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CHOI-WILLIAMS ANALYSIS OF THE NON-LINEAR ULTRAWIDEBAND SIGNALS

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Abstract

Choi-Williams analysis application for the non-linear ultrawideband signal describing was proposed. The advantages and the peculiarities of Choi-Wiliiams transform were discussed. The non-linear ultrawideband signal models in time domain were given. The Choi-Williams analysis for the analysis of the non-linear ultrawideband signals was performed.

Keywords: Non-linear ultrawideband signals, digital processing, Choi-Williams transform, Cohen's class non-linear transforms.

1. INTRODUCTION

The fast development and implantation of ultrawideband (UWB) signals and technologies in different branches of science and engineering are carried to appearance of new types of the UWB signals [1]. In particular, the non-linear ultrawideband (NLUWB) signals are one of them. At the same time new kind of signals demands the new processing method application.

The Cohen's class transforms [2-4], well known as non-linear integral transforms, allow obtaining more information about investigated signals in comparison with traditional Fourier transforms. Furthermore, the Choi-Williams transform (ChWT) allow finding compromise between time-frequency resolution as the main advantage of the Cohen's class transforms and the value of cross terms as the greatest disadvantage that class transforms. Thus, the application of ChWT for analysis of the NLUWB signals is appears to be interest, useful and actual.

2. CHOI-WILLIAMS TRANSFORM

The ChWT called by authors as Exponential Distribution has been proposed by H. L. Choi and W. J. Williams in 1989 [2, 3, 5]. This non-linear transform with Gaussian kernel function refers to the Cohen's class transforms. The basic idea placed by the ChWT authors is the Wigner transform (WiT) averaging with presents of scaling factor allowing to control by the cross-term level.

As well known, the ChWT spectral density function (SDF) $P_{CW}f(\tau,\omega)$ of the signal $s(t)$ is given by

$$P_{CW}f(\tau,\omega) =$$
$$= \int_{-\infty}^{\infty} \exp(-i\omega t)\left[\int_{-\infty}^{\infty} \sqrt{\frac{\sigma}{4\pi t^2}} \exp\left(-\frac{(\mu-\tau)^2}{4t^2\,/\,\sigma}\right)\times\right.$$
$$\left.\times s\left(\mu+\frac{t}{2}\right)s^*\left(\mu-\frac{t}{2}\right)d\mu\right]dt,$$

where symbol "*" denotes the complex conjugation operation, σ $(\sigma > 0)$ is the scaling factor named above,. It was shown [1, 2] that when $\sigma \to \infty$, the ChWT changes into the Wigner transform (WiT) given by the relation:

$$P_V f(\tau,\omega) = \int_{-\infty}^{\infty} s\left(\tau+\frac{t}{2}\right)s^*\left(\tau-\frac{t}{2}\right)\exp(-i\omega t)dt,$$

where $P_V f(\tau,\omega)$ is the SDF WiT.

All transforms from Cohen's class, one hand, have good time-frequency properties comparing with linear transforms as their basic advantage, but other hand, they have interfering cross-terms for multicomponent signals as their basic disadvantage. These cross-terms cause redundancy in the information, and they may obscure the true energy distribution over time and frequency. The exponential kernel function application allows to reduce the cross-terms with presents of relatively sharp resolution of auto-terms describing the signal in time and frequency domains. The authors of [5] assert that in this case the ChWT is more effective than smoothed WiT. Furthermore, with the control parameter σ, the researcher can adjust the resolution of the auto-terms and the effects of cross-terms according to the characteristics of the signal to be analyzed.

3. NON-LINEAR ULTRAWIDEBAND SIGNALS

The UWB signal is a signal whose fractional bandwidth μ satisfies the condition $\mu_{\min} \le \mu < 2$. By the definition [6], the fractional bandwidth μ is given by:

$$\mu = \frac{\Delta f}{f_0} = 2\frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}},$$

where f_0, $f_{\min}$, $f_{\max}$ are correspondently the middle, the minimal and the maximal frequencies of SDF of the one-dimensional Fourier transform $\dot{S}(f)$ of signal $s(t)$, Δf given by $\Delta f = f_{\max} - f_{\min}$ is the absolute bandwidth of signal.

The UWB signal is called as the NLUWB signal, when it is a finite solution of the non-linear differential equation [7]. The NLUWB signals unite the behavior of the non-linear waves and the UWB signals. The advantages of these signals are the invariability of wave profile under propagation to a large distance in non-linear medium, on the one hand, and the big carried information volume, on the other hand.

As the NLUWB signal models the real part of the envelope soliton, some periods of the sawtooth wave and the second derivative of the blast wave can be considered.

Being the solution of the non-linear Schrödinger equation given by [8]

$$i\frac{\partial v}{\partial t} + \frac{\partial^2 v}{\partial x^2} + \beta v|v|^2 = 0,$$

where $\beta > 0$, $i = \sqrt{-1}$, the envelope soliton is given by

$$v(x,t) = \frac{v_m}{ch\left(\frac{b}{2}(x - u_1 t)\right)}\exp\left(i\frac{u_1}{2}(x - u_2 t)\right),$$

where $v_m = b/\sqrt{2\beta}$, $b = ((u_1 - 2u_2)u_1)^{1/2}$. The NLUWB signal model having the fractional band width $\mu \approx 2\pi b/(u_2(1+\ln 2))$ is given by relation $s(t) = \mathrm{Re}\, v(x,t)$.

At each period the sawtooth wave is given by [8]:

$$s(t) = v(x,t) = \frac{1}{1+t}\left(x - \pi\,\mathrm{th}\frac{\pi x}{2\gamma(1+t)}\right),$$
$$-\pi < x < \pi,$$

where t is the dimensionless time, x is the dimensionless coordinate. This wave is a non-stationary solution of Burgers' equation [8]:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \gamma\frac{\partial^2 v}{\partial x^2},$$

where γ is the parameter of dissipation (viscosity factor). The fractional bandwidth of this NLUWB signal model is described by $\mu \approx 2/p$, where p is the sawtooth wave period number.

Next NLUWB signal model, well known as the second derivative of the blast wave $v(\xi)$ being the stationary solution of the Burgers' equation and given by

$$v(\xi) = \frac{v_1 + v_2\exp(\xi/\xi_0)}{1 + \exp(\xi/\xi_0)},$$

where $\xi = x - ut$, $v(-\infty) = v_1$, $v(+\infty) = v_2$, $v'(\pm\infty) = 0$, $\xi_0 = 2\gamma/(v_1 - v_2)$, $u = (v_1 + v_2)/2$, ξ_0 is the blast wave width, is defined by

$$s(t) = \frac{d^2 v}{d\xi^2} = \frac{v_2 - v_1}{\xi_0^2}\frac{\exp(\xi/\xi_0)(1 - \exp(\xi/\xi_0))}{(1 + \exp(\xi/\xi_0))^3}.$$

4. SIMULATION RESULTS

At the fig. 1 the results of Choi-Williams analysis of sawtooth wave are shown.

The results of ChWT application to the NLUWB signal description was comparing with ones of WiT and Fourier Spectrogram (FS) were compared..

The Fourier spectrogram is given by

$$P_{FS}f(\tau,\omega) = \left|\dot{S}f(\tau,\omega)\right|^2 =$$
$$= \left|\int_{-\infty}^{+\infty} s(t)\,g(t-\tau)\exp(-i\omega t)\right|^2,$$

where $P_{SF}f(\tau,\omega)$ is the SDF FS.

All results with usage of the computer mathematics system MATLAB, the extending package Time Frequency Toolbox and the original software designed by authors were produced.

The sawtooth wave in time domain at the fig. 1, a. is shown. At the fig. 1, b the WiT SDF is placed.. The ChWT SDF for $\sigma = 1000$, $\sigma = 10$ and $\sigma = 0{,}01$ at the fig. 1, c-e respectively are shown. The FS SDF is given by the fig. 1, d. The energograms for each SDF in the right column near the congruent SDF are located. By the definition, the energogram $Ef(\omega)$ for each non-linear transform SDF $Pf(\tau,\omega)$ is given by the relation:

$$Ef(\omega) = \int_{-\infty}^{\infty} Pf(\tau,\omega)d\tau.$$

The energogram represents the distribution of energy of the signal or the process $s(t)$ on different frequencies ω. Moreover, the integral from function $Ef(\omega)$ in frequency domain in bounds $\omega \in (-\infty : +\infty)$ is appeared to be equal to the signal energy.

As it was shown at the fig. 1, b, the WiT SDF is significantly corrupted by the interference cross-term presence. These cross-terms are greatly suppressed by the ChWT (fig. 1, c-e). But the horizontal line, which corresponds to the frequency of sawtooth pulses (fig. 1, b), is not present on time-frequency plane of ChWT

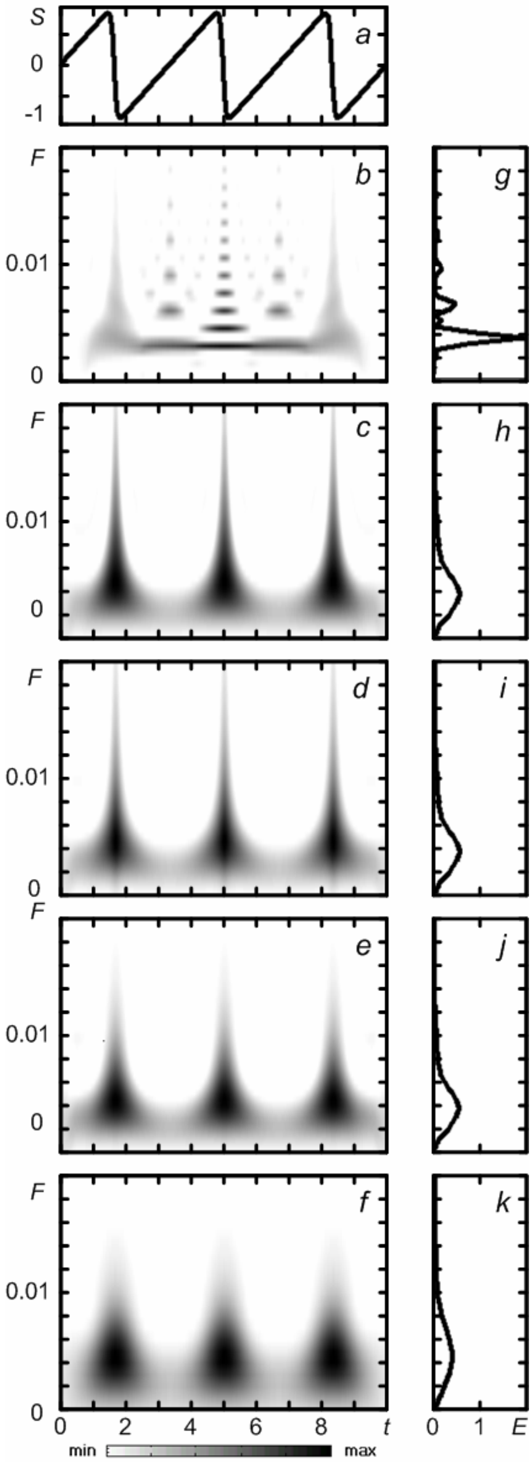

Fig. 1. Analysis of the sawtooth wave: a – the sawtooth wave in time domain, b – the WiT SDF, c – the ChWT SDF ($\sigma = 1000$), d – the ChWT SDF ($\sigma = 10$), e – the ChWT SDF ($\sigma = 0,01$), f – the FS SDF, g – the WiT SDF energogram, h – the ChWT SDF ($\sigma = 1000$) energogram, d – the ChWT SDF ($\sigma = 10$) energogram, e – the ChWT SDF ($\sigma = 0,01$) energogram, f – the FS SDF energogram.

SDF even for big σ values. With decreasing σ the ChWT SDF is spreading, but even for $\sigma = 0,01$ it occurs to be more localized then FS SDF.

The results obtained for the NLUWB signal analysis with the ChWT usage with ones given by the WiT [9] and the wavelet analysis [10] were compared.

Therefore, the ChWT usage without other transforms for the NLUWB signal analysis is appeared to be not recommended. It is necessary to apply simultaneously the ChWT, the WiT and the FS. Such usage allows to obtain more accurate information about the time-frequency distribution of the signal energy.

4. Conclusions

- The ChWT was proposed to apply for analysis of the NLUWB signals and processes.
- The simultaneous usage of the ChWT, the WiT and the FS was shown to be perspective and useful.

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