as a result, to financial losses. Therefore, issues related to the parametric synthesis of forecasting models – particularly single- and two-parameter models – are of particular relevance to managers and analysts, with the aim of ensuring the appropriate quality of forecasts for their effective use in management activities.

Subsequently, the Brown model has undergone numerous variations. For example, the double exponential smoothing model has gained widespread popularity [18]. At the same time, the two fundamental properties of the Brown model are filtering or smoothing and forecasting or predictive [19]. Strictly speaking, both are interpretations of the same property of adaptability, since a fundamental postulate of the BFM is the use of the exponentially weighted or smoothed value of a time series as its forecast estimate. The distinction between these two properties stems primarily from the use of the

Brownian model to solve two classical problems: filtering and forecasting time series [17].

To assess the adequacy of the BFM, as with any other forecasting model, inverse verification is used – that is, testing the adequacy of the forecasting model over a retrospective period.

It should be noted that the concepts of «inverse verification of a forecasting model» [20] and «retrospective analysis of a forecasting model» [19] are absolutely identical; that is, they are two equivalent names for the same procedure. The concepts of «adequacy» and «accuracy» are also identical specifically for predictive models, since the adequacy of a mathematical model is defined as the manifestation of the properties of accuracy and consistency. Consistency for predictive models whose output is a point forecast is satisfied by definition; therefore, in forecasting, the concepts of adequacy and accuracy are easily interchanged, with one used in place of the other without any loss of meaning or mathematical correctness. In this study, we will use the following definition: the adequacy of a predictive model to a real-world process over a given time interval is determined by the accuracy of the retrospective predictive estimates obtained using this model over that time interval.

In addition to the trivial choice of a smoothing constant, there are known search-based [17] and analytical [19] approaches to the parametric identification of the BFM. In both cases, the hypothesis is assumed that the quality of retrospective forecast estimates obtained for time series values at previous time points will be preserved in the future. Clearly, this hypothesis is unavoidable. Similarly, a forecasting model that links a forecast estimate to past values of a time series is based on the hypothesis that such a relationship exists at all. But while in forecasting, the a posteriori determination of forecast accuracy is, in fact, a way to test the forecasting hypothesis, in parametric synthesis such a test does not occur explicitly. However, it is important for the analyst to understand to what extent the hypothesis of parametric synthesis is confirmed, since it determines the maximum level of expected forecasting quality. It is entirely logical for a researcher to want to understand which specific parametric configuration of the forecasting model was optimal at the moment in time preceding the forecasting moment – that is, which specific values of the internal parameters ensured its adequacy at earlier moments in time.

Selecting a single value of makes it possible to perform forecasting by obtaining a point-in-time

predictive estimate for a future time point, but it does not allow the researcher to assess the characteristics of the parametric configuration, given the accumulation of the sample with each forecasting step. The accumulation of the statistical sample – that is, the increase in the parameter – strictly speaking, makes the BFM a two-parameter model. A task that remains unsolved to date is the analytical justification, formulation, and conduct of a statistical experiment for the retrospective analysis of the optimal parametric configuration of the BFM on an extended set. Identifying characteristic regions with specific properties on the internal parameter plane should provide the researcher with additional information useful for the parametric synthesis of the BFM. Solving this pressing scientific and practical problem will improve the soundness of decisions regarding the parametric synthesis of BFM.

1.3. Purpose and Objectives of the Study

The object of the study is the process of parametric synthesis of BFM; the subject is the conditions for its optimal parametric configuration. The aim of the study is to evaluate the relevance of theoretical recommendations regarding the parametric synthesis of BFM through a statistical retrospective analysis of its optimal parametric configuration.

To achieve this aim, the following tasks must be accomplished:

- 1) analyze the characteristics of the feasible set of the BFM smoothing parameter;
- 2) design a statistical experiment for the retrospective analysis of the optimal parametric configuration of the BFM;
- 3) conduct a practical experiment and analyze its results; 4) formulate recommendations for improving the technology of analytical parametric synthesis of the BFM in accordance with the results of the statistical experiment.

2. Materials and Methods

2.1. Analysis of the Characteristics of the Feasible Set of the Internal Parameter of the BFM

A predictive model proposed by R.G. Brown [21] has been known since the middle of the last century:

$$y_t = \sum_{i=1}^n \alpha(1-\alpha)^{i-1} y_{t-i}, \quad (1)$$

where $\hat{y}_t$ is the predictive estimate of the value of the indicator y at time t , y_{t-1} , y_{t-2} , ..., y_{t-n} are the values of

the indicator at the corresponding times; n is the length of the time series sample; α is the smoothing parameter.

The rapid spread and popularity of model (1) among researchers is due, first and foremost, to its algorithmic simplicity under conditions of iterative computations using a constant value for the smoothing parameter α . In this case, when the first element of the time series is chosen as the initial value, formula (1) takes on a recursive form:

$$y_t = \alpha y_{t-1} + (1-\alpha) y_{t-1} \quad (2)$$

or

$$y_t = y_{t-1} - \alpha \Delta_{t-1}, \quad (3)$$

where y_t , y_{t-1} are predictive estimates of the indicator's values for the corresponding time points; $\Delta_{t-1} = y_{t-1} - y_{t-1}$ is the absolute forecast error at time $(t-1)$.

This study considers only the forecasting aspect of the Brown model. The main difference between the forecasting model and the filtering model lies in the fundamental possibility of multiple parametric tuning, that is, the adaptation of the smoothing parameter at each forecasting step. In this sense, the traditional name for the model's internal parameter α – «smoothing constant or parameter» – is more appropriate for the filtering model. In the BFM, the term «adaptation coefficient or parameter» better captures the essence of this parameter; however, to avoid terminological confusion, we will use the traditional name.

The classical set of permissible values for the smoothing parameter is the interval $\alpha \in [0, 1]$, which is determined by the need to ensure that the sequence of weight coefficients in formula (1) converges to unity:

$$\{a_k\}_{k=1}^n = \alpha, \alpha(1-\alpha), \dots, \alpha(1-\alpha)^{n-1}. \quad (4)$$

It is known that the classical set $K_c = \{\alpha : 0 \leq \alpha \leq 1\}$ can be extended to $K_{ext} = \{\alpha : 0 \leq \alpha < 2\}$ without formally violating the conditions for the convergence of the sequence of weight coefficients (4) to unity. In this case, the sequence (4), which is sign-constant on the interval $\alpha \in [0, 1]$, becomes sign-alternating on the interval $\alpha \in (1, 2)$. The set of smoothing parameters $K_{out} = \{\alpha : 1 < \alpha < 2\}$ is called «out-of-bounds».

The widespread practical use of the BFM has led to the emergence of many recommendations regarding the

selection of values for the smoothing parameter α , some of which, according to [20], are shown in Fig. 1.

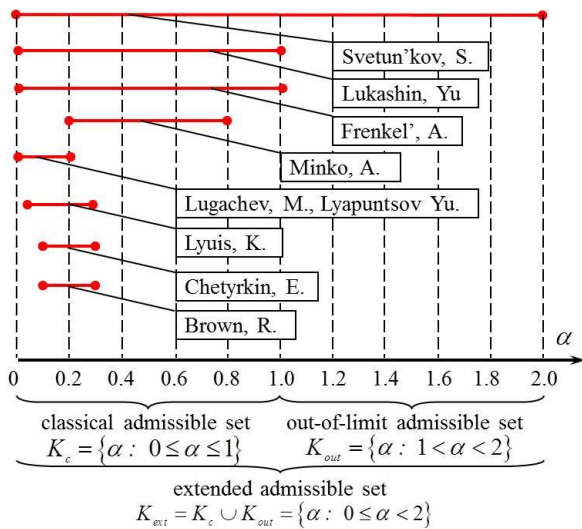


Fig. 1. Some recommendations for selecting values of the smoothing parameter [20]

It is quite clear that the vast majority of these recommendations pertain to the classical set K_c . The out-of-bounds set K_{out} is considered in the literature as a purely theoretical range of valid values for the smoothing parameter α , which has no significant practical application. Nevertheless, any information regarding the characteristics of the admissible parametric configuration of the BFM is important for the informed use of the predictive model as such.

It is important to understand that the extended set of admissible values of the internal parameter K_{ext} is not symmetric and, moreover, exhibits different properties for different sample lengths n . For example, the sum function of the sequence of coefficients (4)

$$S_n(\alpha) = 1 - (1 - \alpha)^n \quad (5)$$

on the «out-of-bounds» set K_{out} for even and odd n has fundamentally different characteristics (Fig. 2). Thus, the hypothesis arises that, when using K_{out} , the optimal parametric configuration will differ for even and odd values of n . When using the extended set K_{ext} in parametric synthesis of BFMs, the researcher must take this fact into account.

We will illustrate the relationship between the internal parameters of the BFM as follows. Let's write the dependence of the values of the elements of the

sequence (4) on the smoothing parameter α and the ordinal number of the element k :

$$a_k(\alpha, k) = \alpha(1 - \alpha)^{k-1}. \quad (6)$$

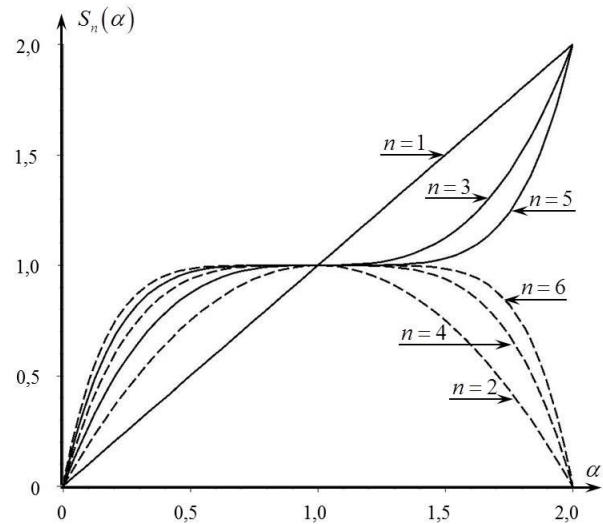


Fig. 2. Graph of the sum function of the sequence of weighting coefficients for the BFM on an extended set K_{ext}

It can be verified that the maxima of the element weights drift as the element index k increases toward the region of small values α , asymptotically approaching zero. Indeed, finding the local maximum of function (6) with respect to the argument α yields

$$\alpha = 1/k, \quad (7)$$

that is, the maxima of the weight coefficients in the BFM are inversely proportional to the ordinal number of the element in the sequence (4).

Relations (5) and (7) provide an analytical basis for the assertion that the parametric synthesis of the BFM is not a trivial choice of the smoothing parameter α . In this process, in addition to searching for characteristic regions on the parameter plane (α, n) , one must also justify the criteria for parametric quality and determine the optimal parametric configuration of the predictive model in accordance with them.

Let us consider how the function of the incomplete sum of the elements of sequence (4), starting from the second one, behaves:

$$S'(\alpha, n) = S_n(\alpha) - \alpha = 1 - \alpha - (1 - \alpha)^n, \quad n \geq 2. \quad (8)$$

The maxima of the incomplete sum of the weight coefficients can be found by differentiating function (8):

$$\frac{\partial S'(\alpha, n)}{\partial \alpha} = -n(1 - \alpha)^{n-1}(-1) - 1 = 0. \quad (9)$$

From this, we obtain

$$(1-\alpha)^{n-1} = n^{-1}. \quad (10)$$

The root of equation (10) is the value

$$\alpha = 1 - \sqrt[n]{n}. \quad (11)$$

Equation (11) analytically relates the number n of observations that are given the greatest arithmetic weight in the formation of the forecast to the smoothing parameter α at which this forecast is generated. The inverse relationship $n(\alpha)$ can be interpreted as the effective memory length of the BFM [21]. It can be argued that relation (11) describes a curve in the parameter space (α, n) , in the vicinity of which the parameter configuration (α', n') ensures the extreme properties of sequence (4). It is close to the empirical relation used by the model's author, R. Brown, himself for selecting the smoothing parameter [21]:

$$\alpha = \frac{2}{n+1}. \quad (12)$$

On the parameter plane (α, n) , a characteristic region Ω (Fig. 3) can be identified with the following characteristics:

- at its upper boundary, the sequence of weight coefficients (4) of the BFM exhibits extreme properties in the sense of a maximum of their incomplete sum;
- it is bounded from below by the Brownian curve;
- within its boundaries, the empirically recommended values of the smoothing parameter are most densely concentrated (Fig. 1);
- the vast majority of short-term forecasting problems, characterized by a sample length of $6 \leq n \leq 30$, fall within its bounds.

Considerations regarding the characteristic region Ω have been formulated for the classical range K_c . The question remains open: does the parameter plane contain a characteristic region similar to Ω on the out-of-range set K_{out} ? Let us attempt to show analytically that such a region exists.

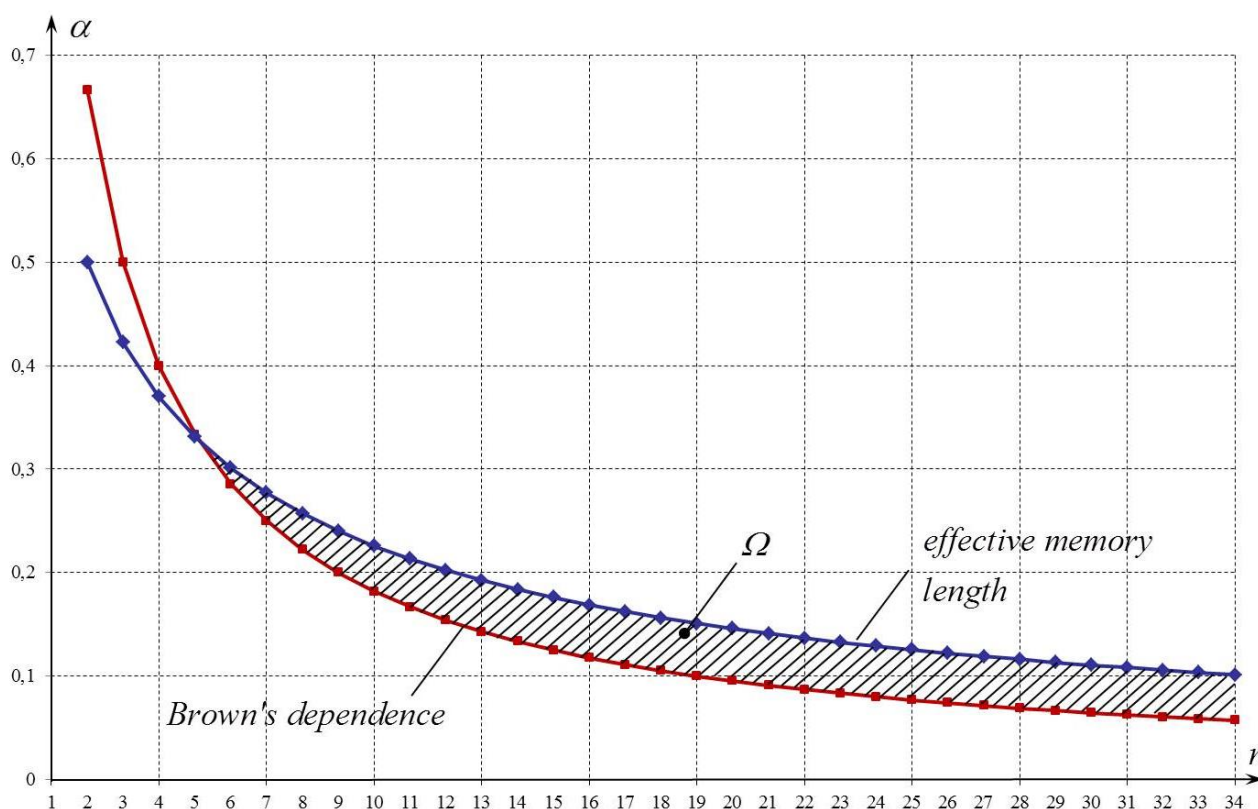


Fig. 3. Characteristic curves on the plane of the BFM internal parameters

Clearly, on the non-bounded set K_{out} , the branches for even and odd n have different behavior, while the branches for odd n have a single extremum.

Indeed, equation (10) has a pair of multiple roots for odd values of n

$$\alpha = 1 \pm \sqrt[n]{n}. \quad (13)$$

Thus, we can conclude that on the parameter plane (α, n) , on the extended set K_{ext} , there exist two symmetric characteristic regions with extreme properties of the incomplete sum of weight coefficients (8).

Let us define the characteristic region Ω' as symmetric to Ω with respect to the line $\alpha = 1$ (Fig. 4):

$$\Omega: \frac{2}{n+1} \leq \alpha \leq 1 - \frac{1}{\sqrt[n]{n}}, \quad (14)$$

$$\Omega': 1 + \frac{1}{\sqrt[n]{n}} \leq \alpha \leq \frac{2n}{n+1} \quad (15)$$

Given the above, we can conclude that when using K_{ext} for odd n , the curve of the effective memory length of the BFM has two symmetric branches. That is, for each odd sample length n , there are two values of the smoothing constant α that endow sequence (4) with extreme properties.

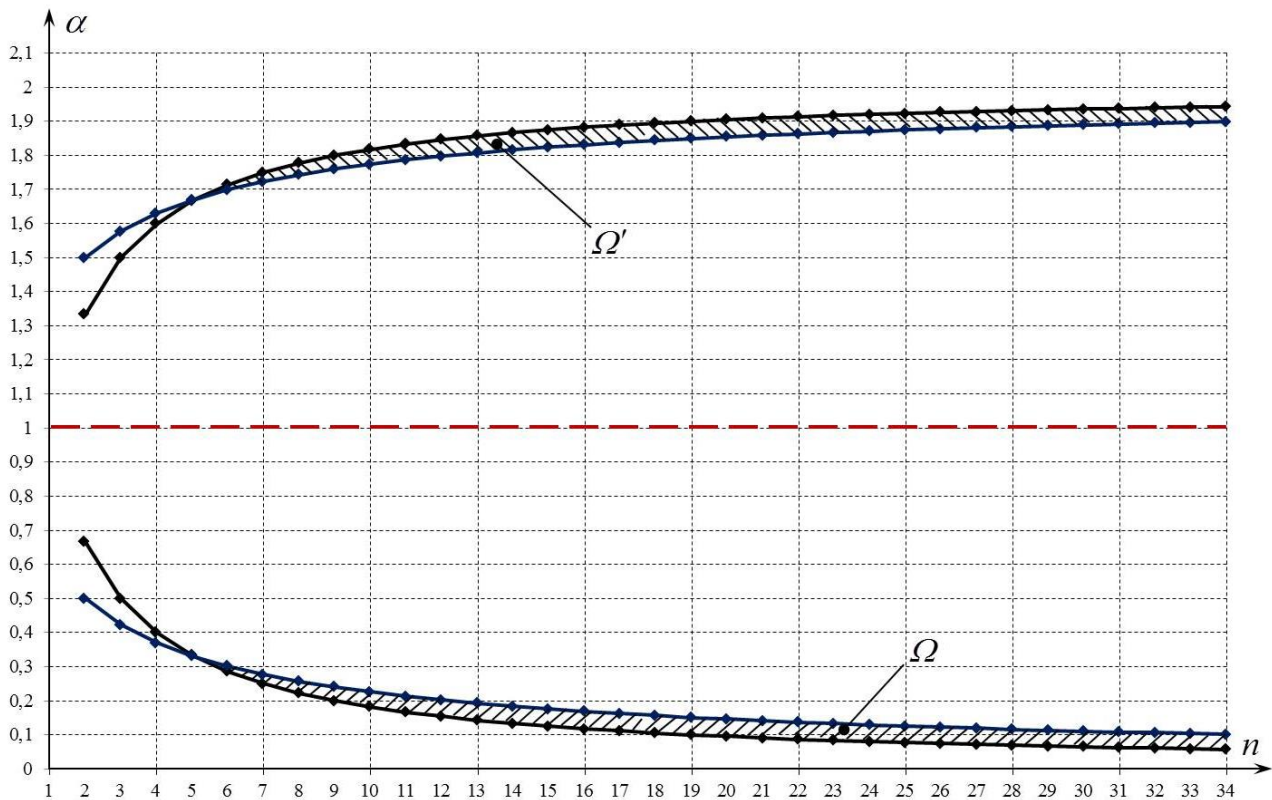


Fig. 4. Characteristic regions with extreme properties of the sequence of BFM weight coefficients

Thus, the presented theoretical calculations allow us to draw the following conclusions:

- on the plane of the internal parameters of the BFM (α, n) , there exist two regions Ω and Ω' that are symmetric with respect to the line $\alpha = 1$; in the vicinity of these regions, the sequence of model coefficients (4) exhibits extreme properties, while the region Ω' exists only for odd values of the sample length n ;
- the concept of the effective memory length of the model can be analytically extended to the out-of-bounds range K_{out} for odd n .

2.2. Planning a Practical Experiment for Retrospective Analysis of the Optimal Parametric Configuration of the BFM

The idea behind the analytical approach to the parametric synthesis of the BFM lies in determining the smoothing parameter α^* among the real roots of the retrospective equations using a solution rule or criterion [19] (Fig. 5).

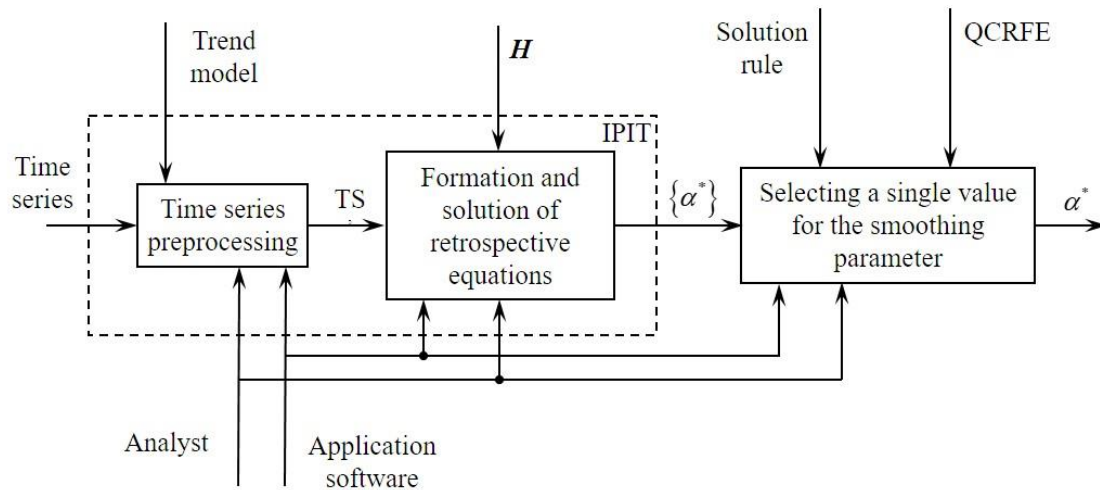


Fig. 5. Flowchart of the analytical approach to the parametric synthesis of BFM

(Notations and abbreviations: IPIT – inverse parametric identification technology; TS’ – time series of residuals

(detrended time series); H – parametric synthesis hypothesis; QCRFE – quality criteria for retrospective forecast estimates;

$\{\alpha^*\}$ – the set of values for the smoothing parameter α^* that ensure optimal quality)

From the real roots of the retrospective equation of the form (17), the set $\{\alpha^*\}$ is formed, provided that they fall either within the classical range $\{\alpha^*\} \subset K_c$ or within the extended range $\{\alpha^*\} \subset K_{ext}$, depending on the selected forecasting technique. The values $\alpha^* \in \{\alpha^*\}$ ensure absolute retrospective accuracy, i.e., maximum adequacy of the BFM. From this set, a single value α^* is selected for forecasting according to defined quality criteria using a decision rule.

To conduct the statistical experiment, we will use the open-access Dataset [22], which researchers use to test forecasting models and train neural networks [23–27]. It contains time series of industrial equipment performance metrics and covers the period from July 2016 to July 2018. All datasets have been preprocessed and saved as files. We will select a time series of a real-valued metric that is long enough to allow for multiple shifted sub-series and conduct a statistical experiment using the method described below.

The time series from the dataset is used as-is, with accuracy to the second decimal place, without detrending or any other preliminary statistical processing. After determining the timestamp that conventionally denotes the time t , we will use a 40-element segment of the time series with the last value y_t . For this segment, we will form retrospective equations of the form (17)

for $n = 7, 8, 9, \dots, 40$ and find their real roots. We will plot these roots on the plane of the internal parameters of the BFM (α, n) , thereby creating a so-called root portrait for the first 40-element segment. By superimposing the images of the characteristic regions Ω and Ω' onto it, we can determine to what extent the hypothesis regarding the extreme properties of the characteristic regions is confirmed. To do this, we will record the appearance of stable root clusters in the vicinity of the characteristic regions Ω and Ω' . We will consider a cluster to be five or more consecutive roots that form a sequence with a stable trend on the parameter plane.

It is recommended to repeat the procedure described above (Fig. 5), shifting one step along the time series, until statistical significance is achieved. Thus, a method for a statistical experiment involving the retrospective analysis of the optimal parametric configuration of the BFM is proposed, which can be applied for the statistical and graphical analysis of the characteristics of the plane of its internal parameters.

2.3. Description of the Experiment and Analysis of Its Results

We will use a time series segment consisting of 140 data points. We will determine the last value in this segment as $y_t = 30.67$ and formulate retrospective equations of the form (17) for $n = 7, 8, \dots, 40$. We will search for real roots using the Maple software package.

We will divide all 34 real roots of the retrospective equations into groups according to Table 1 and construct

a root portrait for the first segment, plotting all root groups separately (Fig. 6).

Table 1. Groups of real roots of the retrospective equations according to their arrangement

Root number from left to right	Sample length n		Symbol
	odd n	even n	
1	first root	first root	●
2	second root	second root	◆
3	third root	third root	■
4	fourth root	fourth root	▲

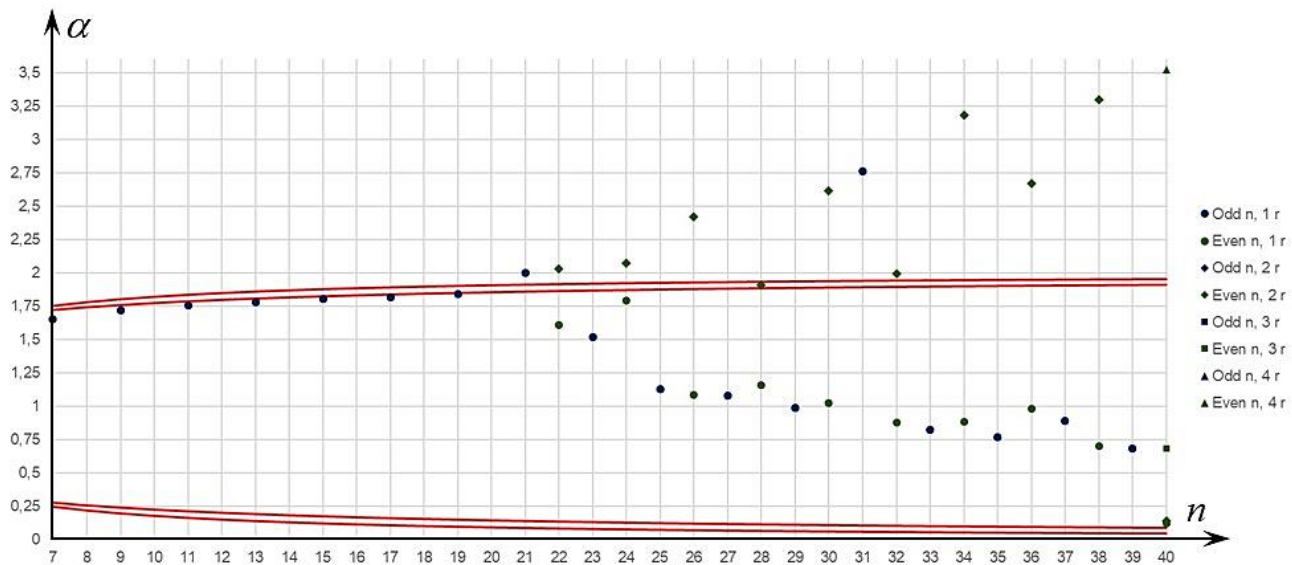


Fig. 6. Root portrait for the first fragment

In Fig. 6, in the range $7 \leq n \leq 19$, a stable root cluster can be seen, formed by the first odd roots, which exhibits a high level of similarity and proximity to the characteristic region Ω' . This may serve as confirmation of the theoretical conclusions presented in Section 2.1.

Let us shift the time series $y_t := y_{t-1}$ and construct a root portrait for the second fragment (Fig. 7).

Analyzing the root portrait in Fig. 7 reveals several additional features. The first odd and first even roots with a high degree of approximation fall within the characteristic region Ω across the entire range $7 \leq n \leq 40$, which indicates the possibility of performing end-to-end parametric synthesis in this case – that is, without separating samples of odd length from those of even length. The second set of even roots forms a stable cluster in the range $8 \leq n \leq 20$, similar to the previous one (Fig. 6), but of lower quality. In this case, we do not record the appearance of a root cluster in the vicinity of the characteristic region Ω'

in the statistics, since the cluster consists of even roots.

By repeating the procedure described above, we will construct 100 root portraits and count the number of those in which corresponding root clusters form in the vicinity of the characteristic regions Ω and Ω' , as well as the number of those that do not contain stable clusters. In doing so, we will count the appearance of clusters in the vicinity of the characteristic region Ω' only for odd n . The results are shown in Fig. 8.

As can be seen in Fig. 8, the frequency of formation of stable root clusters in the vicinity of the characteristic region Ω is 0.55, which is 20% lower than the corresponding frequency in the vicinity of the characteristic region Ω' . The total frequency of formation of stable root clusters in other regions of the BFM internal parameter plane is 0.36. This indicates significant potential for using the extended range K_{ext} for practical forecasting tasks.

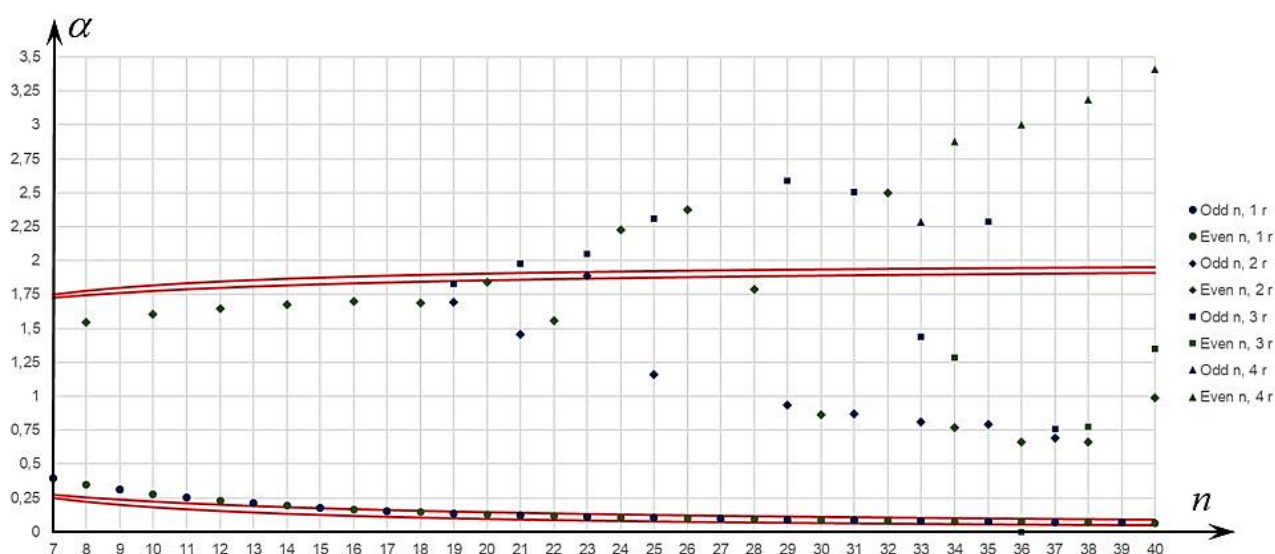


Fig. 7. Root portrait for the second fragment

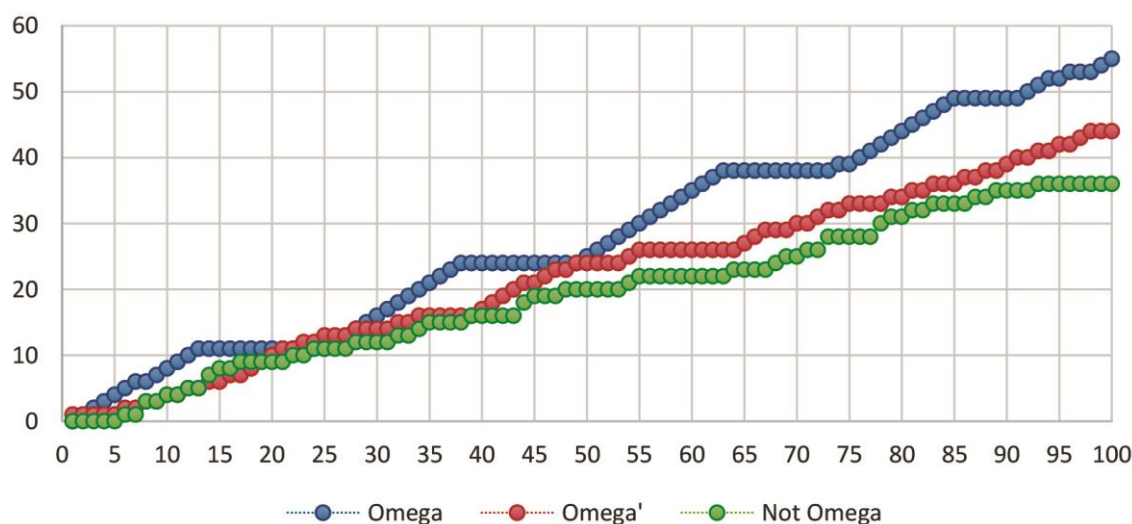


Fig. 8. Frequency of root cluster formation in the vicinity of characteristic regions Ω and Ω'

2.4. Recommendations for Improving the Analytical Parametric Synthesis Technology of BFM

Based on the analytical results presented in Section 2.1 and the statistical results obtained in Section 2.3, recommendations can be formulated for improving the technology of analytical parametric synthesis of BFMs.

First, the retrospective values α^* should not be considered in isolation but in relation to the root of the clusters. The fact that α^* belongs to a stable cluster in the vicinity of the characteristic regions Ω and Ω' makes its selection analytically justified and the model itself parametrically sound in terms of the stability of its own optimal parametric configuration.

Second, the identification of the characteristic region Ω' and the practical confirmation of its

configuration necessitate a separate consideration of even and odd sample lengths n when forming a point forecast. This lays the groundwork for the effective use of the out-of-bounds set of the smoothing parameter K_{out} , for which there are no classical recommendations regarding parametric synthesis.

3. Research Results

Overall, the statistical experiment conducted provides a basis for the following conclusions:

1) The emergence of stable root clusters in the vicinity of the characteristic regions Ω and Ω' provides statistical experimental confirmation of the main hypothesis regarding the existence, on the plane of the

BFM's internal parameters, of characteristic regions in the vicinity of which the BFM's parametric configuration is optimal and stable;

2) The frequency of occurrence of stable root clusters in the vicinity of the characteristic regions Ω and Ω' varies across all experimental runs: for Ω it is 0.55, for Ω' it is 0.44, and for the other regions, the total is 0.36.

3) The formation of all stable root clusters begins in the region of small values of n (in the experiment, $n > 7$);

4) The configuration of stable root clusters can be approximated by a simple function, which will subsequently allow us to obtain a more accurate cluster model in the region of small n ;

5) for even n , a stable root cluster sometimes arises in the vicinity of the characteristic region Ω' , while in the vicinity of the characteristic region Ω there is always a cluster with relatively better characteristics;

6) Numbering the roots from left to right is not entirely correct due to the presence of roots «between» the numbered ones; however, this fact does not alter the general idea of analyzing stable root clusters in the vicinity of the characteristic regions Ω and Ω' .

4. Results Discussion

The main results of the statistical experiment on the retrospective analysis of the optimal parametric configuration of the BFM are presented in Section 2.3; however, it is important to highlight the practical significance of the results, the limitations and shortcomings of the study, and to identify directions for future research.

The practical significance of the statistical experiment's results lies in the practical confirmation of analytical conclusions regarding the characteristics of the BFM's internal parameter space.

The limitations of this study are as follows:

- the time series segments used in the study were not tested for stationarity and were not decomposed into components;
- only one time series from the dataset was used, with 100 realizations;
- the influence of the first linear term was not accounted for in the parametric quality criterion, specifically in the function of the incomplete sum of the sequence of BFM coefficients;
- in addition to the four groups of roots listed in Table 1, fifth, sixth, and subsequent roots may appear,

which were not considered within the scope of the conducted experiment;

- the generalizability of the results depends on the number of realizations and the type of time series dynamics.

The following areas appear to be promising directions for further research:

- approximation of the retrospective optimal parametric configuration of the BFM in the region of small values;
- development of an adaptive detrending technique and comparison of the research results with the results of the analysis of residual time series.

5. Conclusions

The characteristics of the feasible set of the BFM smoothing parameter have been analyzed, and the existence of symmetric characteristic regions with extreme properties on the internal parameter plane has been analytically proven. The concept of the model's effective memory length has been extended to the infeasible set of the smoothing parameter.

A methodology for a practical experiment involving the retrospective analysis of the optimal parametric configuration of the BFM has been proposed, which can be applied for statistical and graphical analysis of the characteristics of the plane of its internal parameters.

A statistical experiment was conducted on real data to perform a retrospective analysis of the optimal parametric configuration of the BFM, and the validity of the analytical results was statistically confirmed. It was found that the frequency of formation of stable root clusters in the vicinity of the characteristic region is 0.55. For the characteristic region itself, this indicator was 20% lower. The frequency of formation of stable root clusters in other regions of the BFM internal parameter space totals 0.36.

Recommendations have been formulated for improving the BFM analytical parametric synthesis technique, namely the need to account for the configuration of root clusters and to consider even and odd sample lengths separately when forming a point forecast.

Thus, the Brown prediction model has been further developed by solving the scientific and practical problem of identifying characteristic regions with specific properties within the non-classical range

of its internal parameters, which provides a basis for the synthesis of modified prediction models.

Conflict of interest

The authors declare that they have no conflicts of interest regarding this study, including those of a financial, personal, authorship, or other nature that could influence the study and its results presented in this article.

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Data availability

Data are available from the corresponding author upon reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies in the creation of this work.

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РЕТРОСПЕКТИВНИЙ АНАЛІЗ ОПТИМАЛЬНОЇ ПАРАМЕТРИЧНОЇ КОНФІГУРАЦІЇ ПРОГНОЗНОЇ МОДЕЛІ БРАУНА В НЕКЛАСИЧНОМУ ДІАПАЗОНІ

Дослідження присвячено виконанню завдань теоретичного обґрунтування, формалізації, планування та проведення статистичного експерименту щодо ретроспективного аналізу оптимальної параметричної конфігурації прогнозової моделі Брауна. **Актуальність дослідження** зумовлена нагальною проблемою прогнозування коротких часових рядів, зокрема таких, що визначають динаміку бізнес-процесів у виробничо-логістичних системах. **Об'єктом дослідження** є процес параметричного синтезу прогнозової моделі Брауна, **предметом** – умови її оптимальної параметричної конфігурації. **Мета дослідження** – оцінити релевантність теоретичних рекомендацій щодо параметричного синтезу прогнозової моделі Брауна способом статистичного ретроспективного аналізу її оптимальної параметричної конфігурації. **Завдання:** аналіз особливості допустимої множини параметра згладжування прогнозової моделі Брауна; планування статистичного експерименту щодо ретроспективного аналізу її оптимальної параметричної конфігурації; проведення статистичного експерименту й аналіз його результатів; формулювання рекомендації щодо вдосконалення технології аналітичного параметричного синтезу прогнозової моделі Брауна відповідно до результатів статистичного експерименту. У дослідженні використано **методи** математичного й статистичного аналізу для обґрунтування існування на площині внутрішніх параметрів двох симетричних характерних ділянок з екстремальними властивостями. **Результати.** Запропоновано технологію статистичного експерименту з ретроспективного аналізу оптимальної параметричної конфігурації прогнозової моделі Брауна, яка може бути застосована для статистичного та графічного аналізу особливостей площини її внутрішніх параметрів. На реальних показниках статистично підтверджено достовірність аналітичних результатів дослідження. Сформульовано умови, за яких поняття ефективної довжини пам'яті прогнозової моделі може бути поширено на позамежну множину параметра згладжування. Основним **висновком** дослідження стала обґрунтована необхідність врахування конфігурації кореневих кластерів та окремого розгляду парних і непарних довжин вибірки в процесі формування точкового прогнозу.

Ключові слова: прогнозна модель Брауна; статистичний аналіз; інверсна параметрична ідентифікація; параметричний синтез; планування статистичного експерименту.

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